

Current & Resistance

12.1

For some materials, the current density \vec{J} is proportional + \parallel to \vec{E}

Ohm's Law

$$\begin{aligned}\vec{J} &\propto \vec{E} \\ &= \underline{\underline{\sigma}} \vec{E} = \underline{\underline{\frac{1}{\rho}}} \vec{E}\end{aligned}$$

where the constant of proportionality σ , $\frac{1}{\rho}$

- σ = "conductivity"

- ρ = "resistivity"

Ohm's Law

12.2

Generally, the expression $\vec{J} = \sigma \vec{E}$ is not directly usable in a circuit.

- we've seen this yield

$$V = I \frac{\rho l}{A}$$

We define 'resistance' $R = \frac{\rho l}{A}$, to get

$$\boxed{V = IR}$$

common form
of Ohm's Law

Resistance is a quantity of a specific device: a 'resistor'

- decreasing (increasing) length (area) reduces R

- reducing radius of cylindrical wire by 2x

- increases R by 4x

Units:

$$1 \text{ ohm } (\Omega) = 1 \frac{\text{Volt}}{\text{ampere}} \left(\frac{V}{A} \right)$$

Example:

12.3

A resistor is a small cylinder of carbon, 0.5cm long and 1mm wide. What is the resistance of this resistor?

$$R = \rho l / A$$

$$\rho_{\text{carbon}} = 3.6 \times 10^{-5} \Omega \cdot \text{m}$$

$$R = \frac{(3.6 \times 10^{-5} \Omega \cdot \text{m})(5 \times 10^{-3} \text{ m})}{\pi (10^{-3} \text{ m})^2}$$

$$= \frac{18 \times 10^{-8} \Omega}{3.1 \times 10^{-6}}$$

$$= \boxed{5.7 \times 10^{-2} \Omega}$$

A battery puts a 120 V potential across this resistor. What is the current in the circuit?

$$V = IR$$

$$I = V/R = 120 \text{ V} / 5.7 \times 10^{-2} \Omega$$

$$= \boxed{2.1 \times 10^3 \text{ A}} !!!$$

this a lot of current,
due to low R

Measuring Temperature

12.33

Use change in R

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

$$\therefore R = R_0 [1 + \alpha(T - T_0)]$$

So $\Delta R (= R - R_0) = \alpha R_0 \Delta T$

$$\boxed{\frac{\Delta R}{\alpha R_0} = \Delta T}$$

Example

Carbon has $\alpha = 5 \times 10^{-3}$. What ΔT if a 100Ω resistor changes by 0.1Ω ?

$$\frac{\Delta R}{\alpha R} = \frac{0.1 \Omega}{(5 \times 10^{-3})(100 \Omega)} = \Delta T$$

$$= \frac{1}{0.5} = \boxed{2 \text{ K} = \Delta T}$$

Power in Electric Circuits

12.35

As work done on charges in a resistive material

- power associated with change in potential energy

$$du = dqV = (i dt) V$$

$$\therefore \frac{du}{dt} = \boxed{p = iV}$$

Power delivered to circuit,
- a general expression, not just for Ohm's Law case

Units:

$$\begin{aligned} 1 \text{ watt (W)} &= 1 \text{ J/s} \\ &= \underline{\underline{1 \text{ V} \cdot \text{A}}} \end{aligned}$$

Power in Electric Circuits

12.4

Resistance is somewhat like friction

→ energy of electrons moving

- some goes into collisions
+ interactions with
atoms

→ vibrations + other
losses of energy

- losses manifest as heat

Using Ohm's Law, we find

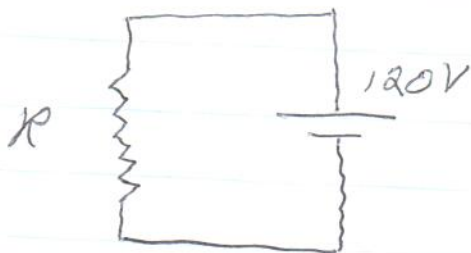
$$\begin{aligned} P &= IV = I(IR) = \left(\frac{V}{R}\right)V \\ &= \boxed{I^2 R} \\ &= \boxed{V^2 / R} \end{aligned}$$

Power lost as heat by
resistors.

Example

12.5

A resistor is connected to a wall outlet.



What is Power lost to heat

for (a) $100\text{m}\Omega$ and (b) $1\text{k}\Omega$ resistors?

$$R_a: P_a = \frac{V^2}{R} = \frac{1.44 \times 10^4 \text{V}^2}{10^{-1} \Omega} \\ = \boxed{1.4 \times 10^5 \text{W}}$$

$$R_b: P_b = \frac{V^2}{R} = \frac{1.4 \times 10^4 \text{V}^2}{10^3 \Omega} \\ = \boxed{14 \text{W}}$$

Other Behaviors

12/10

Not all materials obey Ohm's Law or $R(T)$ being linear.

Semiconductors:

- can have R decrease with increase in T

Superconductors:

- resistance drops to 0 below critical T_c
- many T_c near absolute 0
- some $T_c \sim 100\text{K}$ or warmer
- search on for 'room-temperature T_c '
- importance: no resistance means no heat from current
 - : - no energy losses