Chapter 7 : Hydrogenic Atom

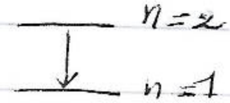
Problem 7.21

Quantum dot - cubic , 3D

$$E_{n_x n_y n_z} = (n_x^2 + n_y^2 + n_z^2)\frac{\pi^2\hbar^2}{2mL^2}$$

the smallest jump will be when two of the n's are (and remain) unity, while the third changes from 2 to 1

The corresponding energy change is



$$\Delta E = (2^2 - 1^2)\frac{\pi^2\hbar^2}{2mL^2} = \frac{3\pi^2\hbar^2}{2mL^2}$$

The photon's Energy $= h\nu = \frac{hc}{\lambda}$ with $\lambda = 450\ 10^{-9}$ m

$$\frac{hc}{\lambda} = \frac{3\pi^2\hbar^2}{2mL^2} \Rightarrow L^2 = \frac{3\pi^2\hbar^2}{2m\,hc}\lambda$$

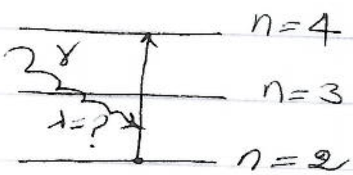
$$L^2 = \frac{3\times\pi^2\times(1.055\times10^{-34}\,\mathrm{J\,s})^2(450\ 10^{-9})}{2\,(9.11\times10^{-31}\,\mathrm{kg})(6.63\times10^{-34}\,\mathrm{J\,s})(3\ 10^{8}\,\mathrm{m/s})}$$

$L = 6.4\ 10^{-10}$ m

$$\boxed{L = 6.4\ \mathring{A}}$$

Problem 7.32.

A hydrogen atom at $n=2$ absorbs a photon. what should be the photon wavelength to cause the electron to jump to an $n=4$ state?



The energy of that photon should be

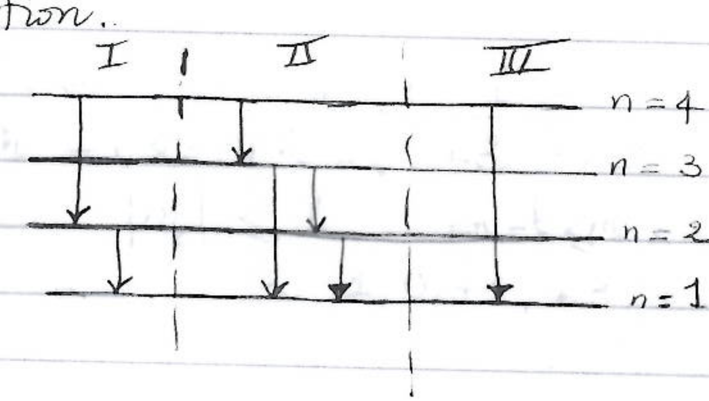
$$\Delta E = E_4 - E_2 = (-13.6\,eV)\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 2.55\,eV$$

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Recalling that: $hc = 1240$ eV.nm

$$\boxed{\lambda = \frac{1240\ eV.nm}{2.55\ eV} = 486\ nm}$$

(b) What wavelength photons might be emitted by the atom following this absorption..



(I): it can go from 4 → 2 by emitting a photon with 486nm then from 2 → 1 by a 122 nm photon. (Example 7.2)

(II)

4 → 3 $\quad \lambda =$ ~~97.2 nm~~ 1875 nm (Use Eq 7.13 $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

3 → 1 $\quad \lambda = 103$ nm

3 → 2 $\quad = 656$ nm

2 → 1 $\quad = 122$ nm

(III)

4 → 1 $\quad \lambda = 97.2$ nm $\left(\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{4^2}\right)\right)$

Problem 7.36

We have:

$$\Phi(\phi) = e^{i\sqrt{D}\phi}$$

if we set $\Phi(0) = \Phi(2\pi)$

$$e^{i\sqrt{D}\,0} = e^{i\sqrt{D}\,2\pi}$$

since $e^{0} = 1$

and $e^{i\theta} = \cos\theta + i\sin\theta$

We will have: $\cos(\sqrt{D}\,2\pi) + i\sin(\sqrt{D}\,2\pi) = 1$

$\Rightarrow \cos(\sqrt{D}\,2\pi) = 1$

Therefore, $\sqrt{D}$ has to be an integer.

Problem 7.37

An electron of a Hydrogen atom is in an $\ell = 3$ state.

