Gauss's Law

We see with Coulomb's Law
- relation between electric force (& field) & charge

Calculations are difficult in many charge distributions

Gauss's Law a more general law
- simplifies some specific calculations
- considered now to be more fundamental than Coulomb's
- provides more insight into underlying physics

For any closed surface:

$$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A} = q_{in}/\epsilon_0}$$

q_{in} → net charge enclosed by A
- if no charge ⇒ no flux & no field

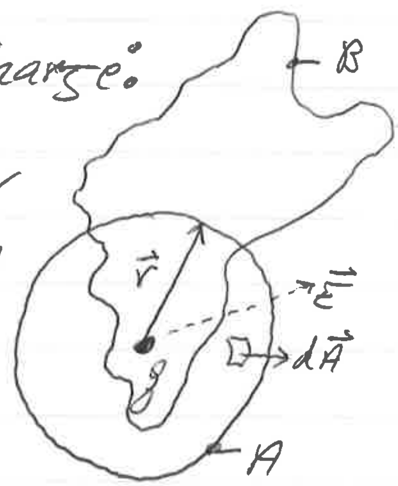
external charge ⇒ no contribution to flux

Gauss's Law & Coulomb's Law

So the net flux thru any closed surface surrounding a point charge, q, = q/ϵ_0
- independent of shape of surface

Used to evaluate $\vec{E}$ most effectively when charge distribution exhibits some symmetry.

Consider point charge:



- spherical symmetry
  - $\rightarrow |E|$ constant for a given $|r|$
  - $\rightarrow d\vec{A} \parallel d\vec{E}$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$
$$= \oint E dA = q_{in}/\epsilon_0$$

Assume we don't know E, & solving

$$E \oint dA = q_{in}/\epsilon_0 = E(4\pi r^2)$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = k \frac{q}{r^2}}$$

Coulomb's Law!!!

for E field

Choice of Surface

We could have ignored symmetry
and chosen surface B

→ same # of crossing field lines
∴ same flux as with sphere
- since same q_{in}

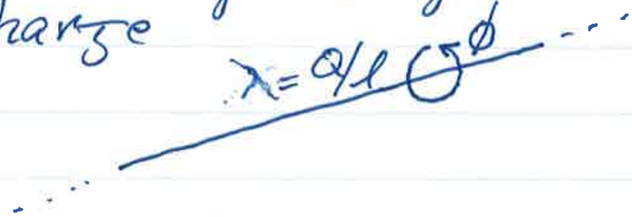
But integral $\oint_B \vec{E} \cdot d\vec{A}$ very
difficult!!

Often we are attempting to
understand E-field
- surface B does not help us here

Example

6.3b

1) For a uniform, infinite line of charge

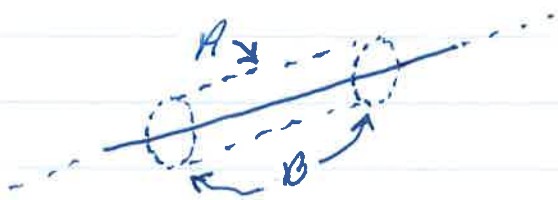


a) what is symmetry?

- same E in all ϕ directions

$\therefore$ cylindrical

b) How draw Gaussian surface?



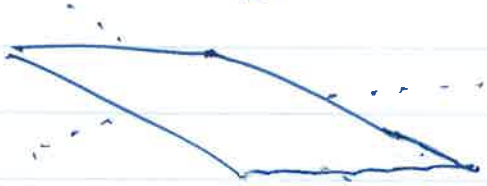
Surface A: $\vec{E} \cdot d\vec{A} = E\,dA \neq 0$

Surfaces B: $\vec{E} \cdot d\vec{A} = 0$

since $\vec{E} \perp d\vec{A}$

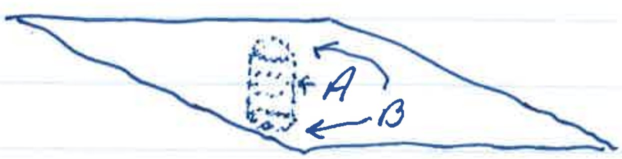
Example (cont.)

2) For an infinite sheet with uniform charge density



a) what is symmetry?
-same E for all (x,y): planar

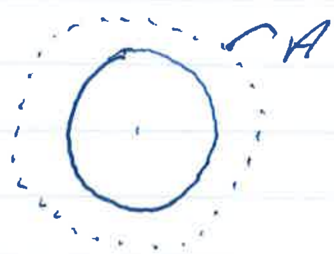
b) what is the Gaussian surface?



B : $\vec{E}\cdot d\vec{A} = E\,dA$
A : $\vec{E}\cdot d\vec{A} = 0$

3) For sphere of uniform charge density

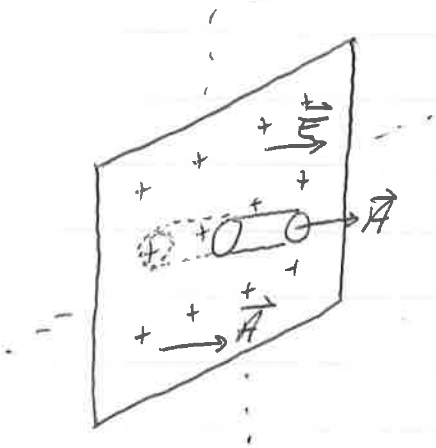
a) symmetry?
E same for all (θ,ϕ):
spherical



b) Gaussian surface? sphere A
$\vec{E}\cdot d\vec{A} = E\,dA$

Infinite Sheet of Charge 6.4

Sometimes symmetry needed is not quite so obvious



- uniform charge density
  $\sigma = Q/A$
- $\vec{E} \perp$ to surface
  - $\parallel$ components cancel by symmetry

