Current & Resistance

For some materials, the current density $\vec{J}$ is proportional & // to $\vec{E}$

Ohm's Law

$$\vec{J} \propto \vec{E}$$

$$= \sigma \vec{E} = \frac{1}{\rho} \vec{E}$$

where the constant of proportionality σ, $\frac{1}{\rho}$

- σ = "conductivity"
- ρ = "resistivity"

Ohm's Law

Generally, the expression $\vec{J} = \sigma \vec{E}$ is not directly usable in a circuit.

- we've seen this yield

$$V = I \frac{\rho l}{A}$$

We define 'resistance' $R = \frac{\rho l}{A}$, to get

$$\boxed{V = IR}$$

common form of Ohm's Law

Resistance is a quantity of a specific device: a 'resistor'

- decreasing (increasing) length (area) reduces R
- reducing radius of cylindrical wire by 2x
  - increases R by 4x

Units:

$$1 \text{ ohm } (\Omega) \equiv 1 \frac{\text{Volt}}{\text{ampere}} \left(\frac{V}{A}\right)$$

Example:

A resistor is a small cylinder of carbon, 0.5cm long and 1mm wide. What is the resistance of this resistor?

$$R = \rho l / A$$

$$\rho_{carbon} = 3.6 \times 10^{-5} \, \Omega \cdot m$$

$$R = \frac{(3.6 \times 10^{-5} \, \Omega m)(5 \times 10^{-3} \, m)}{\pi (10^{-3} \, m)^2}$$

$$= \frac{18 \times 10^{-8} \, \Omega}{3.1 \times 10^{-6}}$$

$$= \boxed{5.7 \times 10^{-2} \, \Omega}$$

A batter puts a 120 V potential across this resistor. What is the current in the circuit?

$$V = IR$$

$$I = V/R = 120 V / 5.7 \times 10^{-2} \, \Omega$$

$$= \boxed{2.1 \times 10^{3} A} \; !!!$$

this a lot of current, due to low R

Measuring Temperature

Use change in R

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

$$\therefore R = R_0 [1 + \alpha(T - T_0)]$$

So $\Delta R (= R - R_0) = \alpha R_0 \Delta T$

$$\boxed{\frac{\Delta R}{\alpha R_0} = \Delta T}$$

Example

Carbon has $\alpha = 5 \times 10^{-3}$. What ΔT if a $100\,\Omega$ resistor changes by $1\,\Omega$?

$$\frac{\Delta R}{\alpha R} = \frac{1\,\Omega}{(5 \times 10^{-3})(100\,\Omega)} = \Delta T$$

$$= \frac{1}{0.5} = \boxed{2\,K = \Delta T}$$

Power in Electric Circuits

12.35

As work done on charges in a resistive material

- power associated with change in potential energy

$$du = dq\,V = (i\,dt)\,V$$

$$\therefore \frac{du}{dt} = \boxed{P = iV}$$

Power delivered to circuit,
- a general expression, not just for Ohm' Law case

Units:

$$1\ \text{watt}\ (W) = 1\ J/s$$
$$= 1\ V \cdot A$$

Power in Electric Circuits

Resistance is somewhat like friction

- energy of electrons moving
  - some goes into collisions + interactions with atoms
    - → vibrations + other losses of energy
  - losses manifest as heat

Using Ohm's Law, we find

$$P = IV = I(IR) = \left(\frac{V}{R}\right)V$$

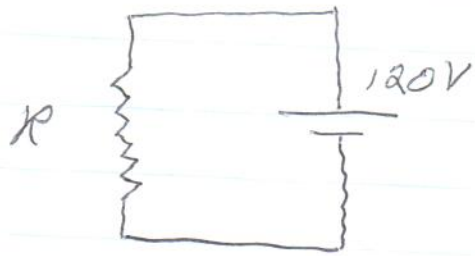
$$= \boxed{I^2 R}$$

$$= \boxed{V^2/R}$$

Power lost as heat by resistors.

Example

A resistor is connected to a wall outlet. What is Power lost to heat for (a) 100 m-Ω and (b) 1 k-Ω resistors?



R_a: $P_a = \frac{V^2}{R} = \frac{1.44\times10^4 V^2}{10^{-1}\Omega}$

$= \boxed{1.4\times10^5 W}$

R_b: $P_b = \frac{V^2}{R} = \frac{1.4\times10^4 V^2}{10^3\Omega}$

$= \boxed{14 W}$

Other Behaviors

Not all materials obey Ohm's Law or $R(T)$ being linear.

Semiconductors:

- can have R decrease with increase in T

Superconductors:

- resistance drops to 0 below critical T, T_c
  - many T_c near absolute 0
  - some $T_c \sim 100K$ or warmer
- search on for 'room-temperature T_c'

- importance: no resistance means no heat from current
  - no energy losses