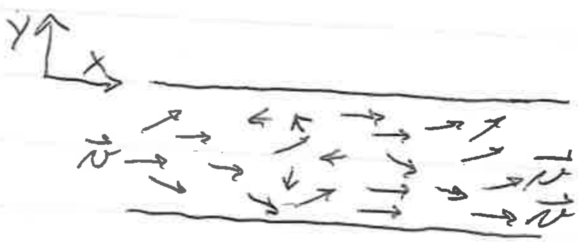
Electric Fields

When using Coulomb's Law
- always have 2 charges
- how study forces on a given charge from all others
  - without specifying charge distribution?

Use a 'field': "any physical quantity which can be specified simultaneously for all points within a given region of interest."

- this is a general concept we can use widely

__Example__: flowing water in a pipe



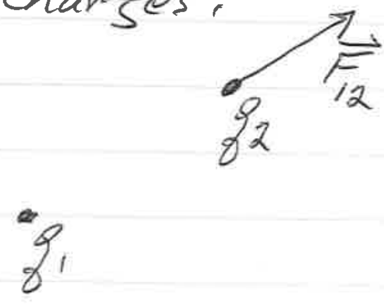
Each point has an associated velocity, $\vec{v}$.

$$\vec{v} = \vec{v}(x, y)$$

Electric Field

Consider a pair of charges:

- q_1 "sets up force" that acts on q_2
- what is this force?



Imagine q_2 is a test charge q_0 we can make arbitrarily small.

- interested in "force per charge" of q_2
- small charge leaves rest of system undisturbed

Define "electric field", $\vec{E}$ as

$$\boxed{\vec{E} \equiv \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}}$$

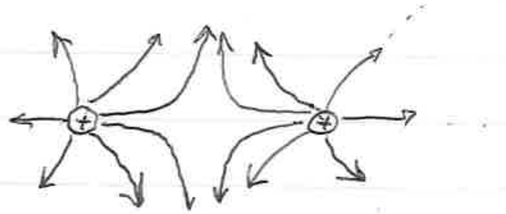
Electric Field Lines

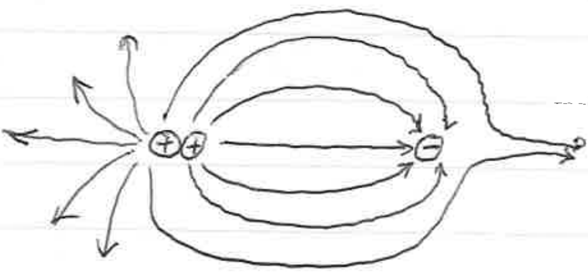
It is often useful to draw a representation of direction and strength of E-field in a region:

- #lines increases with field
- lines emanate from + charges
- lines terminate on − charges

2 equal and opposite charges

2 equal charges



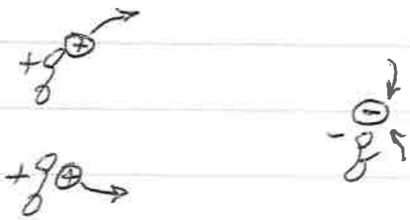


+2q -1q

All look like point or zero charges when seperation distance small

Example:

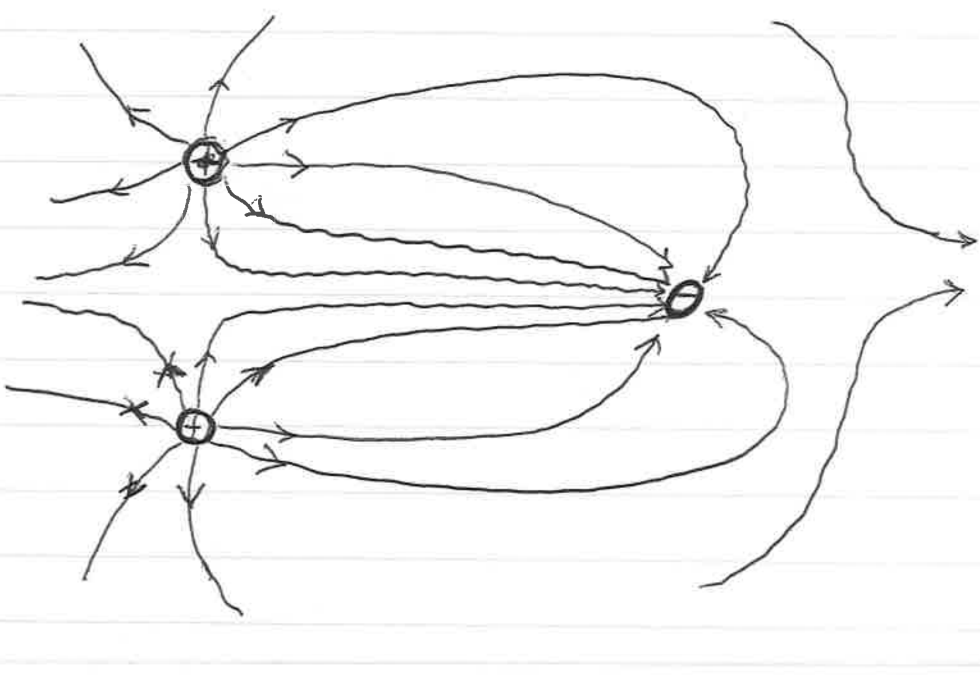
Let's try a more complicated charge distribution:



