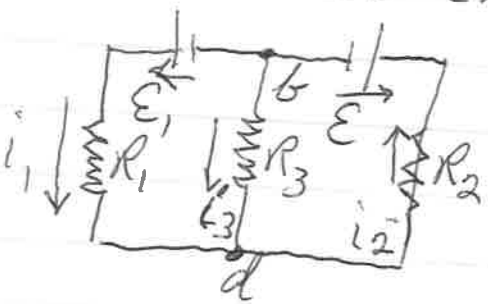
Multiloop Circuits

In a multiloop circuit, current thru resistors may not be the same



→ 2 junctions
b + d

At each junction
- charge flows 'out' on some wires, and 'in' on others

@ b: i_2 "in" while i_3 "out" + i_1

$\therefore\; i_1 + i_3 = i_2$ {conservation of charge}

-or-

$i_1 - i_2 + i_3 = \boxed{\sum i_i = 0}$

Junction theorem:
"At any junction, the algebraic sum of currents must be zero."

How is this useful?
- we want to solve for relationship of currents, $\mathcal{E}_i$ and R_i

<u>left loop, counterclockwise!</u>

$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$$

<u>right loop, clockwise!</u>

need to have consistent direction thru R_3

$$-i_3 R_3 - i R_2 - \mathcal{E}_2 = 0$$

Using the junction rule,

$$i_3 + i_1 - i_2 = 0$$

We can solve for i_1, i_2 and i_3.

14.3

$$i_1 = \frac{\mathcal{E}_1 (R_2 + R_3) - \mathcal{E}_2 R}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$i_2 = \frac{\mathcal{E}_1 R_3 - \mathcal{E}_2 (R_1 + R_3)}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

d

$$i_3 = - \frac{(\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1)}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

means i_3 goes 'up', opposite our choice

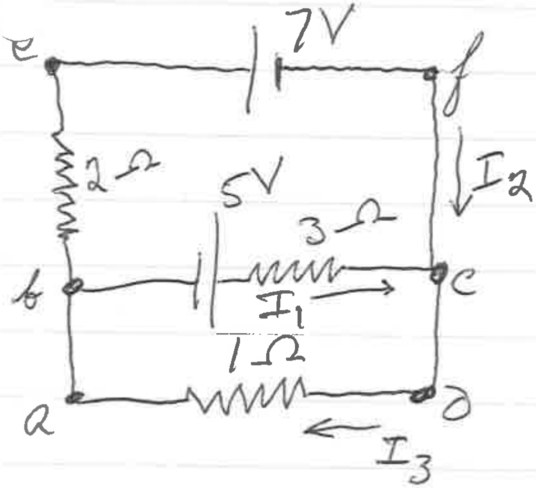
Loop & Junction Rules 14.4

of times can use junction rule = one less than # of junctions
(=1 in prior example)

loops depends on finding new loops that have new unknown currents in each new equation
(=2 for prior example since 1 loop has 2 new currents & 2nd loop adds 3rd current)

The # of independent equations = # of unknown currents

Example



Consider ←

Cannot reduce to R's in series or parallel.

Write down constraint Eq's

(1) @ c) $I_1 + I_2 = I_3$

(2) for abcda) $5V - 3\Omega I_1 - 1\Omega I_3 = 0$

(3) for befcb) $-7V + 3\Omega I_1 - 5V - 2\Omega I_2 = 0$

$-12V = -3\Omega I_1 + 2\Omega I_2$

Using (1) in (2)

