Some Fundamentals

Charge is quantized

→ subatomic particles have charge

electron $q_e = -1.6 \times 10^{-19}$ C

proton $q_p = +1.6 \times 10^{-19}$ C

neutron $q_n = 0$ C

$\underline{q_p = -q_e}$ ↓ to many decimal places

→ made of 3 quarks each

→ charges $\pm \frac{1}{3} q_e$, $\pm \frac{2}{3} q_e$

→ no other values

Electron is smallest isolatable charge (quarks cannot be observed in isolation)

Amazingly $q_p = -q_e$ to very high precision

Charge Conservation

When rub glass rod with silk
- positive charge → rod
- negative charge → silk
  (electrons stripped from atoms)

- total charge conserved

$\boxed{\text{"In a closed system, one can never make or destroy net charge."}}$

At fundamental particle level:
- a "particle" of light ("photon", γ) passing thru matter

$$\gamma \to e^+ e^-$$

↳ positive electron = "positron"

$$0C \to 0C$$

- No cases of change in net electric charge have ever been observed.

Coulomb's Law

electrostatics: describe cases where
- have one or more unmoving electric charges
- calculate force, fields, potential energies

- like sign charges repel
  → opposite signs attract

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\,\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12}}$$

unit vector in direction of $\vec{r}_{12}$

two charges:

$$\frac{1}{4\pi\epsilon_0} = k = 9\times10^9\ Nm^2/C^2$$ – sets strength of force

ϵ_0 "permittivity constant" $= 8.85\times10^{-12}\ C^2/Nm^2$

Some Fundamentals

Coulomb's Law describes a "$1/r^2$" force, like gravity

$$\left(\vec{F}_{12} = -G \frac{m_1 m_2}{r_{12}^{2}} \hat{r}_{12} \right)$$

- for point electric charges
  - exponent r^{n} tested to 1 part in 10^{16}!!!

Also, electrical forces incredibly strong

$k \ggg G$

- actually $\sim \underline{\underline{10^{38}}}$ times stronger!!!

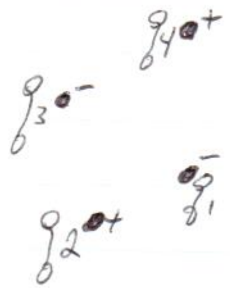
Units:

Coulomb: "amount of charge flowing thru any cross-section of a wire in 1 sec if there's a steady current of 1 Amp."

$$= \underline{\underline{6.3 \times 10^{18}}}$$ electrons

Multiple Charges

If have > 2 charges, use <u>superposition</u>



$\vec{F}_1$ is force on charge q_1

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$

- resultant force is vector sum of all forces from all particles

By <u>component</u>:

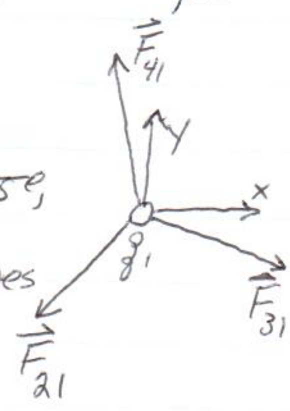
$$|F_{1x}|\hat{i} = |F_{31x}|\hat{i} - |F_{41x}|\hat{i} - |F_{21x}|\hat{i}$$

$$|F_{1y}|\hat{j} = |F_{41y}|\hat{j} - |F_{31y}|\hat{j} - |F_{21y}|\hat{j}$$

<u>Force diagrams</u>:

when multiple forces act on a point charge,

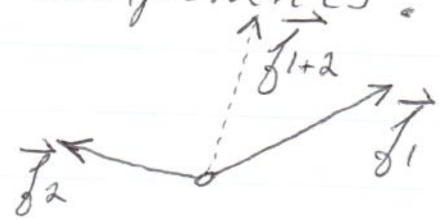
- often useful to draw magnitudes & directions



Vector Addition, Angles & Force Diagrams

Adding by vector components:

$\vec{f}_1 = 4\hat{i} + 3\hat{j}$
$\vec{f}_2 = -3\hat{i} + 1\hat{j}$



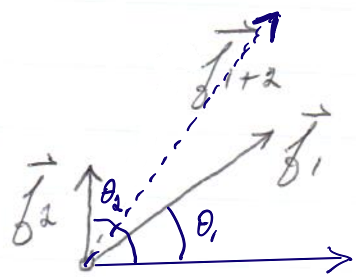
- add in each $\perp$ direction

$$\vec{f}_1 + \vec{f}_2 = (4-3)\hat{i} + (3+1)\hat{j}$$
$$= \boxed{1\hat{i} + 4\hat{j}}$$

↳ consistent with force diagram

Considering angles:

