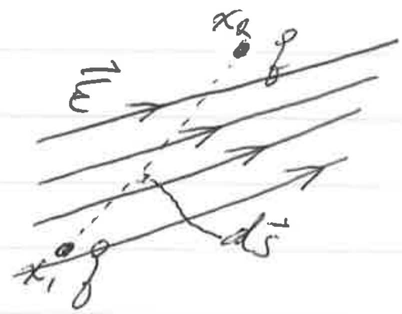
Electric Potential Energy 7.1

It requires work to move charge q in $\vec{E}$ field

Work to move $d\vec{s}$



$dW = \vec{F} \cdot d\vec{s}$

which equals the change in potential energy $dW = -dU$

the difference in potential energy at x_1 and x_2 is

$$\Delta U = -q\int_{x_1}^{x_2} \vec{E} \cdot d\vec{s} = -\int_{x_1}^{x_2} \vec{F} \cdot d\vec{s}$$

- independent of path taken
- only x_1, x_2 matters

Electric Potential

As with electric field + force, we want expression independent of charges, q_0, placed in field

Define 'electric potential'

$$\Delta V = V_{x_1} - V_{x_2} \equiv \Delta U / q_0$$

this gives us an expression based on electric field

$$\underline{\underline{\Delta V}} = V_{x_1} - V_{x_2} = \boxed{-\int_{x_1}^{x_2} \vec{E} \cdot d\vec{s}}$$

Units: work/charge => joule/coulomb

$$1 \text{ Volt} = 1 V = 1 \frac{Nm}{C}$$

$\vec{E}$-field units therefore volts/m

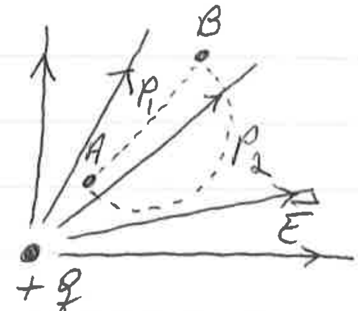
Implications of Conservation of Energy 7.3

$\vec{F}_E$ is a 'conservative' force:

"a force is conservative if the kinetic energy of a particle on which it acts returns to its initial value after any round trip."

Examples:
- gravity
- ideal spring
- E-field

reminder: friction is nonconserving



$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

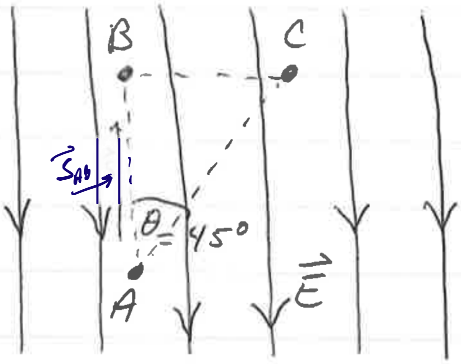
only radial variation in path matters

Paths A + B have same radial extent so $\int$ along P_1 gives same result as P_2

Potential Difference in a Uniform $\vec{E}$ field

7.4

Consider area where $\vec{E}$ constant for all points:



A & B are separated by $|\vec{S}_{AB}| = d$

$$\vec{S}_{AB} \parallel \vec{E}$$

When going from $A \to B$, go to higher potential

$$\underline{\Delta V_{BA}} = -\int_A^B \vec{E} \cdot d\vec{s} = -\int_A^B E \cos 180° \, ds$$

$$= +\int_A^B E\,ds = E\int_A^B ds$$

$$= \boxed{Ed} \quad \text{(a scalar)}$$

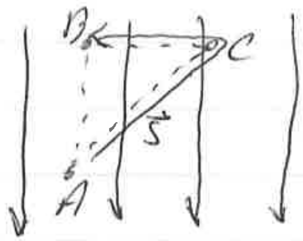
$$V_B > V_A$$

<u>Electric field lines point in direction of decreasing potential</u>

7.5

Consider also path ACB

$\Delta V_{ACB} = \Delta V_{AC} + \Delta V_{CB}$



$\Delta V_{CA} = -\int_A^C \mathcal{E} \cos 135° ds$

$= + \mathcal{E} \frac{\sqrt{2}}{2} \underbrace{\int_A^C d\vec{s}}_{} = \sqrt{2} d$

