# Electromotive Force

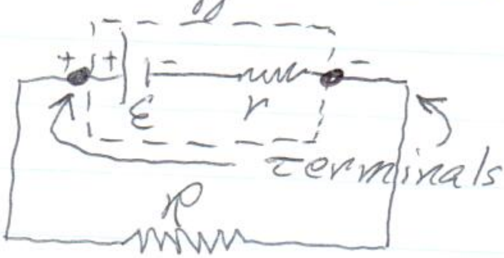


We saw that a battery delivers power to a resistive circuit in the work it does to move charge.

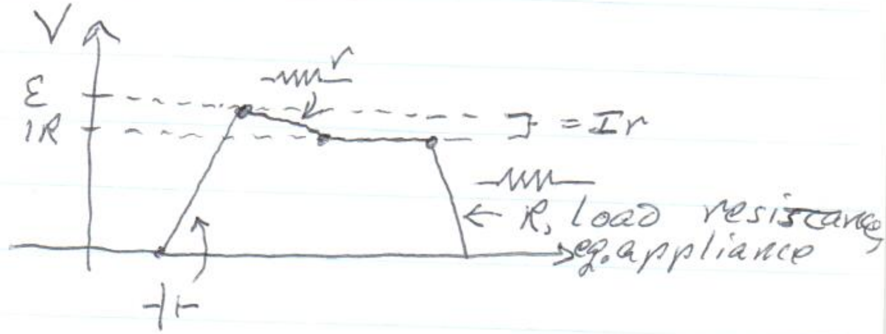
→ total potential difference in battery, "emf"


- not available to outside circuit
- battery has some internal resistance, r

- we need to adjust to determine potential difference across terminals



Consider voltage changes thru circuit




If $r=0$, terminal voltage $= \mathcal{E}$

- $\mathcal{E}$ is maximum possible voltage, $\Delta V$, battery can provide between its terminals

In general

$$\underbrace{\Delta V}_{\text{terminal voltage}} = \overbrace{\mathcal{E}}^{\text{open circuit voltage}} - Ir$$

$$\Delta V = IR = \mathcal{E} - Ir$$

$$\mathcal{E} = I(R+r)$$

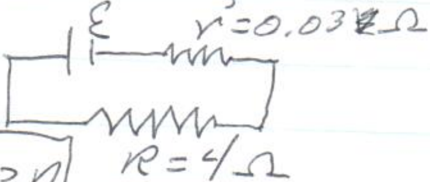
$$\boxed{I = \mathcal{E}/(R+r)}$$

↳ If $R \gg r$, can ignore $r$.

# Example:

Consider battery with 9V emf, + internal resistance of 0.03Ω, placed across a small light having 4Ω resistance.

a) Calculate the current, ΔV,




$$I = \frac{\mathcal{E}}{R+r}$$

$$= \frac{9V}{4.03\,\Omega} = \boxed{2.23A}$$

$$\Delta V = IR = (2.23A)(4\,\Omega)$$

$$= \boxed{8.93V}$$

b) calculate power delivered to load resistor

$$\underline{P} = I^2 R = (2.23A)^2(4\,\Omega)$$

$$= \boxed{19.9\,W}$$

Batteries age → their internal resistance increases.

Consider prior example, but $r = 1\,\Omega$ now:

$$\underline{\underline{I}} = \frac{\mathcal{E}}{R+r} = \frac{9V}{4\Omega + 1\Omega} = \boxed{1.8A}$$

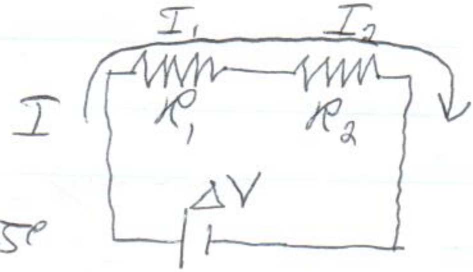
was 2.2A

$$\underline{\underline{\Delta V}} = \mathcal{E} - Ir = 9V - (1.8A)(1\Omega)$$
$$= \boxed{7.2V}$$

was 8.9V

# Resistors in Series

Consider two resistors in series




$I_1 = I_2$

- same charge passes thru both resistors

Potential drop $\Delta V$ divided between resistors

$$\Delta V = IR_1 + IR_2$$
$$= I(R_1 + R_2) = I R_{eff}$$

effective resistance

$R_{eff} = R_1 + R_2$

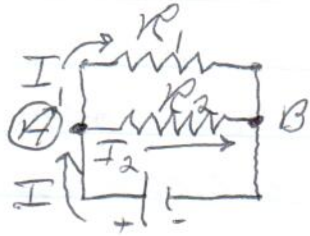
In general

$$\boxed{R_{eff} = R_1 + R_2 + R_3 + \ldots}$$

when all in series

# Resistors in Parallel

Two resistors in parallel.




