Chapter 6

Problem 6.15:

Calculate the reflection probability:

Equation 6-7 reads:

$$R = \frac{\left(\sqrt{E} - \sqrt{E - U_0}\right)^2}{\left(\sqrt{E} + \sqrt{(E - U_0)}\right)^2} \quad \text{and } E = 5\,eV,\ U_0 = -2\,eV$$

$$R = \frac{\left(\sqrt{5} - \sqrt{5-(-2)}\right)^2}{\left(\sqrt{5} + \sqrt{5-(-2)}\right)^2} = 0.00704$$

Problem 6.16:

in case of $U_0 \to -\infty$

We have $T = 4 \dfrac{\sqrt{E(E-U_0)}}{\left(\sqrt{E} + \sqrt{E-U_0}\right)^2}$

$$T \xrightarrow[U_0 \to -\infty]{} \frac{\sqrt{E\,U_0}}{U_0} \longrightarrow 0$$

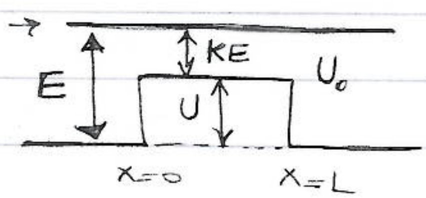
Thus, the particle will definitely reflect

$$\boxed{T + R = 1}$$

Problem 6.24:

$U_0 = 30\,eV$ $\qquad E = 35\,eV$

$E > U_0$



Thus,

$$R = \frac{\sin^2\left(\dfrac{\sqrt{2m(E-U_0)}}{\hbar} L\right)}{\sin^2\left(\dfrac{\sqrt{2m(E-U_0)}}{\hbar} L\right) + 4\dfrac{E}{U_0}\left(\dfrac{E}{U_0} - 1\right)}$$

In case of no Reflection $\Rightarrow R = 0$

So, $\sin\left(\frac{\sqrt{2m(E-U_0)}}{\hbar}L\right) = 0$ or $\frac{\sqrt{2m(E-U_0)}}{\hbar}L = n\pi$

$$\frac{\sqrt{2m(E-U_0)}}{\hbar} = \frac{\sqrt{2(9.11\times10^{-31}kg)((35-30)\times1.6\times10^{-19}J)}}{1.055\times10^{-34}Js}$$

$$= 1.144\times10^{10}$$

Thus, $1.144\ 10^{10}\ L = n\pi$

If $L = 1nm$ is inserted, $n = 3.64$

For $n = 4$: $1.144\ 10^{10}\ L = 4\pi$

$$\Rightarrow \boxed{L = 1.0981\times10^{-9}m = 1.0981\,nm}$$

(b) $E = 36\ eV$

$$\sin^2\left(\frac{\sqrt{2(9.11\times10^{-31}kg)(6\times1.6\times10^{-19}J)}}{1.055\times10^{-34}Js}(1.0981\times10^{-9}m)\right) = 0.8683$$

$$R = \frac{0.8683}{0.8683+4\,\frac{36}{30}\left(\frac{36}{30}-1\right)} = 0.475$$

$\boxed{35}$

For problem 35, please see last page

Problem 6.45

$\lambda = 5mm$, $\omega = \sqrt{(\gamma/\rho)k^3}$, $\rho = 10^3 kg/m^3$

$$v_{phase} = \frac{\omega}{k} = \frac{\sqrt{(\gamma/\rho)k^3}}{k}$$

$$\boxed{v_{phase} = \sqrt{(\gamma/\rho)k}}$$

$$v_{group} = \frac{d\omega}{dk} = \frac{d\left(\sqrt{(\gamma/\rho)k^3}\right)}{dk}$$

$$= \frac{3}{2}\sqrt{(\gamma/\rho)k}$$

But: $k = \frac{2\pi}{\lambda} = \frac{2\pi}{5\times10^{-3}\ m} = 1.26\ 10^{3}\ m^{-1}$

$$v_{phase} = \sqrt{\frac{0.072\ N/m}{10^3\ kg/m^3}\left(1.26\ 10^3 m^{-1}\right)}$$

$$\boxed{v_{phase} = 0.30\ m/s}$$

$$\boxed{v_{group} = 0.45\ m/s}$$

<u>Problem 6.48:</u>

Equation (6.23) we have:

the matter wave dispersion relation: $\omega(k) = \frac{\hbar k^2}{2m}$

Equation (6.29)

$D = \frac{d^2\omega(k)}{dk^2}$: The dispersion Coefficient for matter waves in vacuum.

