Spherical Refracting Surface

Want expression for i given o and r.

First, simplify w/ Snell's Law since both θ_1 & θ_2 will generally be small (object at large distance.)

So $\theta \sim \sin\theta$

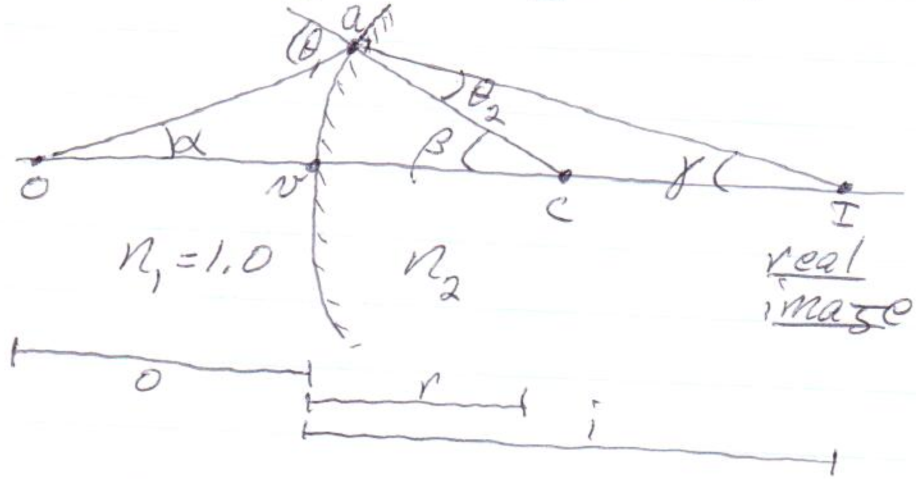
& Snell's Law becomes

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\hookrightarrow \boxed{\theta_2 = \frac{n_1}{n_2}\theta}$$

Considering

Considering triangles in



we have

$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma = \frac{n_1}{n_2}\theta_1 + \gamma$$

$$\therefore \theta_1 = \frac{n_2}{n_1}(\beta - \gamma)$$

combining

$$n_1 \alpha + n_2 \gamma = (n_2 - n_1)\beta$$

As before, these angles are

$$\alpha \simeq \frac{av}{o}\ ;\ \beta \simeq \frac{av}{r}\ ;\ \gamma = \frac{av}{i}$$

$$\therefore \boxed{\frac{n_1}{o} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}}$$

Refraction @ a single spherical surface

Example

For the prior geometry, where is image?

- radius of curvature, $r = \oplus 10\text{cm}$
  - ↳ careful! Opposite mirror
- $n_1 = 1.0$; $n_2 = 2.0$
- object 20cm to left of v.

We know everything but i

$$\frac{n_1}{o} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

$$\frac{1.0}{\oplus 0.2\text{m}} + \frac{2.0}{i} = \frac{2.0 - 1.0}{\oplus 0.1\text{m}}$$

Note sign choices. ~~Image~~ r on 'real' side of interface.

Simplifying

$$\boxed{i = +0.4\text{m}}$$

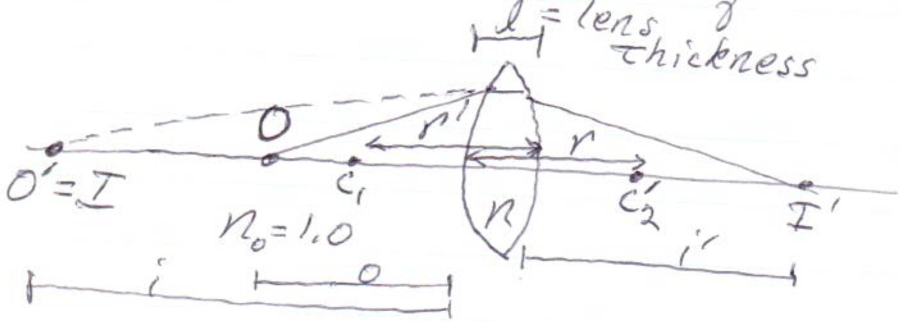
Image is real, as expected.

Thin Lenses

Convex lenses | Concave Lenses

converging | diverging

Two surfaces mean two images + two radii of curvature.



- image I is object for 2nd interface

At first surface

$$\frac{n_0}{o} - \frac{n}{i} = \frac{n-n_0}{r} \qquad (1)$$

since virtual image

For the 2^{nd} surface

- image I is object for 2nd surface

$$\frac{n}{(i+l)} + \frac{n_o}{i'} = \frac{n_o - n}{r'}$$

- with a thin lens, 'l' can be ignored

$$\frac{n}{i} + \frac{n_o}{i'} = \frac{-(n-n_o)}{r'} \quad (2)$$

Adding (1) & (2) provides

$$\frac{n_o}{o} + \frac{n_o}{i'} = (n-n_o)\left[\frac{1}{r} - \frac{1}{r'}\right]$$