- 2 conditions hold

  - at junction A → current splits to go thru 2 resistors

$$\underline{I = I_1 + I_2} \quad \left\{\begin{matrix}\text{conservation}\\ \text{of charge}\end{matrix}\right\}$$

  - also, same potential difference from A → B

$$\underline{\Delta V = \Delta V_1 = \Delta V_2}$$

So

$$I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

$$= \Delta V \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \Delta V R_{eff}$$

In general,

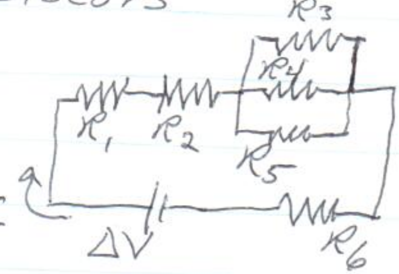
$$\underline{\frac{1}{R_{eff}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

$$= \boxed{\sum_i \frac{1}{R_i}}$$

($R_{eff}$ < minimum of $R_i$)

# Example

A circuit includes a complex array of resistors




$\Delta V = 120V$

$R_1 = 5\Omega$

$R_2 = R_6 = 8\Omega$

$R_3 = R_4 = 6\Omega$ ; $R_5 = 3\Omega$

Calculate the currents.

a)
$$R_{eff} = R_1 + R_2 + R_6 + \left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right]^{-1}$$
$$= 21\Omega + \left[\frac{2}{6\Omega} + \frac{1}{3\Omega}\right]^{-1}$$
$$= 21\Omega + \frac{3}{2}\Omega = \boxed{22.5\Omega}$$

$$I = \Delta V / R_{eff} = 120V / 22.5\Omega = \boxed{5.33A}$$
$$\boxed{= \boxed{I_1 = I_2 = I_6}}$$

For $I_3$ & $I_4$ & $I_5$ $\Delta V_3 = \Delta V_4 = \Delta V_5$

1) $(6\Omega)I_3 = (6\Omega)I_4 = (3\Omega)I_5$

$$I_3 = I_4 \text{ ; } I_5 = 2I_4$$

$$I = 5.33A = I_3 + I_3 + 2I_3 = 4I_3$$

$$\boxed{\boxed{I_3 = 1.33A} = I_4}$$

$$\boxed{I_5 = 2.67A}$$

## Loop theorem:



algebraic sum of changes in potential encountered in a complete circuit loop must be zero.


equi-potential + assume conservative field in wire

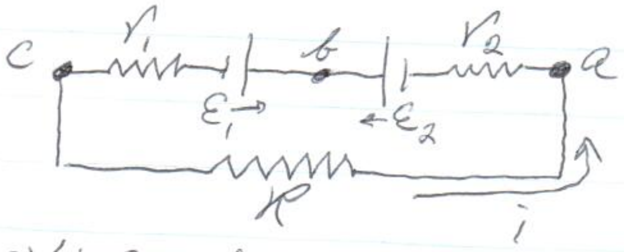
$$-iR + \mathcal{E} = 0$$

- equivalent to conservation of energy

For a resistor traversed in current direction → $\Delta V = -IR$
- equals $+IR$ in opposite direction

When seat of emf traversed in emf direction (ie. ⊖ to ⊕)
$\Delta V = +\mathcal{E}$ ($-\mathcal{E}$ if opposite direction)

## Example




$\mathcal{E}_1 = 2.0V$; $\mathcal{E}_2 = 4.0V$
$r_1 = 1.0\Omega$; $r_2 = 2.0\Omega$; $R = 5.0\Omega$

What is the current $i$?
Since 2 emfs and $\mathcal{E}_2 > \mathcal{E}_1$,
- $i$ will flow counterclockwise
  Note: don't actually need to know as we'll see

Using loop theorem:

$$-\mathcal{E}_2 + ir_2 + iR + ir_1 + \mathcal{E}_1 = 0$$

Note signs!

$$i(r_2 + R + r_1) = \mathcal{E}_2 - \mathcal{E}_1$$

$$i = \frac{(\mathcal{E}_2 - \mathcal{E}_1)}{(r_2 + R + r_1)} = \frac{2V}{8\Omega}$$

$$= \boxed{+0.25A}$$

The + sign means our choice of direction for "$i$" is correct.

Let's assume current going clockwise:

loop rule → $-\mathcal{E}_2 - ir_2 - iR - ir_1 + \mathcal{E}_1 = 0$

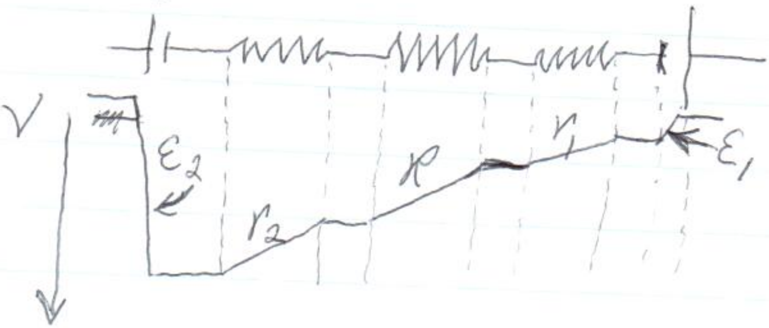
$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R}$$

$$= \boxed{-0.25A}$$

i.e. current flows 'counter' clockwise as saw in previous page

Potential Differences

Going from (b) to (a)




$$-ir_2 + \mathcal{E}_2 = -(0.25A)(2\Omega) + 4\Omega$$

$$= +3.5V$$