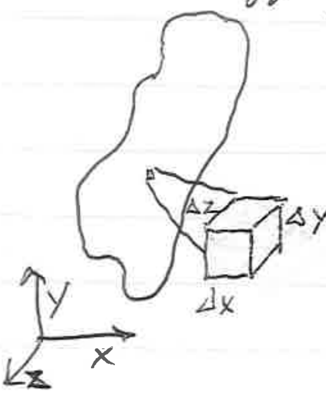
Electric Field of a Continuous Charge Distribution

Continuous charge distributions
- effectively result when have huge # of charges



Each small region with $\Delta x \Delta y \Delta z$ volume
- holds Δq charge

Total $\vec{E}$-field by summing

$$\vec{E} \simeq k \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

As $\Delta q \to 0$,

$$\boxed{\vec{E} = k \int \frac{dq}{r^2} \hat{r}}$$

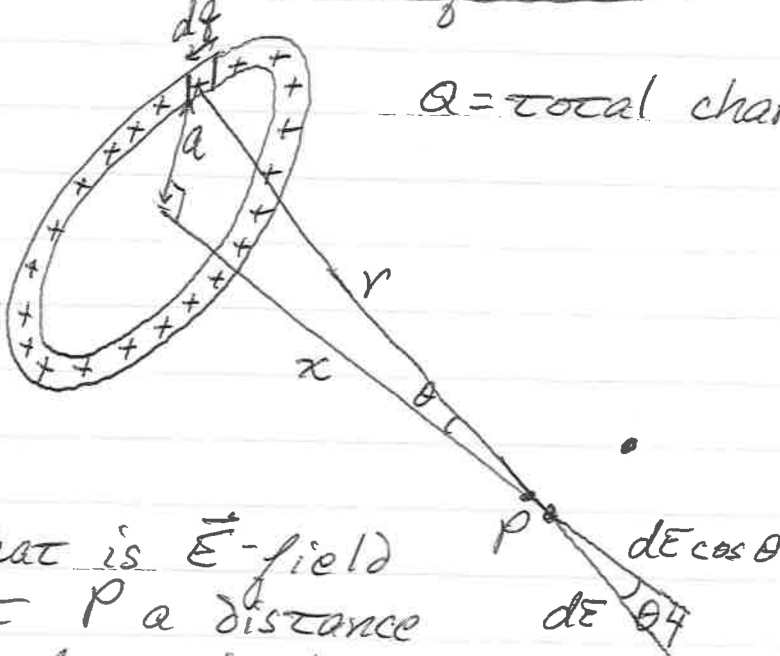
Charge Distributions

Charge is often distributed uniformly in a 1, 2 or 3 dimensional way

	charge density*	dq
line, l	$\lambda = Q/l$	$\lambda\, dl$
surface, A	$\sigma = Q/A$	$\sigma\, dA$
volume, V	$\rho = Q/V$	$\rho\, dV$

* if Q is distributed uniformly, Q = total charge

Electric field from a Ring of Charge



Q = total charge

What is $\vec{E}$-field at P a distance x from ring's center on its axis?

We need to sum $d\vec{E}$ vectorially for all elements dq!

- components $\perp$ to x cancel
- components $\parallel$ to x!

$$dE\cos\theta = k\frac{dq}{r^2}\cos\theta$$

$$= k\frac{dq}{(a^2+x^2)}\frac{x}{\sqrt{a^2+x^2}}$$

$$= k\,x\,dq\big/\left[a^2+x^2\right]^{3/2}$$

Normally, we would integrate over alinear charge density λ.

But we have a finite circumference with known ~~total~~ total charge.

Since x and a are constant fore all dq, we have

$$\underline{\underline{E}} = \int dE \cos\theta$$

$$= \int k \frac{x\, dq}{[a^2+x^2]^{3/2}} = \frac{kx}{[a^2+x^2]^{3/2}} \int dq$$

$dq \to Q$

$$= \boxed{k \frac{xQ}{[a^2+x^2]^{3/2}}}$$

Note: when $x >> a$

$$\underline{\underline{E \approx k \frac{Q}{x^2}}}$$

$\therefore$ acts like a point charge.