$\vec{f}_1 = 30N @ \theta_1 = 45°$
$\vec{f}_2 = 10N @ \theta_2 = 90°$



$$\vec{f}_1 = f_{1x}\hat{i} + f_{1y}\hat{j}$$
$$= |f_1|\cos\theta_1\,\hat{i} + |f_1|\sin\theta_1\,\hat{j}$$
$$= \underline{21.2N\hat{i} + 21.2N\hat{j}}$$
$$\vec{f}_2 = f_{2y}\hat{j} = |f_2|\sin 90°\,\hat{j} = \underline{10N\hat{j}}$$

$$\therefore \boxed{\vec{f}_1 + \vec{f}_2 = 21.2N\hat{i} + 31.2N\hat{j}}$$

Example: Multiple Charges

Three charges,

$q_1 = -1.0 \times 10^{-6}\,C$) $r_{12} = 15cm$) $r_{13} = 10cm$

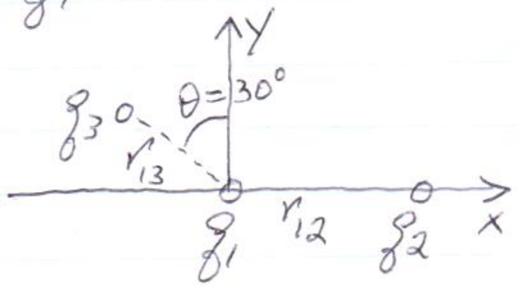
$q_2 = +3.0 \times 10^{-6}\,C$

$q_3 = -2.0 \times 10^{-6}\,C$

Charge q_2 is to right of q_1 and q_3 is on the left 30° away from vertical.

* What is resultant force + direction on q_1?

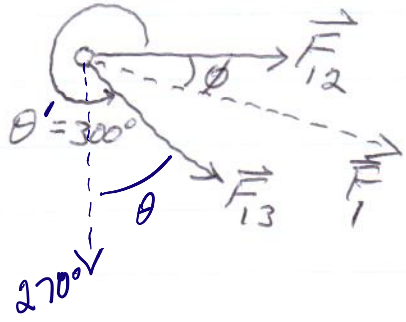
First, diagram



Force Diagram:

F_{12} attractive

F_{13} repulsive



Example (cont.)

$$|F_{12}| = kq_1q_2/r_{12}^2$$
$$= 9\times10^9\,Nm^2/C^2\left[\frac{(+1.0\times10^{-6}C)(+3.0\times10^{-6}C)}{(0.15m)^2}\right]$$
$$= \underline{1.2N}$$

$$|F_{13}| = kq_1q_2/r_{13}^2$$
$$= 9\times10^9\,Nm^2/C^2\left[\frac{(-1.0\times10^{-6}C)(-2.0\times10^{-6}C)}{(0.1m)^2}\right]$$
$$= \underline{1.8N}$$

Adding vectors,

$$F_{1x} = F_{12x} + F_{13x} = |F_{12}| + |F_{13}|\cos\theta'$$
$$= \underline{2.1N}$$

$$F_{1y} = F_{12y} + F_{13y} = \cancel{|F_{12y}|} + |F_{13}|\sin\theta'$$
$$= \underline{1.6N}$$

$$\underline{|F_1|} = \sqrt{(2.1N)^2 + (1.6N)^2}$$
$$= \boxed{2.6N}$$

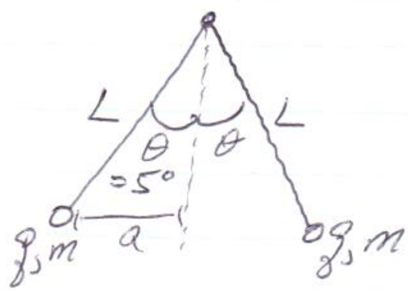
$$\underline{\phi} = \tan^{-1}\left(-\frac{1.6}{2.1}\right)$$
$$= \boxed{-37^\circ}$$

Example:

Two charges, q, are suspended by loose cables of length $L = 0.15\,m$. The charges are let go and fall such that they hang each with angle $5°$ to the vertical. They are not touching.

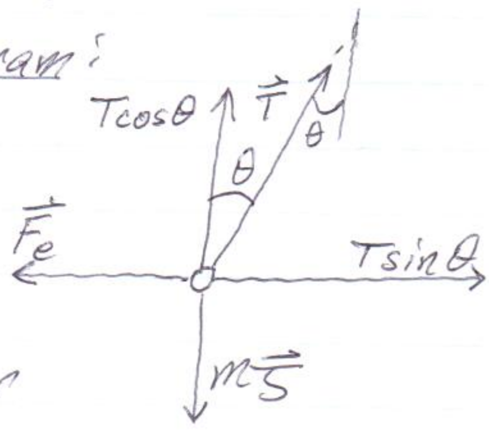
What is the charge q if each has mass $0.03\,kg$?

Diagram:



Force Diagram:

Each sphere in equilibrium by tension, T, weight and Force from other charge, F_e



Example: (cont.)

We don't know $\vec{F}_e, \vec{T}$ or a.
Consider equilibrium:

$$\Sigma F_x = T\sin\theta - F_e = 0 \qquad (1)$$

$$\Sigma F_y = T\cos\theta - mg = 0$$

$$\hookrightarrow T = mg/\cos\theta \qquad (2)$$

Using (2) in (1) provides

$$\frac{mg}{\cos\theta}\sin\theta - F_e = 0$$

$$F_e = mg\tan\theta \qquad \left(\frac{\sin\theta}{\cos\theta} = \tan\theta\right)$$

$$= \underline{2.6 \times 10^{-2}\,N}$$

Coulomb's Law gives

$$F_e = k\,q^2/(2a)^2$$

-or- $\underline{q^2 = 4F_e a^2/k}$ we need 'a'

Using Diagram:

$$\sin\theta = a/L$$

$$\underline{a} = L\sin\theta = (0.15m)(\sin 5°)$$

$$= \underline{1.3\,cm}$$

And we get

$$\underline{q} = 2a\sqrt{F_e/k} = \boxed{4.4 \times 10^{-8}C}$$