(4) $5V = 4\Omega I_1 + 1\Omega I_2$

Taking 2×(4) − (3) yields $\boxed{I_1 = 2A}$

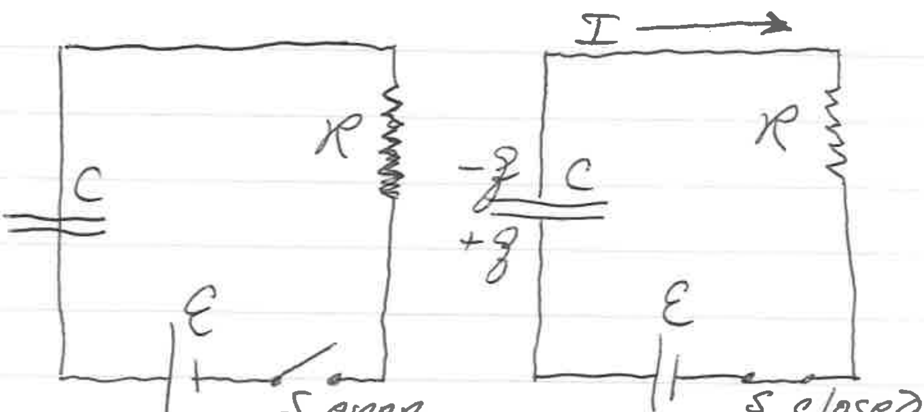
in (3) get: $\boxed{I_2 = -3.0A}$

in (1) get: $\boxed{I_3 = -1A}$

RC Circuits

Consider circuits containing R and C in series

- consider *dynamic* time when capacitor charging



- $\mathcal{E}$ sets up $\vec{E}$ in wires
  - charge exchanged between plates and connecting wires
  - when capacitor fully charged →
    - max. charge depends on voltage

→ $I=0$

→ no more current

Consider case when switch 's' is closed,

$$\mathcal{E} - \frac{q}{C} - IR = 0$$

$\to q(t)$ is instantaneous value of charge on capacitor

@ $t = 0$ $\quad I_0 = \frac{\mathcal{E}}{R}$

→ I is a maximum

→ no $\vec{E}$ in capacitor yet

→ ΔV entirely felt across resistor

When charged fully

→ $I_0 = 0$

- $Q = C\mathcal{E}$ (maximum charge)

- ΔV all across capacitor

Time-Dependend Charge

We want analytic expression

use $I = dq/dt$

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{(q - C\mathcal{E})}{RC}$$

$$\frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\,dt$$

Integrating + using $q = 0$ @ $t = 0$

$$\int_0^q \frac{dq}{(q - C\mathcal{E})} = -\frac{1}{RC}\int_0^t dt$$

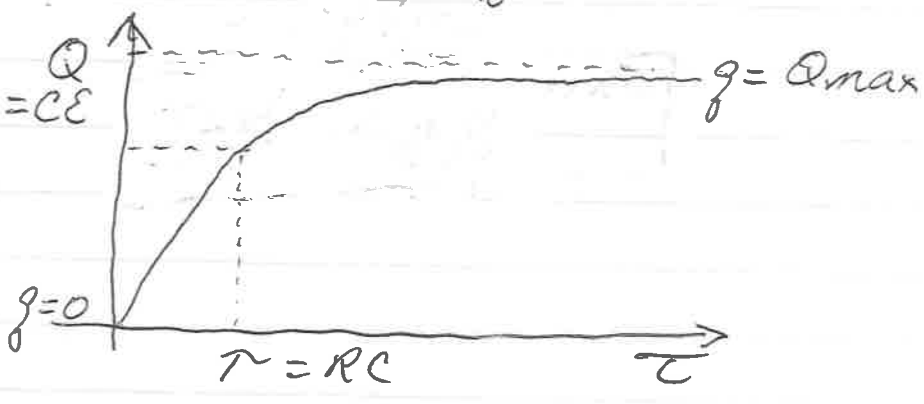
$$\ln\left(\frac{q - \mathcal{E}C}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

