Ampere's Law

In electrostatics

Coulomb's Law: • calculate $\vec{E}$ field • from point charges

- for highly symmetric charge distributions
  - can solve more easily with Gauss's Law

For magnetism

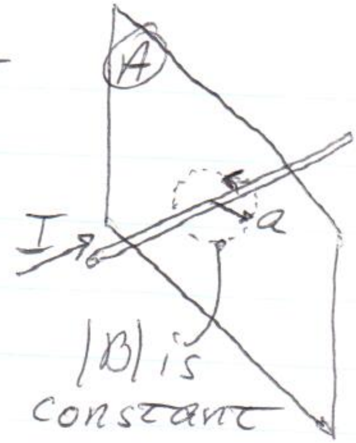
Biot-Savart Law • calculate $\vec{B}$ field • from current segment

- Consider analogy w/ $\vec{E}$ field

  Is there a magnetic analog to Gauss's Law?

Consider a straight wire

- lines of B form circles around wire



$B \propto I$

$B \propto a^{-1}$

We seek relationship between $\vec{B}$ and I we can determine
- for certain symmetric current distributions
- above example → circular symmetry around wire in plane (A)

$\vec{B} \cdot d\vec{s}$ for wire $= B\,ds$ ($\vec{B} \parallel d\vec{s}$)

"Path integral" around wire:

$$\oint \vec{B} \cdot d\vec{s} = \oint \underbrace{B}_{\text{constant on our chosen path}} ds = B \oint ds = \frac{\mu_0 I}{\cancel{2\pi a}} (\cancel{2\pi a})$$

$$\underbrace{\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 I}}_{\text{generally applicable}}$$

Ampere's Law

Note,

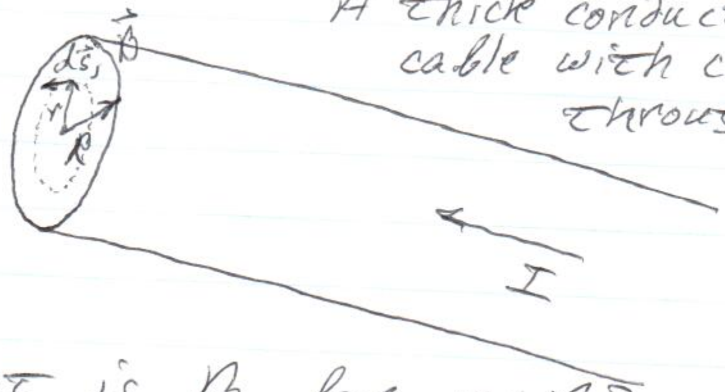
$\vec{B}\cdot d\vec{s}$ around any closed path proportional to the current passing thru the surface bounded by the closed path

Similar to Gauss's Law case,

- we are only concerned with sources of field inside path
  - these are the current-carrying wires enclosed

Magnetic Field from a Cylinder of Current: (18.4)

$d\vec{s} \parallel \vec{B}$



A thick conducting cable with current throughout.

What is B for $r < R$?

Current, i, thru surface bounded by path
- scales as area of surface

$$i_{enclosed} = \frac{\cancel{\pi} r^2}{\cancel{\pi} R^2} I = \frac{r^2}{R^2} I$$

From Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_o i_{enclosed}$$

$$B(2\pi r) = \mu_o \left(\frac{r^2}{R^2} I \right)$$

$$\boxed{B = \frac{\mu_o I r}{2\pi R^2}}$$

18.5

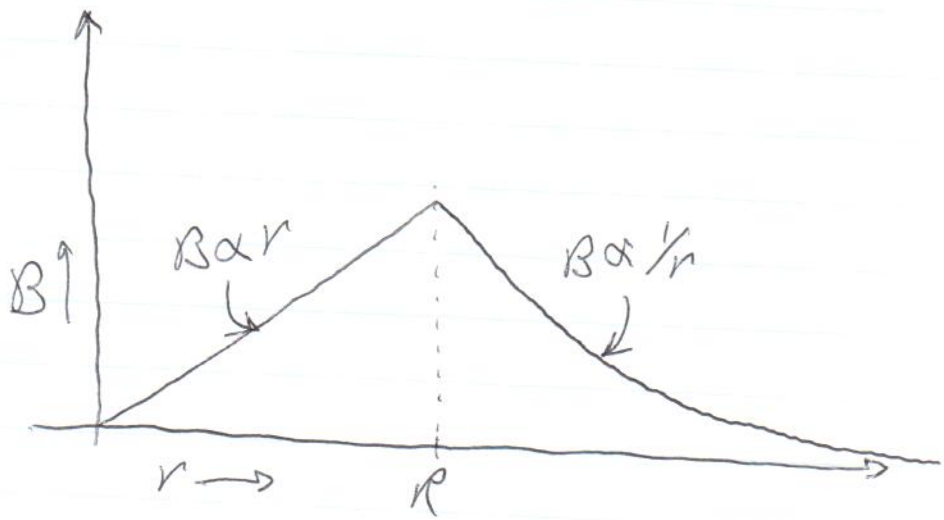
$\therefore$ $B \propto r$ when $r < R$

When $r > R$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0}{2\pi r} I}$$



Example

18.6

A thick wire with current 0.2A registers a magnetic field of 1 mT at a radius half of its outer radius. What is the outer radius?

$$B = \frac{\mu_0}{2\pi} I \left(\frac{r}{R}\right) \frac{1}{R}$$

$$10^{-3}\,T = \frac{\cancel{4\pi}^{2} \times 10^{-7}\,Tm}{\cancel{2\pi}\,\cancel{A}}(0.2\cancel{A})(0.5)\frac{1}{R}$$

$$\underline{R} = \frac{2\times 10^{-4}\,\cancel{T}m}{\cancel{10^{-3}}\,\cancel{T}}(0.1)$$

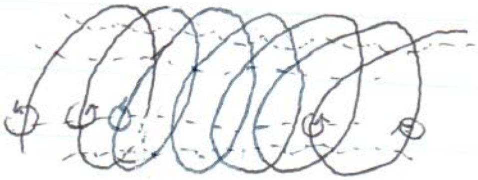
$$= 2\times 10^{-5}\,m$$

– or –

$$= \boxed{20\,\mu m}$$

Solenoids:

- Definition: a long wire wound in a close-packed helix and carrying current I.



- when wires close
  - almost like a sheet of current

inside, E field constant + uniform

Ideal Case of Solenoid

Pick paths $\perp$ or $\parallel$ to $\vec{B}$ field.

uniform field inside
- analogous to capacitor

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \cancelto{0}{\int_b^c \vec{B}\cdot d\vec{s}}$$

$$+ \cancelto{0}{\int_c^d \vec{B}\cdot d\vec{s}} + \cancelto{0}{\int_d^a \vec{B}\cdot d\vec{s}}$$

Since $h >>$ width of wire, $i = i_0 n h$
where $n = \#$ turns/length

$$\int_a^b \vec{B}\cdot d\vec{s} = \int_a^b B\,ds = Bh = \mu_0 i$$

$$\underline{B} = \frac{\mu_0 (i_0 n \cancel{h})}{\cancel{h}}$$

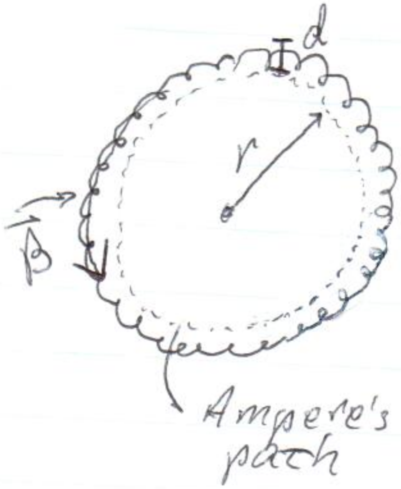
<u>$\vec{B}$ field for a solenoid</u>

$$= \boxed{\mu_0 i_0 n}$$

- uniform $\vec{B}$ field inside solenoid
- not a bad approximation for actual solenoids near axis

Toroids:

Like a solenoid bent closed to a donut shape



We want to know $\vec{B}$ field at r from center.

→ Note circular symmetry

Using Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

$$B(2\pi r) = \mu_0 i_0 \underbrace{N}_{\text{\# turns total}}$$

$$\boxed{B = \frac{\mu_0 i_0 N}{2\pi r}}$$

So B is not constant inside ring.

Example:

A toroid has 1000 turns over its circumference. The current is 7mA in the wire. What is the magnetic field at a radius 1m, which is less than the toroid radius.

$$B = \frac{\mu_0 i_0 N}{2\pi r}$$

$$= \frac{\overset{2}{\cancel{4\pi}} \times 10^{-7} \frac{T\cdot\cancel{m}}{\cancel{A}} (\cancel{7\times10^{-3} A})(\cancel{10^{3}})}{\cancel{2\pi}(1\cancel{m})}$$

$$= 2\times10^{-7} T \ (\times 7)$$

$$= 1.4\times10^{-6} T$$

- or -

$$= \boxed{1.4 \mu T}$$