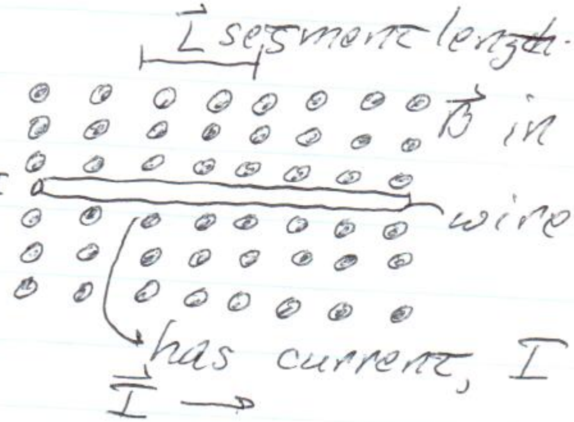
Magnetic Force on a Current Carrying Wire

$A = \pi\left(\frac{d}{2}\right)^2$
cross sectional area of wire in uniform $\vec{B}$



$\vec{F}$ on each q in current is $q\vec{v}\times\vec{B}$

For a given density of electrons

$n = \#$ per volume $(A \times L)$

we have total force

$$\vec{F}_B = q\vec{v}\times\vec{B}(nAL)$$

Since $I = nqvA$,

$$\vec{F}_B^{TOT} = (nqvA)(L\hat{v})\times\vec{B}$$

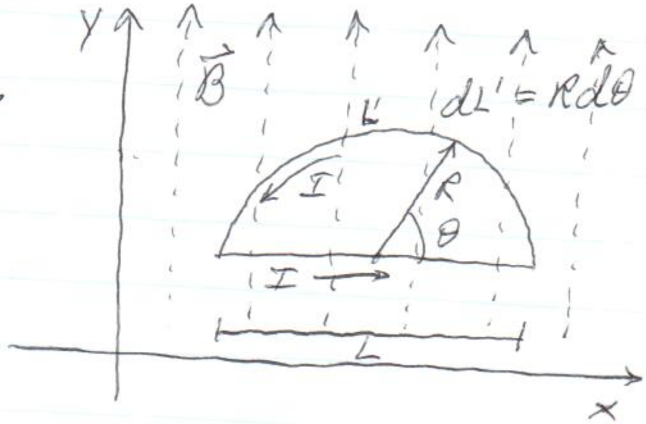
$$\boxed{= I\vec{L}\times\vec{B}}$$

where $\vec{L}$ is vector in direction of current, with length L.

Example:

Current loop in semicircle shape.

Consider 2 segments:



$$F_{straight} = F_1 = I\vec{L} \times B$$
$$= +\underline{\underline{ILB\hat{k}}} \text{ (out of page)}$$

$$F_{curve} = F_c = I\underbrace{\vec{L}' \times \vec{B}}_{\text{need to integrate over } L'}$$

$$dF_c = IBdL = IB(Rd\theta)$$

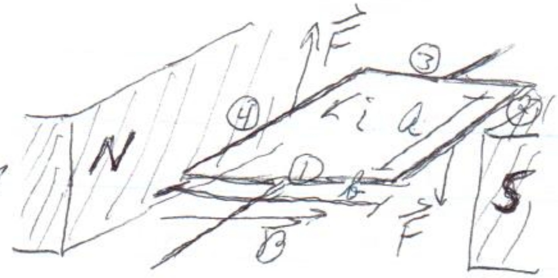
$$\underline{F_c} = \int_0^{\pi} dF_c \sin\theta = IBR\int_0^{\pi} \sin\theta d\theta$$
$$= IBR[\cos\pi - \cos 0] = -2IBR$$
$$= \underline{\underline{-IBL\hat{k}}} \text{ (into page)}$$

$$\vec{F}_{TOT} = \vec{F}_1 + \vec{F}_c =$$
$$= \underline{\underline{0}}$$

∴ Force on a current loop is zero.

Torque on a Current Loop

uniform $\vec{B}$ between $N + S$ poles



Consider Force on each of 4 sides

$$\vec{F}_{1} = i\,\cancel{\vec{L}\times\vec{B}}^{\to 0} = \vec{F}_{3} = 0$$

$$\vec{F}_{2} = iaB = -\vec{F}_{4}$$

$$\therefore \vec{F}_{TOTAL} = 0$$

Since each side 2 and 4 are a distance $b/2$ from center, there is a torque associated with each

$$\tau = iaB\left(\frac{b}{2}\sin\theta\right) + iaB\left(\frac{b}{2}\sin\theta\right)$$

where $b/2$ is the lever arm, and θ is the angle of sides 1 or 3 w/respect to $\vec{B}$.

So

$$\boxed{\tau = iabB\sin\theta}$$

Magnetic Dipole Moment 16.4

Wire loop has torque when in $\vec{B}$ field. If have a coil of multiple turns

$$\tau_{coil} = N\tau_{loop} = N\underbrace{iab}_{A,\text{ area of loop}}B\sin\theta$$

$$= NiAB\sin\theta$$

$$= \boxed{\mu B\sin\theta}$$

where μ is magnetic dipole moment $\mu = NiA$.

Most generally

$$\boxed{\vec{\tau} = \vec{\mu}\times\vec{B}}$$ ($\vec{\mu}$ direction is $\perp$ to loop)

Energy associated with coil in $\vec{B}$ field

$$\boxed{U = -\vec{\mu}\cdot\vec{B}}$$

Example:

A circular coil with 100 turns of wire is placed in a uniform magnetic field such that its axis is $\perp$ to $\vec{B}$. If the diameter of the coil is 2.5 cm and the magnetic field is 0.1 Tesla, what is the torque on the coil if it has $1\,\mu A$?

$$\mu = NiA = Ni\pi\left(\frac{D}{2}\right)^2$$
$$= 100(10^{-6}A)\left(\pi \times \left(\frac{2.5\times10^{-2}m}{2}\right)^2\right)$$
$$= 10^{-4}A(4.8\times10^{-4}m^2)$$
$$= 4.8\times10^{-8}A\cdot m$$

$$\vec{\tau} = \vec{\mu}\times\vec{B} = \mu B = 4.8\times10^{-8}A\cdot m\,(0.1T)$$
$$= \boxed{4.8\times10^{-9}N\cdot m}$$