Placing Charges in an Electric Field

We've considered electric field resulting from charge or a charge distribution.

Now we want to understand what happens to charges when they are placed in a pre-existing "external electric field", E.

For a single charged particle, q,

$$\boxed{\vec{F} = q\vec{E}}$$

this is not the field from q

Charged Particle Motion in Electric Field

Since force is exerted on q, there is acceleration

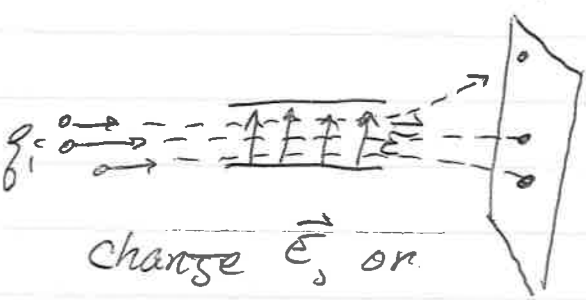
$$\vec{F} = q\vec{E} = m\vec{a}$$

$$\boxed{\vec{a} = \frac{q}{m}\vec{E}}$$

- when $q > 0$ $\quad \vec{a} \parallel \vec{E}$
- when $q < 0$ $\quad \vec{a} \uparrow\downarrow \vec{E}$

anti-parallel

So by passing charge thru a field, it will change motion

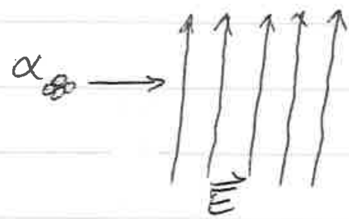


Ink-Jet Printer

change $\vec{E}$, or q_i + aim ink droplets

Example:

An alpha ($\alpha = 2p + 2n$) travels horizontally from a decaying radioactive nucleus. To prevent it it from contacting living tissue in a sample, a constant, uniform field of 1N/C is directed upward. What acceleration does the proton feel? Give magnitude & direction.



$m_\alpha = 2m_p + 2m_n \approx 37 \times 10^{-27}$ kg

$$\vec{E} = 1\frac{N}{C}\hat{j}$$

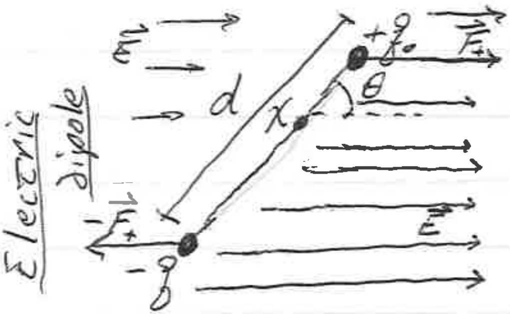
$$\vec{a} = \frac{q}{m}\vec{E}$$

$$= \frac{2(1.6\times10^{-19}\,\cancel{C})}{3.7\times10^{-26}\,\cancel{kg}}\left(1\,\frac{\cancel{kg}\,m/s^2}{\cancel{C}}\right)\hat{j}$$

$$\boxed{\vec{a} = 8.5\times10^{6}\,m/s^2\,\hat{j}}$$

Electric Dipole in an Electric Field

When we have a uniform E-field



electric dipole moment, $p_d = qd$

- No net force since $\sum q_i = 0$
- Net torque
  - nonzero force at ends
  - equal & opposite directions

Consider center-of-mass (ie. center-of-rotation) at position x along dipole axis.

$$\tau_+ = F_+(d-x)\sin\theta$$

$$\tau_- = -F_+(-x)\sin\theta = F_+ x \sin\theta$$

$$\tau = \tau_+ + \tau_- = \underline{Fd\sin\theta}$$

↳ no need to know dipole structure details

Since dipole moment $p_d = qd$
and force on each charge
is $F = qE$

We have

$$\tau = qE(p_d/q)\sin\theta$$
$$\boxed{= p_d E \sin\theta}$$

Generalizing + in vector form:

$$\boxed{\vec{\tau} = \vec{p}_d \times \vec{E}}$$

↳ zero when $\vec{p}_d \parallel \vec{E}$

<u>Microwave Oven</u>:

Applied E-field causes H_2O to align with field. Switching field rapidly causes molecules to keep switching, thereby breaking groups and eventally thermal energy (heat) that cooks food.

Flux

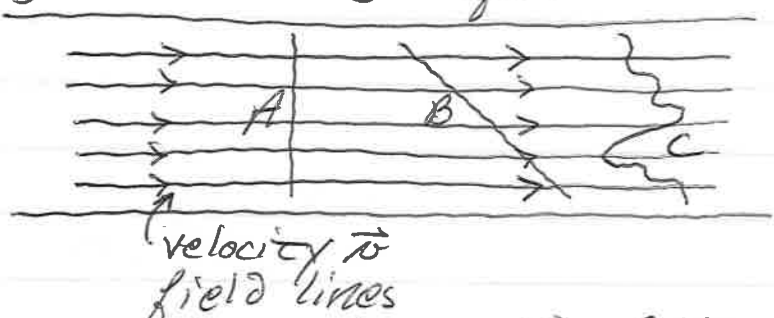
flux: A property of all vector fields. [fr. L. fluere = to flow]

- related essentially to amount of field passing thru an area, A

  $\propto$ # field lines

Let's go back to case of water flowing thru a pipe:
↳ a velocity vector field

3 intersecting surfaces:



A → flat, $\perp$ to $\vec{v}$, area A

B ⇒ flat, angle to $\vec{v}$, area A

C ⇒ irregular shaped surface

Which sees more flux?

