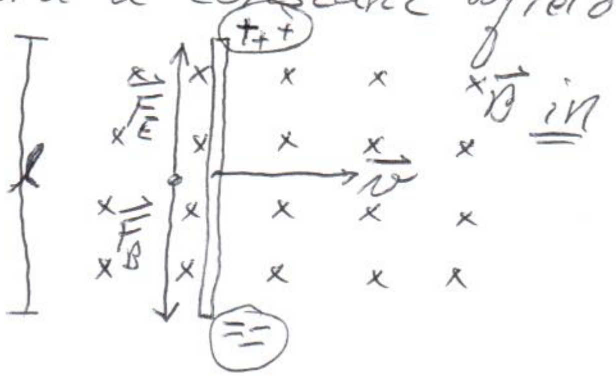
Motional Emf

$\mathcal{E}$ induced in a conductor moving thru a constant $\vec{B}$ field

straight conductor
$\vec{v} \perp \vec{B}$



Electrons $(q<0)$ feel force

$$\vec{F}_B = q\vec{v}\times\vec{B}$$

$\therefore$ electrons pushed to bottom

Since rod becomes 'polarized'
- and + at opposite ends
- electrons feel force

$$\vec{F}_e = q\vec{E} \text{ upwards}$$

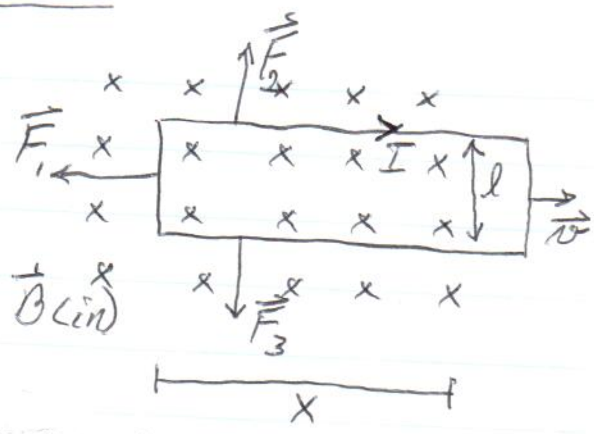
Equilibrium condition:

$$\vec{F}_B = q\vec{v}\times\vec{B} = qvB = \vec{F}_E = q\vec{E}$$

$$\boxed{vB = E}$$

Loop in a $\vec{B}$ Field

Rectangular loop pulled out of B field w/velocity, $\vec{v}$



Flux, ϕ_B, thru loop changes with time $(x = x(t))$

$$\phi_B = BA = Blx$$

Use Faraday's Law,

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -Bl\frac{dx}{dt} = \underline{\ominus Blv}$$

- Negative sign means
  - induced I clockwise
  - sets up field // existing

$F_2 + F_3$ equal & opposite $\therefore$ cancel

F_1 counters our effort to move loop

$$\vec{F}_1 = I\vec{l} \times \vec{B} = IlB\sin 90° (-\hat{i})$$

$$= \boxed{-BlI\,\hat{i}}$$

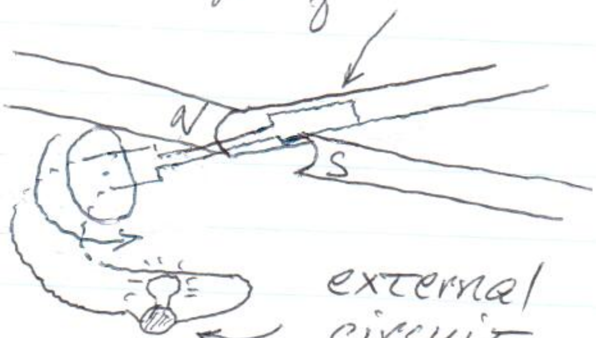
Magnetic Brake

Generators & Motors

20.3

Electric generators
- energy created by work
  - transferred by electrical transmission

AC Generator
- loop of wire rotated in B field



external circuit

Φ_B thru loop changes with time $\rightarrow \mathcal{E}$

$\underline{\Phi_B = BA\cos\omega t}$

where 'ω' is rotation frequency

We can calculate induced $\mathcal{E}$

$$\underline{\underline{\mathcal{E}}} = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t)$$

$$= \underline{NAB\omega\sin(\omega t)}$$

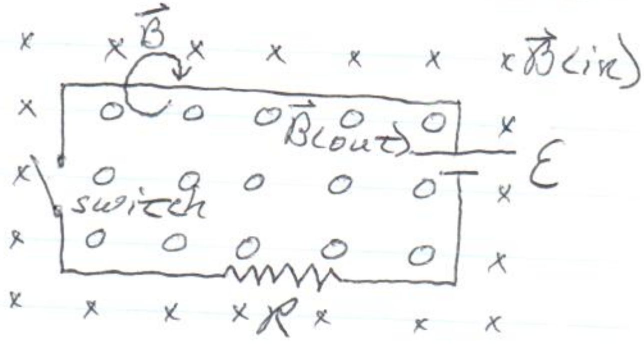
where N is number of turns in loop

Self Inductance

Consider a circuit with current and emf,

$I_{max} = \mathcal{E}/R$ after throw switch at $t=0$

$(I(t=0)=0)$



When @ $I_{max} \rightarrow$ have $B = B_{max}$ (fr. Biot-Savart)

$$\therefore \Delta\phi_B > 0$$

So, we'll have induced emf + current

$$\mathcal{E}_L = - \frac{d\phi_B}{dt}$$

happens on same wire where $\mathcal{E}$ — Lenz's law means $\mathcal{E}_L$ will oppose $\Delta\phi_B$ and induce 'back-current' + emf.