① lines from ⊕ to ⊖

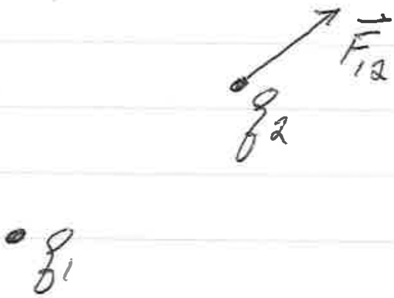
② No connecting lines from two +q charges

③ more lines from ⊕'s than to ⊖ charge



E-field due to a Point Charge

Consider again the 2 charge example:



Coulomb's Law gives

$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

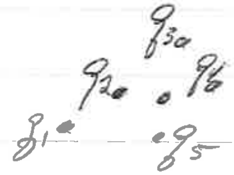
Taking q_2 as our test charge, the field due to q_1 is

$$\vec{E}_1 = \lim_{q_2 \to 0} \vec{F}_{12}/q_2 = \boxed{k\frac{q_1}{r^2}\hat{r}}$$

~~no depend~~ ence on ~~test~~ charge

$\vec{r}$ is vector to any point in region of interest

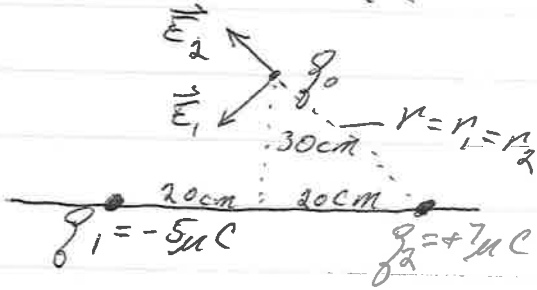
For multiple charges



$$\vec{E}_{TOT} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \ldots$$
$$= \sum \vec{E}_i$$

Example:

Calculate the field of two charges $-5\mu C$ and $+7\mu C$ separated by $0.4m$, evaluated at a point $30\,cm$ above the halfway point between them.



$$|E_1| = k q_1 / r_1^2 = 9\times10^9 \frac{Nm^2}{C^2}\left[\frac{|-5\mu C|}{(0.2m)^2 + (0.3m)^2}\right]$$

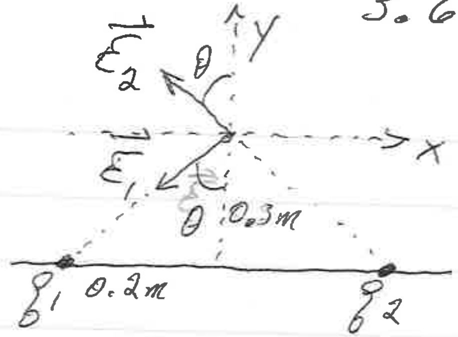
$$= 9\times10^9 \frac{N}{C}\left[\frac{5\times10^{-6}}{0.13}\right]$$

$$= \boxed{3.5\times10^5\,N/C}$$

$$|E_2| = k q_2 / r_2^2 = 9\times10 \frac{Nm^2}{C^2}\left[\frac{7\mu C}{0.13m^2}\right]$$

$$= \boxed{4.8\times10^5\,N/C}$$

Example: (cont.)