For us, this is equivalent to 'which surface intersects the most field lines?'

- In all 3, five lines are crossed

The # field lines thru surface the same if density, ρ, uniform

We can speak of the "mass flux" thru these areas

For A + B:

$$\underline{\phi = \rho \vec{v} \cdot \vec{A}}$$

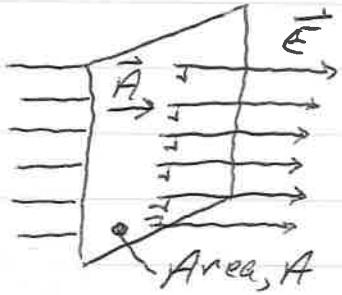
(a scalar)

The dot product means that regardless of orientation, only the part of $\vec{A} \parallel \vec{v}$ is important.

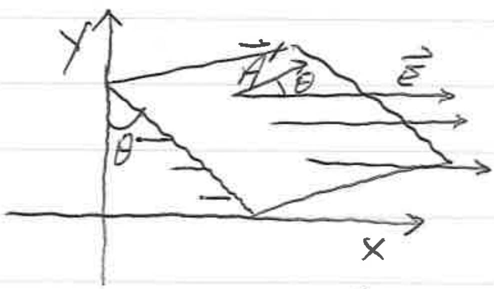
→ also true for C → same flux thru as A + B

Flux of an Electric Field

Electric field a vector field, like $\vec{v}$



$$\phi_E = \vec{E} \cdot \vec{A} = EA$$



$$\phi_E = \vec{E} \cdot \vec{A}' = EA' \cos\theta$$

If $A' = A/\cos\theta$ (i.e. length in y same)

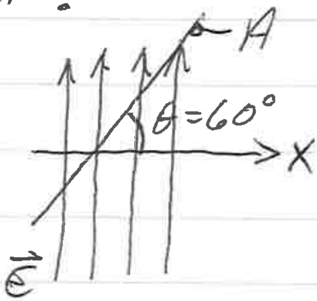
$$\phi_E = EA = \phi_{E'} = E(A/\cos\theta)\cos\theta = EA$$

If $\vec{A}$ + $\vec{E}$ are $\parallel$ → when $\theta = 0$
- flux is maximum

If $\vec{A}$ + E are $\perp$ → when $\theta = 90°$
- flux = 0

Example:

Consider a constant $\vec{E}$-field directed upward. How much flux would pass thru a surface with area $0.5\,m^2$ and oriented $60°$ to the horizontal?



$$\Phi_E = \vec{E} \cdot \vec{A}$$

$$\Phi_E = 3\,\frac{N}{C}\,\hat{j} \cdot \left(\cancel{0.5m^2 \cos 60\,\hat{i}}^{\,0} + 0.5m^2 \sin 60\,\hat{j}\right)$$

$\hat{i} \cdot \hat{j} = 0$

$$= 1.5\,\frac{Nm^2}{C} \sin 60°$$

$$\boxed{\Phi_E = 1.3\,\frac{Nm^2}{C}}$$

Flux from Complex Surfaces

$\Delta\vec{A}$ is $\perp$ surface locally

$\Delta\phi_{E_i} = \vec{E}_i \cdot \Delta\vec{A}_i$

$= E_i \, \Delta A_i \times \cos\theta_i$

To get flux over a large part of surface

$$\phi_E = \sum_i \vec{E}_i \cdot \Delta\vec{A}_i$$

For a very complicated, or continuously varying surface, need to reduce ΔA_i

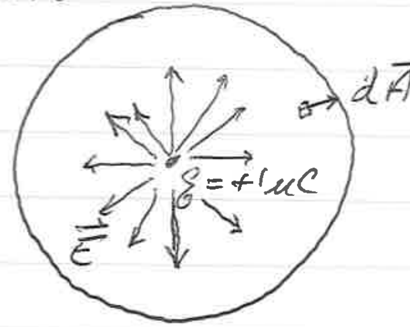
$$\phi_E = \lim_{\Delta A_i \to 0} \sum_i \vec{E} \cdot \Delta\vec{A}_i$$

$$= \boxed{\int_{\text{surface } A} \vec{E} \cdot d\vec{A}}$$

Closed Surfaces

A closed surface completely envelopes a volume
- consider a sphere

- around a point source



$E = k\dfrac{q}{r^2}$

$A = 4\pi r^2$ (sphere)

$$\underline{\underline{\Phi_E}} = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA$$

$$= EA = \frac{1}{\cancel{4\pi}\epsilon_0}\left(\frac{q}{\cancel{r^2}}\right)\left(\cancel{4\pi r^2}\right) = \boxed{\frac{q}{\epsilon_0}}$$

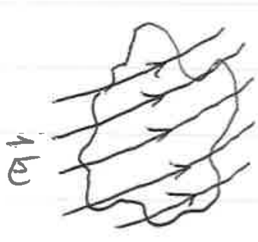
$= 1.1 \times 10^5\ \underline{\underline{Nm^2/C}}$

↳ units of flux

<u>Note!</u>

- if $q = 0$ (charge inside surface)

$\rightarrow \boxed{\phi = 0}$



#lines in = # lines out