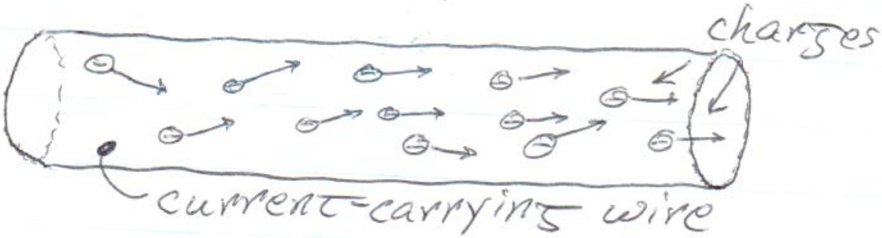
Current & Resistance

So far, we've focused on electrostatics. Now, let's allow charges to move!!

Current: the rate at which charge moves past a hypothetical plane.



$$\boxed{i(t) = \frac{dq}{dt}}$$

$i(t)$ → instantaneous current

Conversely, the total charge thru an area is

$$q = \int_{t_1}^{t_2} i(t)\,dt$$

Example

A current rises linearly from 0 to 1 amp over 10 seconds in a conducting wire. What is the total charge passing thru a cross-sectional surface of the wire during this time?

$$i(t) = 0.1\frac{A}{s}t \qquad \frac{\Delta I}{\Delta t} = \frac{(1-0)\text{ Amp}}{10\text{ s}} = 0.1\,A/s$$

Generally,

$$q = \int_{t_1}^{t_2} i(t)\,dt = \int_0^{10s} 0.1\frac{A}{s}\,t\,dt$$

$$= \left[\frac{(0.1\,A/s)t^2}{2}\right]_0^{10s}$$

$$\boxed{q = 5C}$$

Or, since $i(t)$ is linear,

$$q = \underbrace{\left(\frac{I_{t=10} - I_{t=0}}{2}\right)}_{I_{average}} \Delta t$$

$$= \frac{1}{2}\left(1\frac{A}{s} - 0\right)(10s)$$

$$= \underline{\underline{5C}}$$

Current Units + Direction

units are among fundamental physics units

meter, kilogram, second, ampere
m, kg, s, A

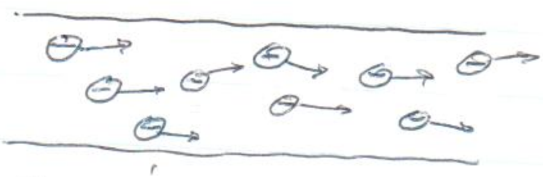
1 ampere = 1 Coulomb/sec

Current usually labelled 'I', is a scalar

- but has a direction

⇒ direction of flow positive charges appear to go

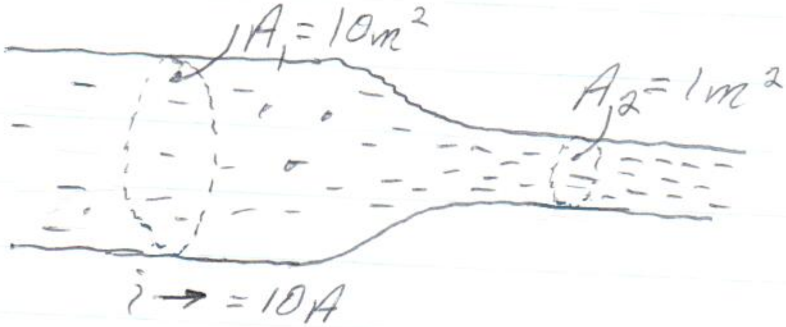
- even though electrons (−) are what's moving!!



← i, current arrow

Current Density

Consider a tube of variable width



Define current density

$$|J| \equiv i/\text{Area}$$

but $\vec{J}$ is also a vector

- direction same as direction of $\vec{E}$

  Regardless of sign of charge carriers !!!

$$\underline{J_1} = i/A_1 = \underline{1 A/m^2}$$

$$\underline{J_2} = i/A_2 = \underline{10 A/m^2}$$

Electric Field in Conductor?

11.4

We learned $\mathcal{E}=0$ in conductor.