$\phi_B \propto B \propto I$: $\boxed{\mathcal{E}_L = -L\frac{dI}{dt}}$

"inductance"

Inductance

- constant of proportionality relating induced emf and change in current
- a measure of how much emf induced by loop (or coil — see later)

Units: 'henrys' $\boxed{1H = 1 V \cdot s/A}$

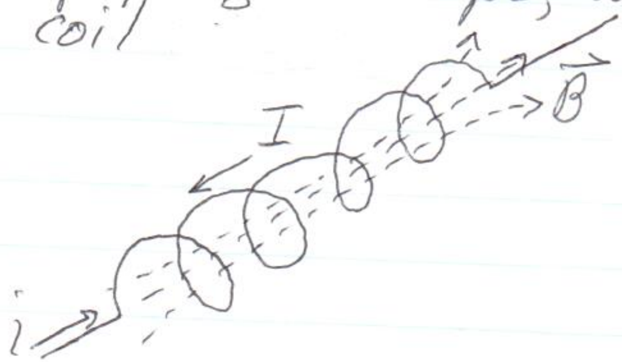
Causes a slow down in change in current, I

$$I = I_{\mathcal{E}}(t) + i_i(t)$$

Inductors

20.6

We can increase inductance by multiplying # loops, as for coil



Giving us $\underline{\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}}$

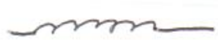
Equating both sides + integrating over time

$$N\Phi_B = LI$$

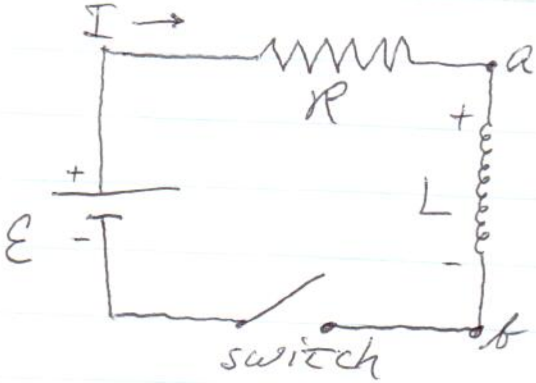
$$\therefore \boxed{L = N\Phi_B / I}$$

An 'inductor' has large self-inductance, L

- opposes changes in current in a circuit

Symbol: 

RL Circuits



When close switch I increases, $\frac{dI}{dt} > 0$

$$\therefore \mathcal{E}_L = -L\frac{dI}{dt} < 0$$

Electric potential decreases over $a \to b$

From Kirchoff's loop rule:

$$\frac{\mathcal{E}}{R} - \frac{IR}{R} - \frac{L\,dI/dt}{R} = 0$$

Using $X = \mathcal{E}/R - I$, we have

$$X = -\frac{L}{R}\frac{dX}{dt}$$

$$\frac{R}{L}dt = -\frac{dX}{X}$$

Integrating

$$\int_0^t \frac{R}{L}dt = -\int_{X_0}^{X}\frac{dX}{X}$$

We then have

$$-\frac{R}{L}\tau = \ln\left(\frac{X}{X_0}\right)$$

Exponentiating gives

$$X = X_0\, e^{-R\tau/L} \qquad \left(\begin{matrix}\text{where}\\ \tau = L/R\end{matrix}\right)$$

$$\frac{\mathcal{E}}{R} - I = \left(\frac{\mathcal{E}}{R} - \cancelto{0}{I_0}\right) e^{-R\tau/L}$$

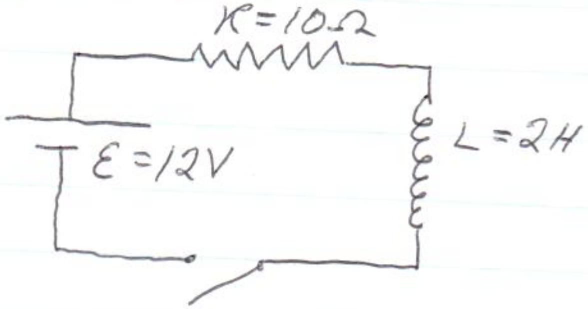
$$-I = -\frac{\mathcal{E}}{R} + \frac{\mathcal{E}}{R}\, e^{-R\tau/L}$$

$$\boxed{I = \frac{\mathcal{E}}{R}\left(1 - e^{-R\tau/L}\right)}$$

equilibrium value of the current

Example

RL circuit



How long until current reaches 50% of its maximum value?

$$\frac{I}{I_{max}} = 0.5 = 1 - e^{-R\tau/L}$$

$$0.5 = e^{-R\tau/L}$$

$$\ln(0.5) = -R\tau/L$$

$$0.69 = R\tau/L$$

$$\underline{\underline{\tau}} = 0.69\tau = 0.69\left(\frac{2H}{10\Omega}\right)$$

$$= \underline{\underline{0.14s}}$$

Energy Stored

20.10

Use loop rule on RL circuit
- quantify energy stored in inductor

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$I\mathcal{E} - I^2R = IL\frac{dI}{dt}$$

In terms of Power

$$P_L = P_{tot}(= I\mathcal{E}) - P_R(= I^2R) = \frac{dU}{dt}$$

- rate energy stored

So

$$\frac{dU}{\cancel{dt}} = LI\frac{dI}{\cancel{dt}}$$

Integrating gives

$$\int dU = \int_0^I LI\,dI$$

Total Energy Stored

$$\boxed{U = \frac{1}{2}LI^2}$$