$= + \mathcal{E} \frac{2}{2} d$

$= \underline{\underline{+\mathcal{E} d}}$ ~~same~~ as

$\Delta V_{BC} = -\int_C^B \mathcal{E} \cos 90° ds = \underline{\underline{0}}$

$\underline{\underline{\Delta V_{ACB}}} = \mathcal{E} d + 0$

$= \boxed{\mathcal{E} d}$ same as ΔV_{BA}

Example

A probe is placed in a uniform electric field of 10^{-3} N/C. It starts at the origin and is moved 2cm to the right, then 5cm upward. It is moved 3cm forward and 1cm down. What is the potential difference over this path?

$\vec{E}$ field is $45°$ w/respect right and upward directions.

- we just care about the net distance, not the path length.

$$\vec{d} = +2\text{cm}\,\hat{i} + (5\text{cm} - 1\text{cm})\,\hat{j} + 3\text{cm}\,\hat{k}$$
$$= 2\text{cm}\,\hat{i} + 4\text{cm}\,\hat{j} + 3\text{cm}\,\hat{k}$$

$$\vec{E} = (\cos 45°\,\hat{i} + \sin 45°\,\hat{j})(10^{-3}\tfrac{N}{C})$$
$$= \left(\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}\right)(10^{-3}\tfrac{N}{C})$$

$$V = \vec{E}\cdot\vec{d} = \frac{\sqrt{2}}{2}\times 10^{-3}\frac{N}{C}(2\text{cm})\,\hat{i}\cdot\hat{i} + \frac{\sqrt{2}}{2}\times 10^{-3}\frac{N}{C}(4\text{cm})\,\hat{j}\cdot\hat{j} + 0$$
$$= 1.4\times 10^{-5}\frac{Nm}{C} + 2.8\times 10^{-5}\frac{Nm}{C}$$

$$V = \boxed{4.2\times 10^{-5}\frac{Nm}{C}}$$

E-field does work:

1) when positive charge moves in direction of $\vec{E}$:

→ q feels force $q\vec{E}$ downward

→ accelerate, gain K.E.

-potential energy drops

→ It requires positive work for us to "go against" by moving q from $A \to B$

2) when negative charges move in direction of $\vec{E}$

→ opposite behavior

→ gain electric potential energy

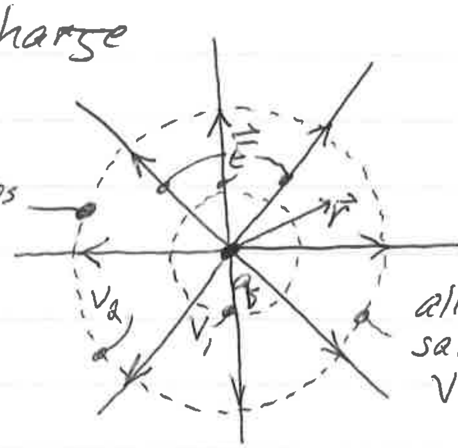
Equipotentials

We're used to drawing field lines, but another line tells about potentials.

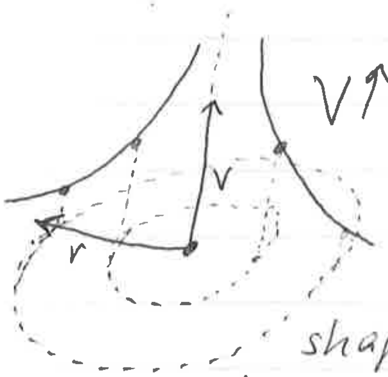
If have point charge

→ radial $\vec{E}$

→ spherical surfaces have constant V



all @ same V



equipotential surfaces ⊥ to E field lines passing thru them

shape of V vs. r for point charge

Potential difference for 2 points $d\vec{s}$ apart

$$dV = -\vec{\mathcal{E}} \cdot d\vec{s} \quad \text{(general)}$$
$$= -\mathcal{E}\,dr \quad \text{(point charge)}$$
$$\boxed{\mathcal{E}_r = -\frac{dV}{dr}}$$

$\therefore$ Electric field is a measure of the rate of change with position of the electric potential