Problem 2-45 (Cont.) Home Work #2

In Bob's frame,

He will wait for Anna to get to W planet

$$t = \frac{12\,yr \times \cancel{c}}{0.6\,\cancel{c}} = 20\,yr$$

and another 20 yr for her to come back

But, He will age by $\underline{\underline{40\,yrs}}$

Thus, $\Delta t = \gamma\,\Delta t'$

$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{40}{1.25} = 32\,yrs$$

Thus Anna will be aged by $\underline{\underline{32\,yrs}}$

At the meeting time:

Bob will be 60 yrs

Anna will be 52 yrs

Problem 2-51

Equation 2.17 reads $f_{obs} = f_{source}\dfrac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}\cos\theta}$

If the source is moving toward an observer, $\theta = 180°$

i.e. $\cos\theta = -1$

$$f_{obs} = f_{source}\frac{\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}} = \frac{\sqrt{\cancel{\left(1-\frac{v}{c}\right)}\left(1+\frac{v}{c}\right)}}{\sqrt{\cancel{\left(1-\frac{v}{c}\right)}\left(1-\frac{v}{c}\right)}}$$

$$\boxed{f_{obs} = f_{source}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}}$$

Problem 2-54

Since the observed frequency is larger, than the source, the galaxy must be moving toward Earth

$$f_{obs} = f_{source}\sqrt{\frac{1+v/c}{1-v/c}}$$

$$f = \frac{c}{\lambda}$$

$$\frac{f_{obs}}{f_{source}} = \sqrt{\frac{1+v/c}{1-v/c}} = \frac{c/\lambda_{obs}}{c/\lambda_{source}} = \sqrt{\frac{1+v/c}{1-v/c}}$$

$$\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 = \frac{1+v/c}{1-v/c}$$

$$\left(1-\frac{v}{c}\right)\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 = 1+\frac{v}{c}$$

$$\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 - \frac{v}{c}\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 = 1+\frac{v}{c}$$

$$\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 - 1 = \frac{v}{c}\left(1+\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2\right)$$

$$\boxed{\frac{v}{c} = \frac{\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 - 1}{\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 + 1}}$$

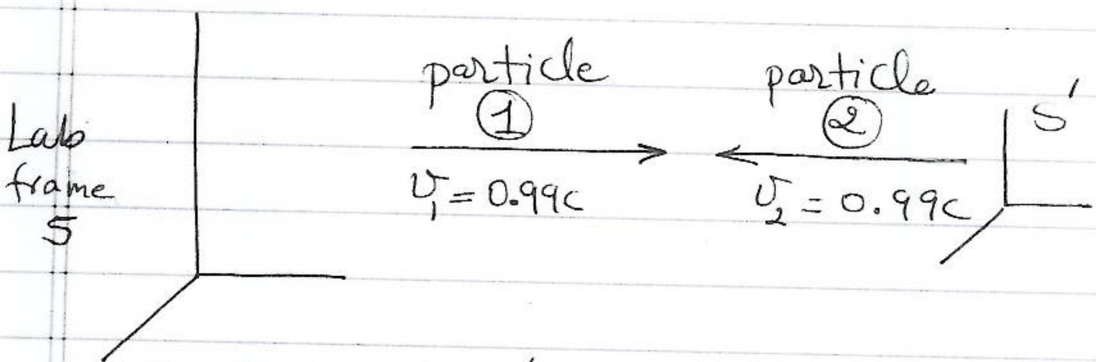
$$\left(\frac{\lambda_{source}}{\lambda_{obs}}\right)^2 = \left(\frac{396.85}{396.58}\right)^2 = 1.00136$$

~~wavele~~ wavelength shift is about one part in a thousand

$$\boxed{\frac{v}{c} = 0.00068}$$

The speed is roughly one-thousandth that of light

Problem 2-62 : Particle Collider



Particle 2 is S' moving at $v = 0.99c$ relative to the Lab and particle 1 is the object which moves at $u = -0.99c$ in Lab

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{-0.99c - 0.99c}{1-(-0.99)(0.99)}$$

$$\boxed{u' = -0.9995c}$$

Problem 2-70

Consider a proton moving with at $v = 0.8c$

a) Momentum of the proton is:

$$p = \gamma m_e u = \underbrace{\frac{1}{\sqrt{1-(0.8)^2}}}_{\gamma_{0.8c}} \underbrace{(1.67\ 10^{-27} \text{ Kg})}_{m_e} \left(0.8 \times 3 \times 10^8 \text{ m/s}\right)$$

$$\boxed{p = 6.68\ 10^{-19} \text{ Kg m/s}}$$

b) $$E = \gamma m_e c^2 = \frac{1}{\sqrt{1-(0.8)^2}} (1.67\ 10^{-27} \text{ Kg})\left(3 \times 10^8 \text{ m/s}\right)^2$$

$$\boxed{E = 2.51\ 10^{-10} \text{ J}}$$

Energy can also be expressed in eV (electron Volt) units.

$1\,eV = 1.6\ 10^{-19}\,J$ (1.6 10^{-19} is also the charge of an electron.)