$$-(q - \mathcal{E}C)/(\mathcal{E}C) = e^{-t/RC}$$

$$-q/\mathcal{E}C = -1 + e^{-t/RC}$$

charge for charging capacitor

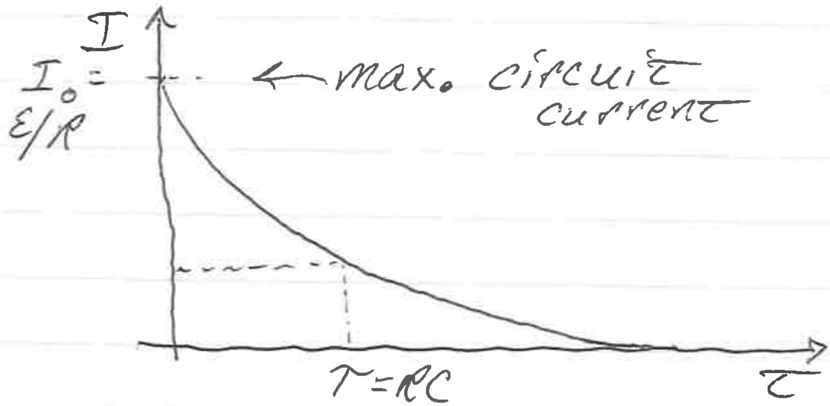
$$\boxed{q(t) = \underbrace{\mathcal{E}C}_{Q_0}\left(1 - e^{-t/RC}\right)}$$



Current $= dq/dt$

$$\underline{\underline{I}} = \frac{dq}{dt} = \frac{\mathcal{E}}{R}\left(e^{-t/RC}\right)$$

$$= \boxed{I_0\, e^{-t/\tau}}$$

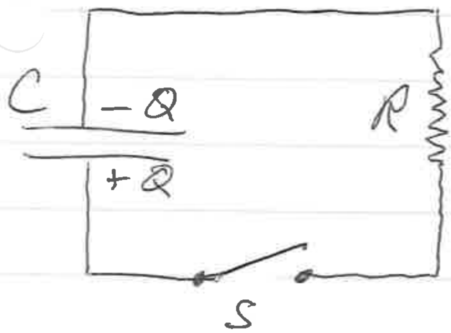


Time constant, $\tau = RC$

- time by which I goes to $1/e$ of initial I_0
  ↳ $\underline{\underline{= 0.368\, I_0}}$

- in 2τ time,
  $\underline{I} = e^{-2} I_0 = \underline{\underline{0.135\, I_0}}$

Discharging a Capacitor



How do Q, I vary?

$$-\frac{q}{C} - IR = 0$$
(loop eq.)

Using $I = dq/dt$

$$-R\frac{dq}{dt} = \frac{q}{C}$$

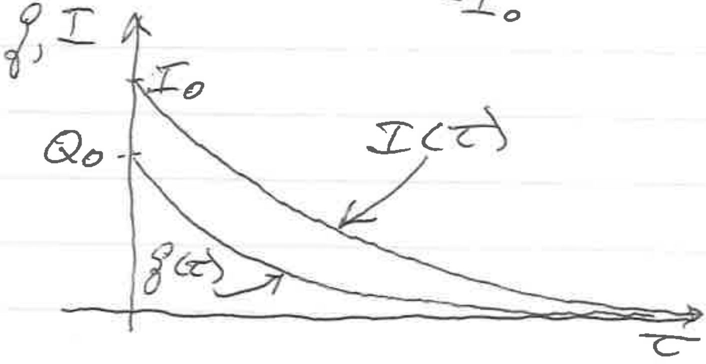
$$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -t/RC$$

therefore $\boxed{q(t) = Q_0\, e^{-t/\tau}}$

$$\boxed{I = \frac{dq}{dt} = -\underbrace{\frac{Q}{RC}}_{I_0}\, e^{-t/RC}}$$



Example:

a circuit with a charged capacitor and a resister in series is open. If $R = 3\Omega$ and $C = 10nF$, how long will it take for the charge to reach $\frac{1}{4}$ of it's initial value?

$$q(t) = Q_0 e^{-t/RC}$$

$$\frac{Q_0}{4} = Q_0 e^{-t/RC}$$

$$\ln\left(\frac{1}{4}\right) = \ln\left(e^{-t/RC}\right)$$

$$+\ln 4 = +t/RC$$

$$t = RC \ln 4 = 1.39\,\tau$$

$$= 1.39(3\Omega)(10 \times 10^{-12} F)$$

$$= 4.2 \times 10^{-8}\ \text{sec}$$

$$\boxed{t = 42\,\text{nsec}}\ !!$$