Components of $\vec{E}_1$ & $\vec{E}_2$:

$$\vec{E}_1 = -|E_1|\sin\theta\,\hat{i} - |E_1|\cos\theta\,\hat{j}$$
$$\vec{E}_2 = -|E_2|\sin\theta\,\hat{i} + |E_2|\cos\theta\,\hat{j}$$

$$\sin\theta = \frac{0.2\,m}{\sqrt{(0.2m)^2 + (0.3m)^2}} = \underline{0.55}$$

$$\cos\theta = \frac{0.3m}{\sqrt{0.13m^2}} = \underline{0.83}$$

$$\vec{E} = -(|E_1| + |E_2|)\sin\theta\,\hat{i} + (|E_2| - |E_1|)\cos\theta\,\hat{j}$$

$$= -(3.5\times10^5 + 4.8\times10^5)\,0.55\,\hat{i}$$
$$+ (4.8\times10^5 - 3.5\times10^5)\,0.83\,\hat{j}$$

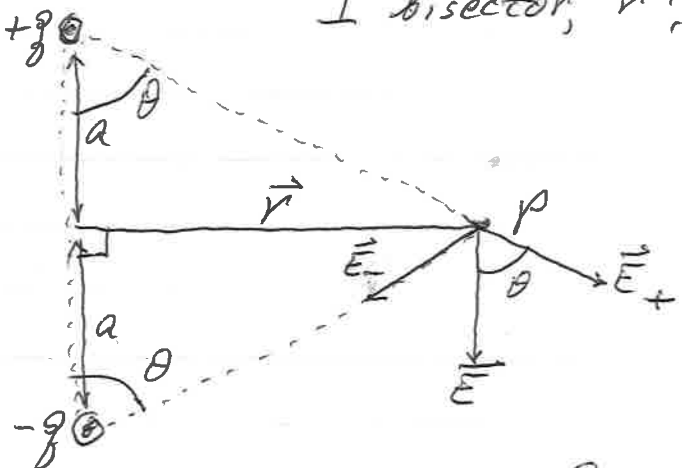
$$\boxed{\vec{E} = -4.6\times10^5\,\frac{N}{C}\,\hat{i} + 1.2\times10^5\,\frac{N}{C}\,\hat{j}}$$

Electric Dipole Field

Dipole is two equal, opposite sign charges a distance apart.

What is electric field @ P along ⊥ bisector, r?

$|r| \gg a$



$\vec{E} = \vec{E}_+ + \vec{E}_-$ $\qquad |E_+| = |E_-| = k\frac{q}{a^2+r^2}$

$\vec{E}$ points down with magnitude

horizontal components cancel

$$|E| = 2|E_+|$$
$$= 2\left(k\frac{q}{a^2+r^2}\right)\left[\frac{a}{\sqrt{a^2+r^2}}\right]$$
$$= 2kaq/[a^2+r^2]^{3/2}$$

Since $a \ll r$, $\boxed{E = 2kaq/r^3}$

So $E(r) \propto 1/r^3$, not $1/r^2$, because 2 charges' fields increasingly with distance.

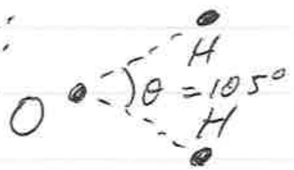
Electric Dipole Moment

In $E = 2kaq/r^3$, term '$2aq$' is termed 'electric dipole moment'.

- reflects that seperation & charge can't be measured seperately in many cases.

Key to many chemical bonds & properties of some liquids:

H_2O:



H more negative charge

O positive charge

→ A strong dipole moment to H_2O acts on molecules dissolved.

If H_2O generates an E field, why don't we notice it?

Molecules enormous & randomly aligned. The $\vec{E}$ fields all cancel on average.

Example

Consider two opposite charges at a separation of $2\,\text{Å}$ (= angstrom $= 10^{-10}$ m). What is the distance to the halfway point between the charges if each charge has magnitude 1.6×10^{-19} C and the electric field at this distance is 3×10^{10} N/C.?

$E = 3 \times 10^{6}$ N/C

$2a = 2 \times 10^{-10}$ m

$q = 1.6 \times 10^{-19}$ C

$E = k\,aq/r^3$ $\left((a^2+r^2)^{3/2} \text{ if } a \sim r\right)$

We know E and a, q → invert to calculate → is $a \ll r$? Don't know

So start generally

$E = 2kaq / [a^2 + r^2]^{3/2}$

$3 \times 10^6 \, N/C = 2 \times 9 \times 10^9 \, Nm^2/C^2 \, \frac{(10^{-10} m)(1.6 \times 10^{-19} C)}{[a^2 + r^2]^{3/2}}$

$[a^2 + r^2]^{3/2} = 6 \times 10^3 m^3 \times 1.6 \times 10^{-29}$

$[a^2 + r^2]^{3/2} = 9.6 \times 10^{-26} m^3$

- squaring both sides & cube-rooting

$a^2 + r^2 = 2.1 \times 10^{-17} m^2$

$r^2 = 2.1 \times 10^{-17} m^2 - 10^{-20} m^2$

$\approx 2.1 \times 10^{-17} m^2$

$\therefore a \ll r$

$\boxed{r = 4.6 \times 10^{-9} m}$