

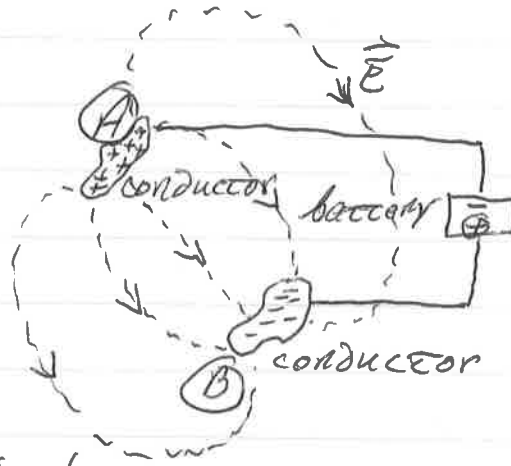
Capacitance

9.1

Consider
2 conductors:

- separated by
insulator
(e.g. air)

- attach to
battery terminals



→ A + B become charged, Q

- all $\vec{E}$ field lines from A
terminate on B

- a potential difference ΔV
develops between A + B

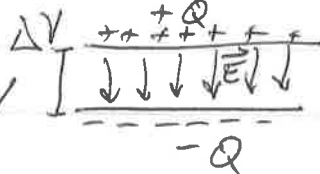
2 charged // plates:

- uniform $\vec{E}$
observe

$$Q \propto \Delta V$$

$$= \text{const. } \Delta V$$

$$= C \Delta V$$



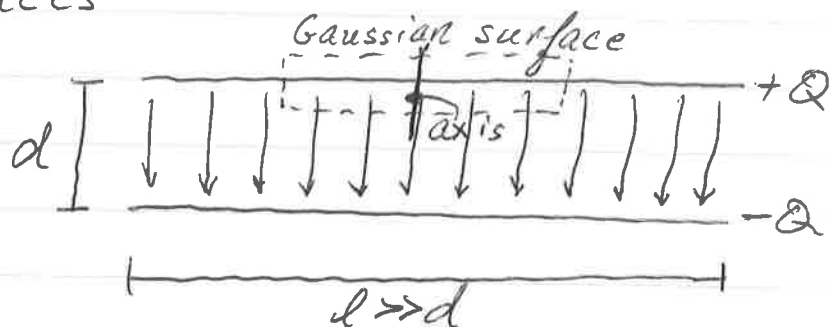
$$C \equiv Q/\Delta V \Rightarrow \text{capacitance}$$

"amount of charge a
'capacitor' can store per
unit potential
difference."

units 'farad' = coulomb/volt

Calculating the Capacitance 9.3

Use as example 2// conducting plates



We already applied Gauss's Law to this configuration.

- cylindrical symmetry + surface: axis $\perp$ plates
- top surface inside conductor
 $\rightarrow$ no field flux
- side cylinder $\vec{E} \cdot \vec{A} = 0$
- bottom surface $\vec{E} \parallel \vec{A}$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

Calculating the Potential Difference 9.4

For a uniform $\vec{E}$ field, we also know

$$\Delta V = Ed$$

Considering our expression for E , we have

$$\Delta V = \left(\frac{Q}{\epsilon_0}\right)d = \left(\frac{Q/A}{\epsilon_0}\right)d$$

$$\boxed{\Delta V = \frac{Qd}{\epsilon_0 A}}$$

Q = total charge on plate

A = area of plate (not ^{just} Gaussian area)

We can calculate capacitance for // plates:

$$C = Q/\Delta V = Q/\left(\frac{Qd}{\epsilon_0 A}\right)$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

Note! only geometry of plates enter

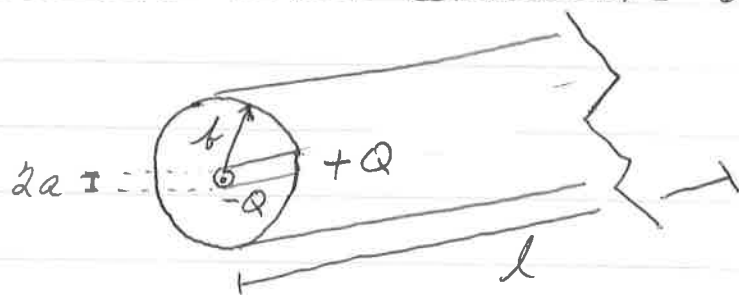
- bigger plates!
more Q & C

- larger separation! less C

Example

9.46

A cylindrical capacitor 1cm long has a wire of radius 100 μ m and outer conductor radius 1mm. When 2nC of charge are deposited on this capacitor, what is the potential difference between these conductors?