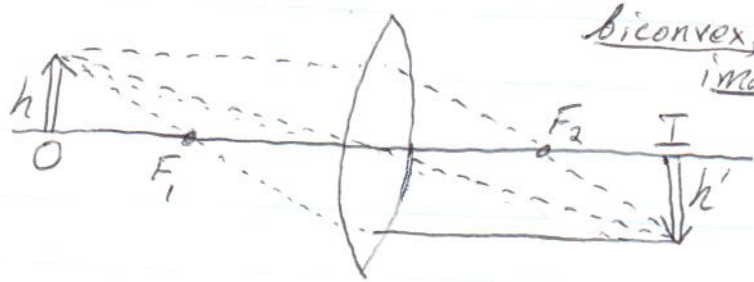
For a thin lens in air or vacuum

$$\boxed{\frac{1}{o} + \frac{1}{i} = (n-1)\left[\frac{1}{r} - \frac{1}{r'}\right]}$$

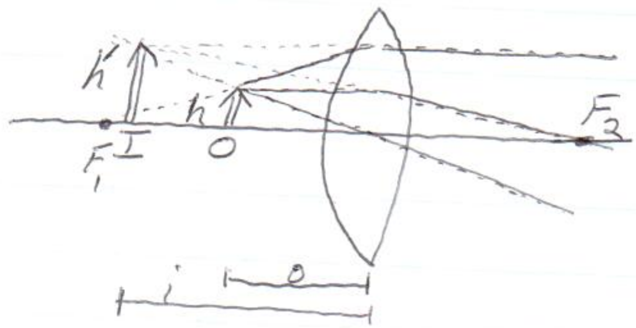
now image distance for overall lens

Magnification

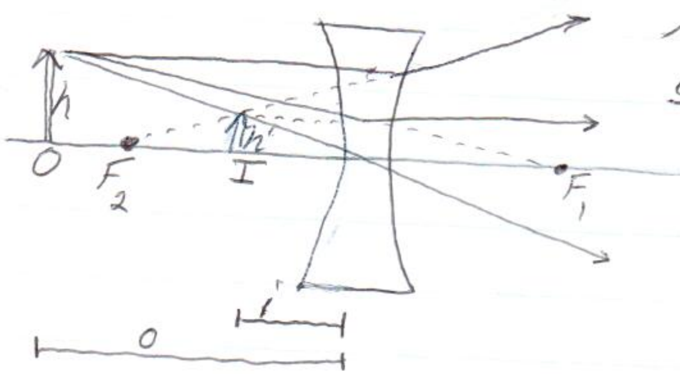
Consider three lenses



biconvex, real image



biconvex, virtual image



biconcave lens

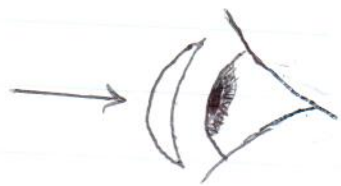
$$\boxed{M = -\frac{i}{o}} \left(= \frac{h'}{h} \right)$$ in all cases

Example

Contact lens has $n=1.5$ and radii of curvature $+2.0cm$ and $+2.5cm$.

What is focal length?

Note: both radii >0



$$\frac{1}{f} = (n-1)\left[\frac{1}{r} - \frac{1}{r'}\right]$$

$$= (1.5-1)\left[\frac{1}{0.02m} - \frac{1}{0.025m}\right]$$

$$= 0.05m^{-1}$$

$$\boxed{f = 20cm}$$

Example 26.9

Consider a biconvex glass ($n = 1.44$) lens with radii of curvature of 12 cm (18 cm) on the left (right) face.

Biconvex, so $r_1 > 0$ and $r_2 < 0$.

What is the focal length?

a) L → Right

$$\frac{1}{f} = (n-1)\left(\frac{1}{r} - \frac{1}{r'}\right)$$

$$= 0.44\left(\frac{1}{0.12\,m} - \frac{1}{0.18\,m}\right)$$

$$\boxed{f = 16.4\,cm}$$

b) "R → Left" (actually flip it)

$$\frac{1}{f} = (n-1)\left[\frac{1}{r'} - \frac{1}{r}\right]$$ but now $r < 0$, $r' > 0$

$$= 0.44\left(\frac{1}{0.18\,m} - \frac{1}{0.12\,m}\right)$$

$$\boxed{f = 16.4\,cm}$$ again

Example

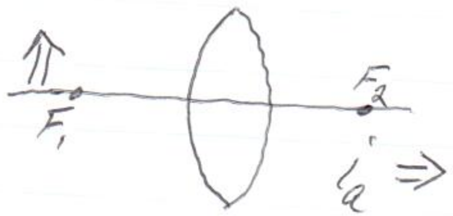
26.10

You have a thin lens with 25cm focal length.

a) Describe image when object @ 26 cm.

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o}$$

$$= \frac{1}{0.25m} - \frac{1}{0.26m}$$



$$\boxed{i = +6.5m}$$

$$\boxed{M} = -i/o = \frac{-6.5m}{0.25m} = \boxed{-26} < 0$$

Image is real, inverted and very distant.

b) Describe image when o = 24cm.

$$\frac{1}{i} = \frac{1}{0.25m} - \frac{1}{0.24m} =$$

$$\boxed{i = -6.0\,m}$$

Magnification $M = -\frac{(-6.00)}{0.25}$

$$\boxed{M = +24}$$

Image virtual, upright and enlarged.