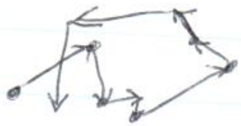
Electrostatics

- charges move around to equalize field

Current in a wire: electro-dynamics

- A gradient on conductor causes charge to move

In solids:



no $\vec{\mathcal{E}}$, no $\langle \vec{v} \rangle$

$\vec{\mathcal{E}} \neq 0$, $\langle \vec{v} \rangle \neq 0$

Electrons collide with atoms + only move systematically if $\vec{\mathcal{E}} \neq 0$

Drift Velocity

Random velocities of electrons in typical case:

$$\sim 10^{6}\, m/s$$

Does not quite average out if $\vec{E} \neq 0$! Get net velocity

$$\underline{v_{drift} \sim 10^{-4}\, m/s}$$

(or $\sim$ 10cm/hr)

- a snail can go faster!

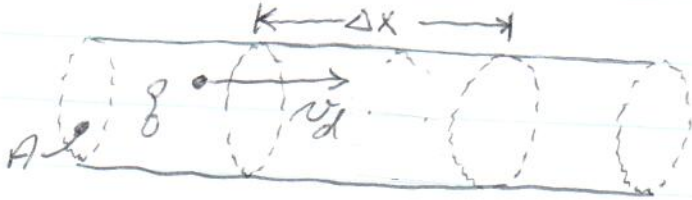
Confused? Why lights go on instantly?

The "crest" of the wave associated with the change in E-field

- travels ~ speed of light

Calculating Drift Velocity

Consider solid



Want v_d in terms of macroscopically measurable current.

n = # mobile charge carriers per volume $V = \Delta x\, A$

ΔQ = charge for n carriers ($n \cdot q$)

Note: $\Delta x = v_d \Delta t$ and $\Delta Q = nVq$

We have

$$I = \frac{\Delta Q}{\Delta t} = \frac{n v_d A \cancel{\Delta t}\, q}{\cancel{\Delta t}} = \underline{n A v_d q}$$

Solving for v_d gives

$$v_d = \frac{I}{nAq}$$

$$\boxed{v_d = J/nq}$$

Current & Resistance

For some materials
- current density, J

$$\vec{J} \propto \vec{E}$$

Ohm's Law

$$= \underline{\sigma} \vec{E} = \frac{1}{\rho} \vec{E}$$

'conductivity'

- so $\vec{J}$ also $\parallel \vec{E}$

Some other materials do not obey this rule

→ diodes: conductors at some $\vec{E}$, but insulators suddenly at some other $\vec{E}$

We know $E = \Delta V / \Delta l = V/l$

Substituting, we get

$$V = \frac{I}{\sigma} l = \frac{I \rho l}{A}$$

where ρ ('resistivity'), units $\Omega \cdot m$
- characteristic of a material
- 'Ω' (ohm) is unit of resistance

Example:

For a carbon material, having resistivity $3.6\times10^{-5}\,\Omega\cdot m$, how long would a 2mm wide cylinder of this material be if it had 1A when 120V were applied?

$$V=\frac{I\rho l}{A}\Rightarrow l=\frac{VA}{I\rho}$$

$$\underline{\underline{l}}=\frac{(120V)(\pi(10^{-3}m)^2)}{(1A)(3.6\times10^{-5}\,\Omega m)}$$

$$=\frac{3.8\times10^{-4}m^2\cancel{V}}{3.6\times10^{-5}\,\cancel{\Omega A}\cancel{m}}$$

$$=\boxed{10.5m}\ !!!$$

Resistivity & Temperature

Resistivities very low for conductors:

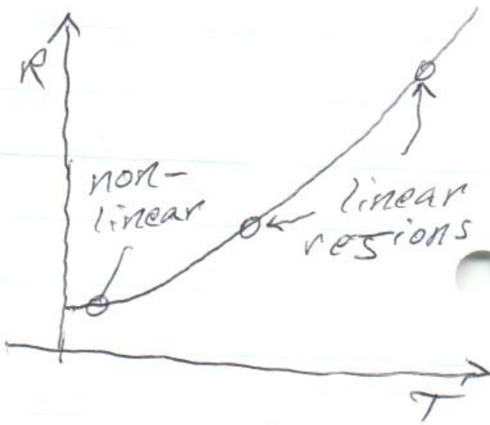
$$\rho_{copper} = 1.69 \times 10^{-8}\ \Omega \cdot m$$

$$\rho_{iron} = 9.7 \times 10^{-8}\ \Omega \cdot m$$

There is a variation of ρ with temperature

- often linear over some range

$$\boxed{\rho = \rho_0 \left[1 + \alpha (T - T_0)\right]}$$



- if T rises, so does R
  - → coefficient α gives rate.