$$\frac{d\omega(k)}{dk} = \frac{\cancel{2}\hbar k}{\cancel{2}m}$$

$$D = \frac{d^2\omega(k)}{dk^2} = \frac{\hbar}{m}$$

Equation (6.28):

$$|\psi(x,t)|^2 = \frac{C^2}{\sqrt{1 + D^2t^2/4\epsilon^4}} \exp\left[\frac{-(x-st)^2}{2\epsilon^2(1 + D^2t^2/4\epsilon^4)}\right]$$

will become: (After replacing D by $\hbar/m$)

$$|\psi(x,t)|^2 = \frac{C^2}{\sqrt{1 + \frac{\hbar^2 t^2}{4\epsilon^4 m^2}}} \exp\left[\frac{-(x-st)^2}{2\epsilon^2\left(1 + \frac{\hbar^2t^2}{4m^2\epsilon^4}\right)}\right]$$

Problem 6.56

The condition is $2s = \frac{\beta}{k}$

where $\beta = \tan^{-1}\left(\frac{2\alpha k}{k^2-\alpha^2}\coth(\alpha L)\right)$.

In the limit $L \to \infty$, $\coth x \to 1$, so

$$\beta = \tan^{-1}\left(\frac{2\alpha k}{k^2-\alpha^2}\right)$$

thus $2sk = \tan^{-1}\left(\frac{2\alpha k}{k^2-\alpha^2}\right)$

or $\tan(2sk) = \frac{2\alpha k}{k^2-\alpha^2}$

$$\cot(2sk) = \frac{k^2-\alpha^2}{2\alpha k}$$

$$2\cot(2sk) = \frac{k^2-\alpha^2}{\alpha k} = \frac{k}{\alpha} - \frac{\alpha}{k}$$

Note that, $2s$ is the distance between the barriers.

Eq(5.22) $2\cot(kL) = \frac{k}{\alpha} - \frac{\alpha}{k}$

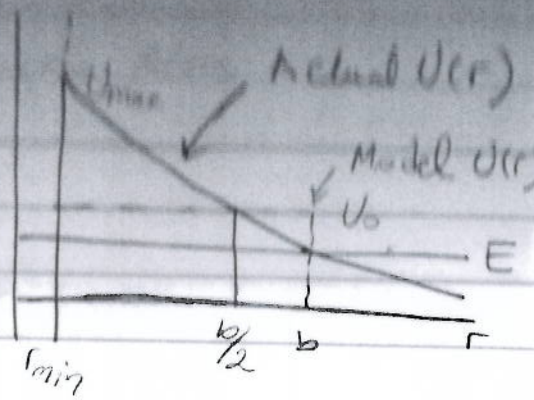
Problem 6.38

a) $E_T = \frac{3}{2} k_B T$ (thermal energy)

$U = \frac{ke^2}{r}$ Coulomb potential

if $r = 2\ \text{fm} = 2\ 10^{-15}\ m$

show that $T = 10^9 K$



$$E_T = U \Rightarrow \frac{3}{2} k_B T = \frac{ke^2}{r}$$

$$T = \frac{2\,ke^2}{3\,k_B\,r} = \frac{2\,(9 \times 10^9\ Nm^2/C^2)(1.6\ 10^{-19}\ C)^2}{3 \times (1.38\ 10^{-23}\ J/K) \times 2\ 10^{-15}}$$

$$\boxed{T \cong 6 \times 10^9 K}$$

b) Let $r = b$ is where the Energy E equals the Coulomb potential

$$4\,\frac{3}{2} k_B T = \frac{ke^2}{b}, \quad E = 6 k_B T$$

$$\Rightarrow \boxed{b = \frac{ke^2}{6 k_B T}}$$

The potential Energy at $\frac{1}{2} b$ will be twice its value at b

$$U = 8\,\frac{3}{2} k_B T = 12\,k_B T$$

Thus, the expression $e^{-2\frac{\sqrt{2m(U_0 - E)}}{\hbar}L}$ will be

$$e^{-2\frac{\sqrt{2m(U_0-E)}}{\hbar}L} \cong e^{-2\frac{\sqrt{2m(12 k_B T - 6 k_B T)}}{\hbar}\frac{ke^2}{6 k_B T}}$$

⚠ $b = L$

$$= e^{-\frac{ke^2}{\hbar}\sqrt{\frac{4m}{3 k_B T}}}$$

c) At $T = 10^7 K$

$$e^{-\frac{(9 \times 10^9)(1.6\ 10^{-19})^2}{1.055\ 10^{-34}}\sqrt{\frac{4(1.67\ 10^{-27})}{3(1.38\ 10^{-23})(10^7)}}} = 0.00015$$

At $T = 3000 K$ $\Rightarrow$ it is 10^{-223}

Even at high Earthly temperatures won't initiate fusion, but tunneling is significant at higher temperatures