Thus $E_p = 2.51\ 10^{-10}\,J = \frac{2.51\ 10^{-10}\,\cancel{J}}{1.6\ 10^{-19}\,\cancel{J}}\,eV$

$= 1568\ 10^{6}\ eV$

$\boxed{E_p = 1568\ MeV}$, $1\,MeV = 10^{6}\ eV$

(c) Kinetic Energy

$KE = T = (\gamma - 1)\, m_p c^2$

$= \left(\frac{1}{\sqrt{1-(0.8)^2}} - 1\right)\left(1.67\ 10^{-27}\,kg\right)\left(3\times10^{8}\ m/s\right)^2$

$= 1.00\ 10^{-10}\ J$

$= 6.25\ 10^{8}\ eV$

$\boxed{KE = T = 625\ 10^{6}\,eV = 625\ MeV}$

Problem - 2.84

$W = \Delta KE = KE_{final} - KE_{initial}$

$= (\gamma_{u_f} - 1)\, mc^2 - (\gamma_{u_i} - 1)\, mc^2$

$\boxed{W = \Delta KE = (\gamma_{u_f} - \gamma_{u_i})\, mc^2}$

a) The work required to accelerate an electron from 0.3c to 0.6c

$W = \left(\frac{1}{\sqrt{1-(0.6)^2}} - \frac{1}{\sqrt{1-(0.3)^2}}\right)\left(9.11\ 10^{-31}\,kg\right)\left(3\times10^{8}\,m/s\right)^2$

$$\boxed{W = 1.65\ 10^{-14}\ J}$$

in eV: $W = \dfrac{1.65\ 10^{-14}}{1.6\ 10^{-19}} = 103125\ eV = 0.103\ MeV$

$= 103.125\ keV$

$1\ MeV = 10^6\ eV$

$1\ keV = 10^3\ eV$

(b) from 0.6c to 0.9c

$$W = \left(\frac{1}{\sqrt{1-(0.9)^2}} - \frac{1}{\sqrt{1-(0.6)^2}}\right)(9.11\ 10^{-31}\ kg)\left(3 \times 10^8\ m/s\right)^2$$

$= 8.56\ 10^{-14}\ J$

$= 0.535\ MeV$

$$\boxed{W = 535\ keV}$$

Problem 2-94

Consider the decay of a kaon: $K^0 \longrightarrow \pi^+ + \pi^-$

$m_k = 8.87\ 10^{-28}\ kg$, $m_\pi = 2.49\ 10^{-28}\ kg$

Kaon moving in the positive x direction

(a) Speed of kaon before decay

- Use Conservation of Energy

$\pi^+: \gamma_{0.9c} = \dfrac{1}{\sqrt{1-(0.9)^2}}$

$\pi^-: \gamma_{0.8c} = \dfrac{1}{\sqrt{1-(0.8)^2}}$

$$E_{k^0} = E_{\pi^+} + E_{\pi^-}$$

$$\gamma_u m_k \cancel{c^2} = \gamma_{0.9} m_\pi \cancel{c^2} + \gamma_{0.8} m_\pi \cancel{c^2}$$

$$\gamma_u = \frac{(\gamma_{0.9} + \gamma_{0.8})\, m_\pi}{m_k} = \frac{(2.294 + 1.67)\, 2.49\ \cancel{10^{-28}}}{8.87\ \cancel{10^{-28}}}$$

$$\frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = 1.11278$$

$$\frac{1}{1-\left(\frac{u}{c}\right)^2} = 1.2382 \Rightarrow 1-\left(\frac{u}{c}\right)^2 = 0.8075$$

Thus, $\boxed{U = 0.437c}$

(b) The directions of the pions after the decay.

. Use conservation of Momentum.

$$\gamma_u m_k u = \gamma_{0.9} m_{\pi^+}(0.9c) + \gamma_{0.8} m_{\pi^-}(0.8c)$$

Now, we need to project this on x and y axis.

• Along the x axis.

$$\gamma_{0.437}(8.87\, \cancel{10^{-28}})(0.437c) = \gamma_{0.9}(2.49\, \cancel{10^{-28}})(0.9c)\cos\theta_1 + \gamma_{0.8}(2.49\, \cancel{10^{-28}})(0.8c)\cos\theta_2$$

Thus

(1) $$\boxed{\gamma_{0.437}(8.87)(0.437c) = \gamma_{0.9}(2.49)(0.9c)\cos\theta_1 + \gamma_{0.8}(2.49)(0.8c)\cos\theta_2}$$

Along the y-axis

(2) $$\boxed{0 = \gamma_{0.9}(2.49)(0.9c\sin\theta_1) - \gamma_{0.8}(2.49)(0.8c\sin\theta_2)}$$

$$\gamma_{0.437} = \frac{1}{\sqrt{1-(0.437)^2}} = 1.11178$$

$$\gamma_{0.9} = \frac{1}{\sqrt{1-(0.9)^2}} = 2.29416$$

$$\gamma_{0.8} = \frac{1}{\sqrt{1-(0.8)^2}} = 1.6667$$

Thus, Equation (1) will be

$$\boxed{4.312 - 5.141\cos\theta_1 = 3.32\cos\theta_2}$$

and Equation (2) will be

$$5.141 \sin\theta_1 = 3.32 \sin\theta_2$$

Square both equations

$$(1)\ 18.591 - 44.335\cos\theta_1 + 26.432\cos^2\theta_1 = 11.022\cos^2\theta_2$$
$$(2)\ 26.432\sin^2\theta_1 = 11.022\sin^2\theta_2$$

if we add (1) and (2) $(\sin^2\theta + \cos^2\theta = 1)$

$$18.591 - 44.335\cos\theta_1 + 26.432 = 11.022$$

Solving this for θ_1

$$\boxed{\theta_1 = 39.9^\circ} \text{ for the } 0.9c\ \pi^+$$

plug the value of θ_1 in $5.41\sin\theta_1 = 3.32\sin\theta_2$ will give us

$$\boxed{\theta_2 = 83.6^\circ} \text{ for the } 0.8c\ \pi^-$$