Physics 3305 - Home Work - chapter 4

Exercise 4.17

$$\lambda = \frac{h}{p} = \frac{h}{m_e c} = \frac{6.63\ 10^{-34} J.s}{(9.11\ 10^{-31} kg)(3\ 10^8 m/s)} = 2.43\ 10^{-12} m$$

Exercise 4.18

$$E = \frac{p^2}{2m} = \frac{3}{2} k_B T \quad \text{with } p = \frac{h}{\lambda}$$

$$\frac{(h/\lambda)^2}{2m} = \frac{3}{2} k_B T \Rightarrow \frac{h^2}{\lambda^2 m} = 3 k_B T \Rightarrow$$

$$\boxed{\lambda = \frac{h}{\sqrt{3 m k_B T}}}$$

Exercise 4.19

$$\lambda = \frac{h}{\sqrt{3 m k_B T}} \quad , T(°K) = T(°C) + 273$$

a) For an electron:

$$\lambda = \frac{6.63\ 10^{-34} J.s}{\sqrt{3 (9.11 . 10^{-31})(1.38\ 10^{-23} J/K)(295 K)}} = 6.29\ nm$$

b) For a proton

$$\lambda = \frac{6.63\ 10^{-34} J.s}{\sqrt{3 (1.67\ 10^{-27} kg)(1.38\ 10^{-23} J/K)(295 K)}} = 0.147 nm$$

Although the proton's speed would be smaller, its mass is so much larger that it's momentum is large thus smaller wave length.
In situations in which dimensions are smaller or comparable to ~ 6.29nm the electron will exhibit its wave nature.
At the same (T) dimensions would have to be smaller by a factor (42) for a proton to exhibits a wave nature

Exercise 22

If the maximum nonrelativistic speed is taken to be $c/10$, the wavelength would be

$$\lambda = \frac{h}{p} = \frac{6.63 \cdot 10^{-34} J \cdot s}{(9.11\ 10^{-31} kg)(3\ 10^{7} m/s)} = 2.43 \times 10^{-11} m$$

wavelengths this small or smaller would imply relativistic motion

For the accelerating potential. Use formula example 4.3

$$V = \frac{h^2}{2mq\lambda^2} = \frac{(6.63\ 10^{-34} J\ s)^2}{2(9.11\ 10^{-31} kg)(1.6\ 10^{-19})(2.43\ 10^{-11} m)^2}$$

$$\boxed{V \simeq 2500V}$$

Exercise 24

Find the speed of the moon $F = ma$ $a = \frac{v^2}{R}$

$$F = G \frac{m_{earth} m_{moon}}{r^2} = m_{moon} \frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{G m_{earth}}{r}} = \sqrt{\frac{(6.67\ 10^{-11} N\ m^2/kg^2)(5.98\ 10^{24} kg)}{3.84\ 10^{8} m}}$$

$$\boxed{v = 1.02\ 10^{3} m/s}$$

Thus $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\ 10^{-34} Js}{(7.35\ 10^{22} kg)(1.02\ 10^{3} m/s)}$

$= 8.85\ 10^{-60} m$

This is much smaller than the dimensions of the region in which it moves. In fact, ~~[illegible]~~ it is smaller than the atomic nucleus.

The moon certainly orbits as a classical particle

Exercise 4.41

Heisenberg principle: $\Delta x \, \Delta p \geq \frac{\hbar}{2}$

$\Delta x = 1 \mu m = 10^{-6} m$, $m = 0.145 \, kg$, $\hbar = \frac{h}{2\pi} = 1.055 \, 10^{-34} J.s$

$$\Delta x \, \Delta p \geq \frac{\hbar}{2} \Rightarrow (10^{-6} m)(0.145 kg) \, \Delta v \geq \frac{1}{2}(1.055 \, 10^{-34} Js)$$

$$\Delta v \geq 3.6 \, 10^{-28} \, m/s$$

It is theoretically impossible to say whether it might not be moving at $\sim 10^{-22}$ m/s.

Exercise 4.43

$$\Delta x \, \Delta p \geq \frac{1}{2}\hbar \rightarrow (5 \times 10^{-15} m)(1.67 \, 10^{-27} kg) \, \Delta v \geq \frac{1}{2}(1.055 \, 10^{-34} J.s)$$

$$\Rightarrow \Delta v \geq 6.3 \, 10^{6} m/s$$

its $KE = \frac{1}{2}(1.67 \, 10^{-27} kg)(6.3 \, 10^{6} m/s)^2 \, J \cong 0.2 \, MeV$

Exercise 4.48

$\Delta E = 150 \, MeV = 150 \, 10^{6} \times 1.6 \, 10^{-19} \, J$

$\Delta t \, \Delta E \geq \frac{\hbar}{2}$ $\qquad \Delta t \geq \frac{\hbar/2}{\Delta E}$

$$\Delta t \geq \frac{\frac{1}{2}(1.055 \times 10^{-34} Js)}{150 \, 10^{-13} J}$$

$$\boxed{\Delta t \geq 2.2 \times 10^{-24} s}$$

Exercise 4.62

Eq. 4-27 $\quad \Delta\omega\,\Delta t \geq \frac{1}{2} \Rightarrow \Delta\omega \geq \frac{1}{2}\frac{1}{\Delta t}$

$\omega = 2\pi f$

Angular frequency

$$\Delta f = \frac{1}{2\pi}\Delta\omega \quad , \Delta t = 1ns = 10^{-9}s$$

$$\Delta f = \frac{1}{2\pi}\Delta\omega \cong \frac{1}{2\pi}\left(\frac{1}{2}\frac{1}{\Delta t}\right) = \frac{1}{4\pi}\left(\frac{1}{10^{-9}s}\right)$$

$$\boxed{\Delta f = 7.96\ 10^{7}\ Hz}$$

(C) For the 1060 nm laser

$$f = \frac{c}{\lambda} = \frac{3\ 10^{8} m/s}{1060\ 10^{-9} m} = 2.83\ 10^{14}\ Hz$$

The relative uncertainty is significant for 100MHz radioware, but is very small compared to the frequency of the light.

Exercise 4.63

$\Delta x = 0.3\ \mu m$

$$\Delta k \cong \frac{1}{2}\frac{1}{\Delta x} = \frac{1}{2}\ \frac{1}{0.3\times10^{-6}} = \underbrace{1.67\times10^{6}\ m^{-1}}$$

$$k = \frac{2\pi}{\lambda} \quad (\text{wave vector}) \Rightarrow \Delta k = \frac{2\pi}{\lambda^2}\Delta\lambda =$$

$$\Delta\lambda = \frac{\lambda^2 \Delta k}{2\pi} = \frac{(6\ 10^{-7}m)^2\ 1.67\ 10^{6}}{2\pi}$$

$$\Delta\lambda = 95\ nm$$

A 1 femtosecond pulse of 600 nm light is not just 600nm light