'Gaussian surface': construct cyinder $\perp$ to sheet plane
- no flux thru cyinder sides only thru ends

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$$

$$E_{front} \cancel{A} + E_{back} \cancel{A} = \sigma \cancel{A}/\epsilon_0$$

$$\boxed{E = \sigma/2\epsilon_0}$$

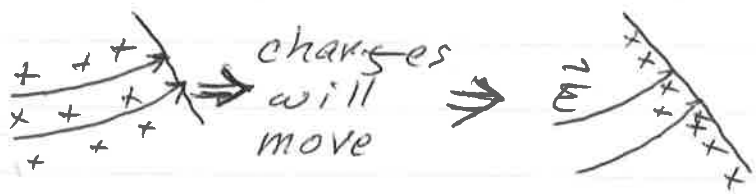
∴ - E field same for all points
- no radial dependence

Charged Conductors in Electrostatic Equilibrium

Electrostatic equilibrium = no <u>net</u> motion of charges

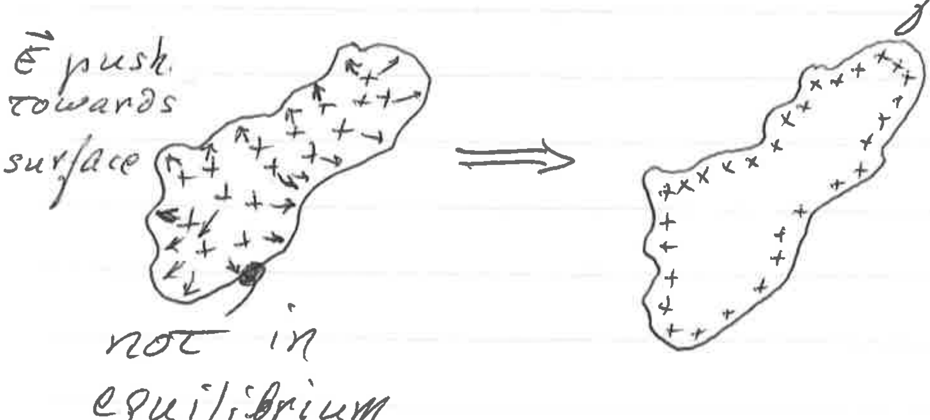
We can identify several properties of these conductors

A) $\vec{E} = 0$ at all point inside conductor



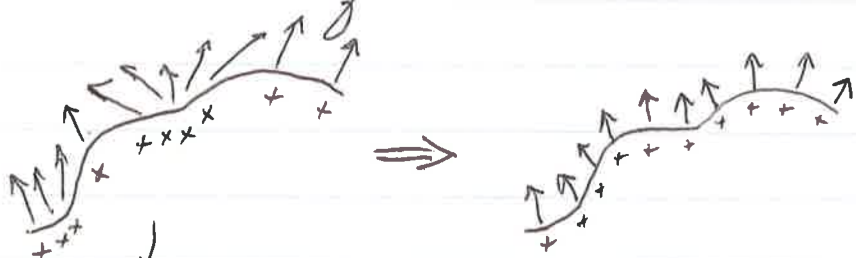
→ charges move until $\vec{E} = 0$

B) for isolated conductors: all <u>net</u> charge distributed on surface



Charged Conductor (cont.)

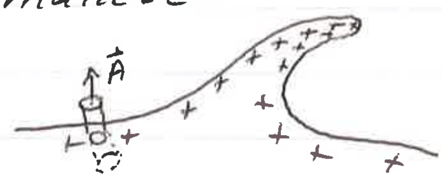
c) $\vec{E}$ is $\perp$ to surface for points close to surface



- non-$\perp$ components of $\vec{E}$ will cause "surface currents" until they vanish

d) irregular shapes: charge density σ maximized where radius of curvature is smallest

What is E near surface?



Like with ∞ sheet of charge

- But, endcap inside, $\phi = 0$

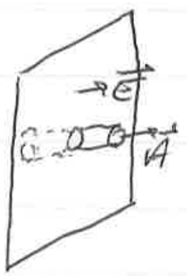
$$\therefore \phi_E = \oint \vec{E}\cdot d\vec{A} = \overset{0}{\cancel{E_{end} A}} + E_{front} A = \frac{\sigma \cancel{A}}{\epsilon_0}$$

$$E \cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0}$$

$$\boxed{E = \sigma/\epsilon_0}$$

<u>Example</u>

A nonconducting wall has $8.6\,\mu C$ distributed over $1\,m^2$. If we get $7\,cm$ from the wall's center, what is the electric field?



Using ∞ charge sheet as an approximation,

$$E = \sigma/2\epsilon_0 = (q/A)/2\epsilon_0$$

$$= 8.6\,\mu C/m^2 / 2\epsilon_0$$

$$= \frac{8.6 \times \cancel{10^{-6}}\ \cancel{C}/\cancel{m^2}}{2(8.85 \times 10^{\cancel{-12}\,-6}\ C^{\cancel{2}}/N\cancel{m^2})}$$

$$\sim \frac{1}{2} \times 10^{6}\ N/C = \boxed{5 \times 10^{5}\ N/C}$$

Will this be different at $5\,cm$?

<u>No, ($d \ll 1m$)</u>

What about @ $10m$?

<u>Yes. $d \gg 1m$</u> & sheet will start to approximate a point charge.