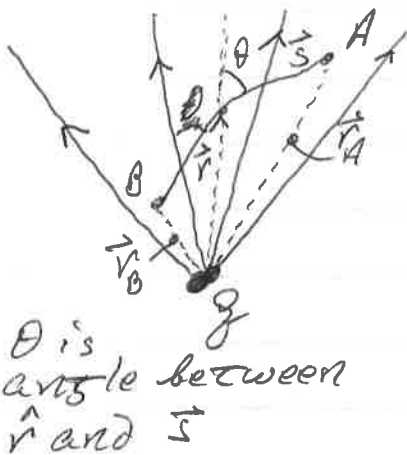


Potential Due to a charged Particle

8.1



Consider potential difference calculated from A to B.

\vec{E} directed radially outward $\parallel \hat{r}$

What is $V(r)$?

$$\begin{aligned} V_B - V_A &= - \int_A^B \vec{E} \cdot d\vec{s} \\ &= - \int_A^B k \frac{q}{r^2} \hat{r} \cdot d\vec{s} \\ &= - \int_A^B k \frac{q}{r^2} dr = -kq \int_A^B \frac{dr}{r^2} \\ &= \underline{kq \left[\frac{1}{r_B} - \frac{1}{r_A} \right]} \end{aligned}$$

so, again, independent of path

We often take the convention $r_A \rightarrow \infty$ and $r_B = r$

$$\boxed{V = kq/r}$$

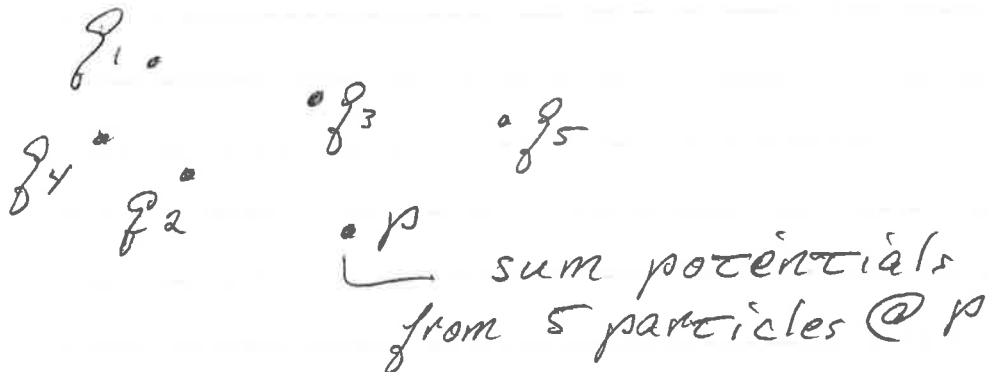
Potential Due to a Group of Charges

8.2

For a group of charges, treat each single charge, & sum potentials.

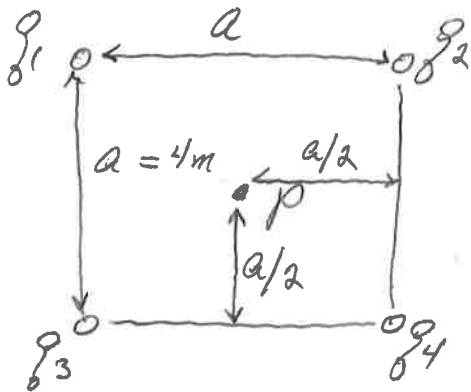
V is a scalar, so no need to keep track of vector components

$$V = k \sum_i \frac{q_i}{r_i}$$



Example

8.3



$$q_1 = +1 \times 10^{-8} \text{ C}$$

$$q_2 = -20 \text{ nC}$$

$$q_3 = +20 \text{ nC}$$

$$q_4 = +30 \text{ nC}$$

What is V @ point P ?

$$r_1 = r_2 = r_3 = r_4 = \sqrt{(a/2)^2 + (a/2)^2} = \sqrt{\frac{2a^2}{4}}$$
$$= \underline{a/\sqrt{2}}$$

$$V = k \sum_i \frac{q_i}{r_i} = \frac{k}{r} [q_1 + q_2 + q_3 + q_4]$$

$$= 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left(\frac{1}{a/\sqrt{2}} \right) [40 \text{ nC}]$$

$$= \boxed{127 \text{ Nm/C}}$$

Electric Potential in 8.4 Continuous Charge Distribution



If charge distribution continuous

$$V = \int dV = \int k \frac{dq}{r^2}$$
$$= \underline{\underline{k \int \frac{dq}{r^2}}}$$

Integrate over whole charge distribution.

May involve ^{uniform} charge densities:

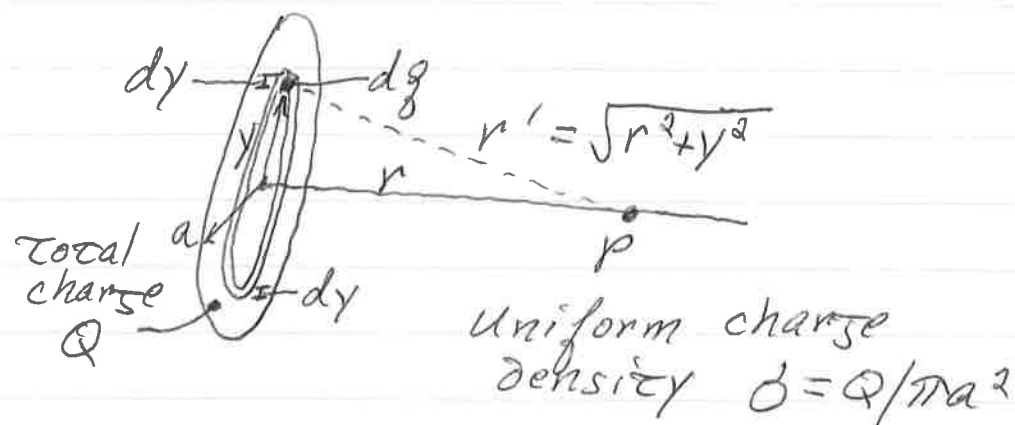
linear $dq = \lambda dx$

areal $dq = \sigma dA$

volume $dq = \rho dV$

Potential from a Uniformly Charged Disk

8.5



Consider annulus of radius ' y '

Area of annulus $dq = \sigma [(2\pi y) dy]$

All dq on annulus give same potential

$$dV = k \frac{dq}{r'^2} = k \frac{\sigma (2\pi y) dy}{\sqrt{y^2 + r^2}}$$

Integrating over all radii y

$$V = k(2\pi\sigma) \int_0^a \frac{y dy}{\sqrt{y^2 + r^2}}$$

$\underbrace{\hspace{10em}}_{\sigma / 2\epsilon_0}$

$$= \boxed{\frac{\sigma}{2\epsilon_0} [\sqrt{a^2 + r^2} - r]}$$

Example

8.6

An insulating disk has a uniform charge density throughout. The disk's outer radius is 12cm, and the total charge is +30nC. What is the electric potential at 30cm along the disk axis?

$$\sigma = \frac{Q}{A} = \frac{30\text{nC}}{(\pi(0.12\text{m})^2)}$$
$$\sigma = 6.7 \times 10^{-6} \text{C/m}^2$$

$$V = \frac{\sigma}{2\epsilon_0} \left[\sqrt{a^2 + r^2} - r \right]$$
$$= \frac{6.7 \times 10^{-6} \text{C/m}^2}{2(8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2)} \left[\sqrt{0.0144\text{m}^2 + 0.09\text{m}^2} - 0.3\text{m} \right]$$

$$V = 9.2 \frac{\text{Nm}}{\text{C}}$$