$\Rightarrow L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{3(3+1)}\,\hbar = \sqrt{12}\,\hbar \cong 3.46\,\hbar$

Also, as $\ell = 3$ $\quad |m_\ell| \leq \ell \quad \Rightarrow -\ell \leq m_\ell \leq +\ell$

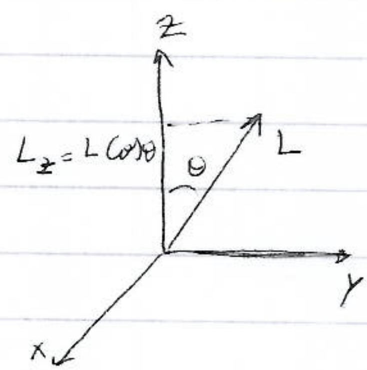
So $m_\ell = \pm\ell \quad m_\ell = -3, -2, -1, 0, +1, +2, +3$

alors que $L_z = m_\ell\hbar = -3\hbar, -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar, 3\hbar$

$$L_z = L\cos\theta \qquad \text{with } L = \sqrt{12}\,\hbar$$

$$\cos\theta = \frac{L_z}{L} \Rightarrow \theta = \cos^{-1}\left(\frac{L_z}{L}\right)$$

L_z	θ
$-3\hbar$	$150°$
$-2\hbar$	$125.3°$
$-1\hbar$	$106.8°$
0	$90°$
$1\hbar$	$73.2°$
$2\hbar$	$54.7°$
$3\hbar$	$30°$



Problem 7.[illegible]

A particle orbiting due to an attractive central force $V(r)$ has angular momentum 1.00×10^{-33} kg.m/s

find the component of L_z possible to detect.

$$L = \sqrt{\ell(\ell+1)}\,\hbar$$

$$\sqrt{\ell(\ell+1)} = \frac{L}{\hbar} = \frac{10^{-33}}{1.055\ 10^{-34}} = 9.47$$

$$\Rightarrow \boxed{\ell = 9} \qquad \text{So } m_\ell = 0, \pm 1, \text{----}, \pm \ell$$

and $L_z = m_\ell \hbar$

Thus: $L_z = 0, \pm \hbar, \pm 2\hbar, \pm 3\hbar, \pm 4\hbar, \pm 5\hbar, \pm 6\hbar, 7\hbar, 8\hbar, 9\hbar.$

Problem 7.44.

In a 3d state.

$\Rightarrow n = 3,\ \ell = 2$

Given that, there are 5 different ~~values~~ allowed values of m_ℓ : $0, \pm 1, \pm 2$

They are distinguished by their z-component of angular momentum L_z

$$L_z = 0, \pm \hbar, \pm 2\hbar$$

Problem 7.58.

From Table 7.4 The Radial solutions of Eq 7-31

$R_{n,\ell}(r)$ incase $n = 3,\ \ell = 1$

$$R_{3,1}(r) = \frac{1}{(3a_0)^{3/2}} \frac{4\sqrt{2}r}{9a_0}\left(1 - \frac{r}{6a_0}\right) e^{-r/3a_0}$$

Thus, the radial probability $P(r)$, a probability per unit radial distance is:

$$P_{3,1}(r) = r^2 R^2(r)$$

$$= \left(\frac{1}{(3a_0)^{3/2}} \frac{4\sqrt{2}\, r}{9\, a_0}\left(1 - \frac{r}{6a_0}\right) e^{-r/3a_0}\right)^2 r^2$$

$$= \frac{32}{2187\, a_0^3} r^4 \left(1 - \frac{r}{6a_0}\right)^2 e^{-2r/3a_0}$$

$$\bar{r} = \int_0^\infty r\, P(r)\, dr = \frac{32}{2187\, a_0^3} \int_0^\infty r^5 \left(1 - \frac{r}{6a_0}\right)^2 e^{-2r/3a_0}\, dr$$

$$= \frac{32}{2187\, a_0^3} \int_0^\infty \left(r^5 - \frac{r^6}{3a_0} + \frac{r^7}{36\, a_0^2}\right) e^{-2r/3a_0} dr$$

$$= \frac{32}{2187\, a_0^3} \left(\frac{5!}{(2/3a_0)^6} - \frac{1}{3a_0}\frac{6!}{(2/3a_0)^7} + \frac{1}{36a_0^2}\frac{7!}{(2/3a_0)^8}\right)$$

$$\boxed{\bar{r} = 12.5\, a_0}$$

(b) This value is greater than $\bar{r}$ for the 3d (Example 7.7)
From the graph of $P(r)$ vs r (figure 7.17)
we see that the 3p state spreads to higher values of r more than the 3d.
The 3d has more rotational energy, is more like a circle, while the 3p has more radial energy and thus shows greater motion in the r-direction

Problem 7.[illegible]

Equation 7.4a hydrogen like $r_n = \frac{1}{Z} n^2 a_0$

the atomic number of Beryllium $Z = 4$ $_4Be$

as an ion (loses one electron)

we have $Z = 3$

So $r_1 = \frac{1}{\cancel{2}3} 1^2 a_0 = \frac{a_0}{3}$ about one-third the radius.

Problem 7.85

$$E_{n_x n_y n_z} = (n_x^2 + n_y^2 + n_z^2) \frac{\pi^2 \hbar^2}{2mL^2}$$

$$E_{1,1,1} = (1^2 + 1^2 + 1^2) \frac{\pi^2 \hbar^2}{2mL^2}$$

$$E_{1,1,1} = 3 \frac{\pi^2 \hbar^2}{2mL^2} = 13.6 \text{ eV}$$

$$L = \frac{\pi (1.055 \times 10^{-34} \text{ J s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(13.6 \times 1.6 \, 10^{-19} \text{ J})/3}}$$

$= 0.29 \text{ nm}$