$$C = \frac{l}{2k \ln(b/a)}$$

$$= \frac{10^{-2} \text{ m}}{2 \times 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \ln\left(\frac{10^{-3} \text{ m}}{10^{-4} \text{ m}}\right)}$$

$$= \frac{10^{-11} \text{ C}^2}{18 \text{ Nm}^2 \ln(10)}$$

$$= 0.024 \times 10^{-11} \text{ C}^2/\text{Nm}$$

$$= 2.4 \times 10^{-13} \text{ farad}$$

$$\underline{\underline{\Delta V = Q/C = 2 \times 10^{-9} \text{ C} / 2.4 \times 10^{-13} \text{ farad}}}$$

$$= 0.8 \times 10^4 \text{ Volt}$$

$$= \boxed{8,000 \text{ Volt}}$$

$$a = 100 \mu\text{m}$$

$$b = 1 \text{ mm}$$

$$l = 1 \text{ cm}$$

$$Q = 2 \text{ nC}$$

Cylindrical Capacitor

9.5

Let's consider following geometry:

2 coaxial cylinders

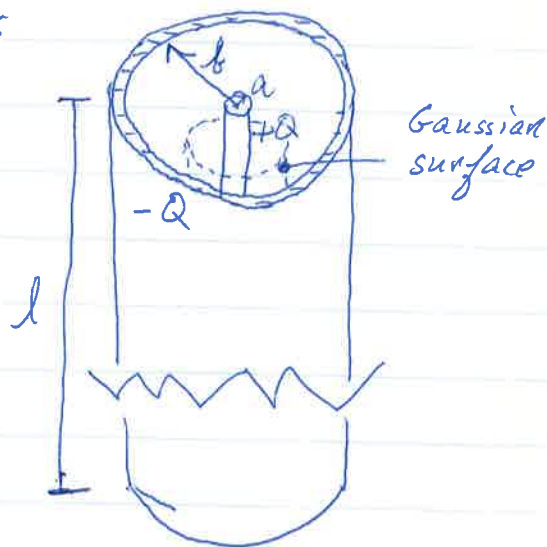
- radii a & b

$b > a$

- length l

$l \gg b$

We stay far from
ends of cylinder



What is $\vec{E}$ field internal to
outer cylinder?

Gauss's Law : cylindrical symmetry

- coaxial with inner conductor

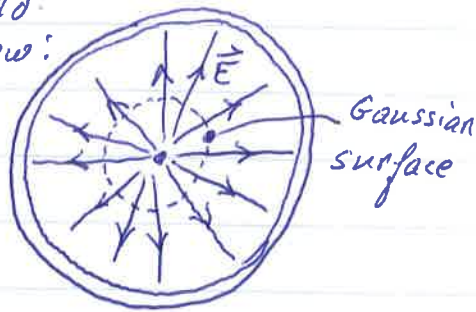
- no flux from outer cylinder

- doesn't contribute to
 E -field

9.6

$\vec{E}$ is $\perp$ to
axis and
 $\parallel$ to $\vec{r}$

End
View:



From the inner
conductor, we have seen

$$E_r = 2k\lambda/r$$

To calculate the potential
difference between (a) and (b), we
have:

$$V_b - V_a = - \int_a^b E_r dr = -2k\lambda \int_a^b \frac{dr}{r}$$

$$\Delta V = -2k\lambda \ln(b/a)$$

By substitution in $C = |Q/\Delta V|$

$$C = \frac{Q}{+2k\lambda \ln(b/a)} = \frac{Q}{2k(Q/l) \ln(b/a)}$$

$$C = l / (2k \ln(b/a))$$

Again, no λ or $Q \rightarrow$ only geometry
 l, b and a

Capacitors in Parallel

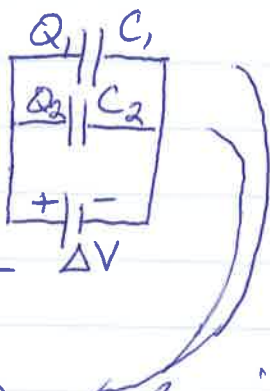
9.7

Consider our first electrical circuit:

$\text{---}||\text{---}$ = capacitor

$\text{+}||\text{---}$ = DC battery

- lines are conducting wire



These capacitors said to be in a "parallel" configuration

ΔV_1 across C_1 same as ΔV_2

Q_{TOTAL} is total charge stored
 $= Q + Q_2$

So we can think of $C_1 + C_2$ together as one capacitor C_{eff}

$$\underline{Q_{\text{TOTAL}} = C_{\text{eff}} \Delta V}$$

To determine C_{eff} from known circuit elements, we recall

$$Q_1 = C_1 \Delta V$$

$$Q_2 = C_2 \Delta V$$

Thus,

$$C_{eff} \Delta V = C_1 \Delta V + C_2 \Delta V$$

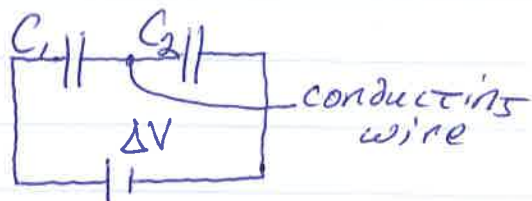
$$\boxed{C_{eff} = C_1 + C_2}$$

Capacitances sum when in parallel.

Capacitors in Series

9.9

Another circuit with 2 capacitors in "series"



Here, $Q_1 = Q_2$

- since charge conservation will not allow creation of charge in intervening conductor

But $\Delta V = \Delta V_1 + \Delta V_2$ since each step changes the potential

The equivalent capacitor to C_1 & C_2 is

$$\Delta V = Q/C_{\text{eff}} = \Delta V_1 + \Delta V_2$$

$$\frac{Q}{C_{\text{eff}}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

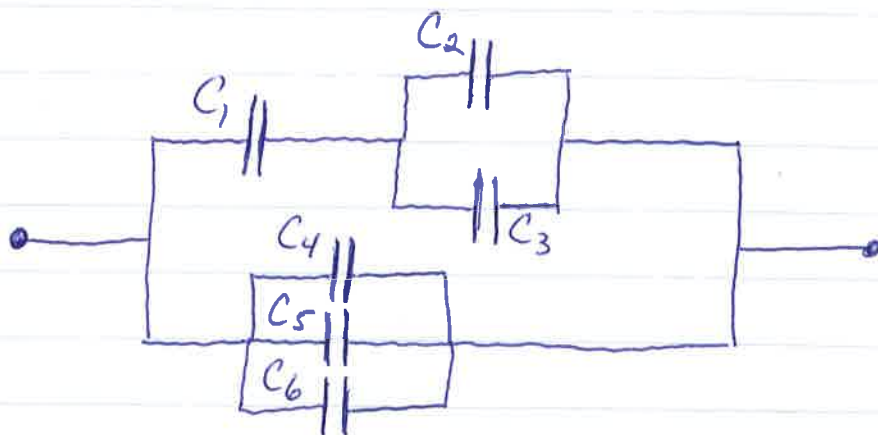
$$\boxed{\frac{1}{C_{\text{eff}}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Sum inverses of Capacitances in series.

NOTE: $C_{\text{eff}} < C_1, C_2$ always

Example

9.10



What is the effective capacitance?

$$C_{23} = C_2 + C_3$$

$$C_{456} = C_4 + C_5 + C_6$$

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_{23}}$$

$$\underline{\underline{C_{eff}}} = C_{123} + C_{456}$$

$$= \left[\frac{1}{C_1} + \frac{1}{C_2 + C_3} \right]^{-1} + C_4 + C_5 + C_6$$