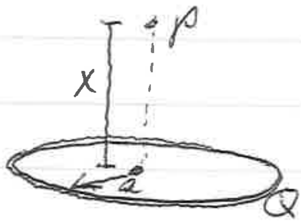
Example

A ring of charge lies flat on a table with total charge $Q = 0.1\,\mu C$ and radius $0.1\,m$. What is the electric field $1\,m$ from the ring?

Is $a \ll x$?

Not sure:

So use more general expression



$a = 0.1\,m$

$$\underline{\underline{E}} = k\frac{xQ}{[a^2+x^2]^{3/2}}$$

$$= 9\times10^{\cancel{9}\,3}\,\frac{N\cancel{m^2}}{\cancel{C^2}}\;\frac{(1\cancel{m}\cdot 0.1\cancel{\mu C})}{[1.01\,\cancel{m^2}]^{3/2}}$$

1.015

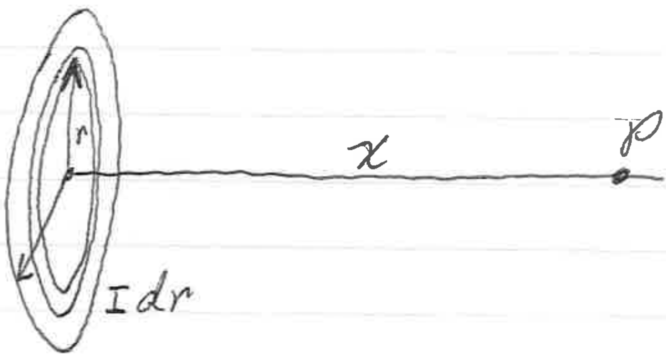
$$= 9\times10^3\,\frac{N}{C}\Big/1.015$$

$$= \boxed{8.91\times10^3\,\frac{N}{C}}$$

So using 'a' in denominator only makes 1.5% difference

Electric Field from a Charged Disk

We can consider a charged disk as a series of concentric charged rings.



We know the field for each ring

$$dE = k\frac{x\,dq}{[r^2+x^2]^{3/2}}$$

To determine the E-field from the disk

- integrate from $r=0$ to R

Using ~~an~~ uniform charge ~~den~~ per area, σ, we can write

$$dq = \sigma\, dA = \sigma(2\pi r)(dr)$$

σ per ring

Setting up the integral

$$E = \int k \frac{x\,(\sigma(2\pi r)dr)}{[r^2+x^2]^{3/2}}$$

$$= \frac{\sigma(\cancel{2\pi})}{4\cancel{\pi}\epsilon_0} x \int_0^R \frac{2r\,dr}{[r^2+x^2]^{3/2}}$$

$$= \frac{\sigma x}{4\epsilon_0}\left[\frac{(r^2+x^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R$$

$$= \frac{-\sigma x}{\cancel{4}\epsilon_0}\left[\frac{+1}{\sqrt{R^2+x^2}} - \frac{1}{x}\right]$$

$$\boxed{= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]}$$

If we go to very large distance $R \ggg x$

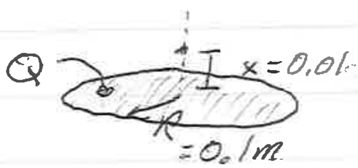
$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

E-field from infinite sheet

Example

Calculate the electric field from a uniformly charged disk a distance along the disk axis of 1cm. The disk has $Q = 5\mu C$ and radius 0.1m.

$x << R$



Calculate charge density

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

$$= \frac{5\times10^{-6}C}{\pi(0.01m^2)}$$

$$\underline{\underline{\sigma = 1.5\times10^{-4}\frac{C}{m^2}}}$$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \underbrace{\frac{x}{\sqrt{R^2+x^2}}}_{\text{need to check}}\right]$$

$$= \frac{1.5\times10^{\cancel{-4}\,+5}\,C/m^2}{2(8.85\times10^{-12}\,C^2/Nm^2)}\left[1 - \frac{0.01m}{\sqrt{0.0101m^2}}\right]$$

$$= 8.5\times10^3\frac{N}{C}\left[1 - \underbrace{0.0995}_{}\right]$$

so edge effects 10%!!

$$\underline{\underline{E = \boxed{7.7\times10^3\frac{N}{C}}}}$$