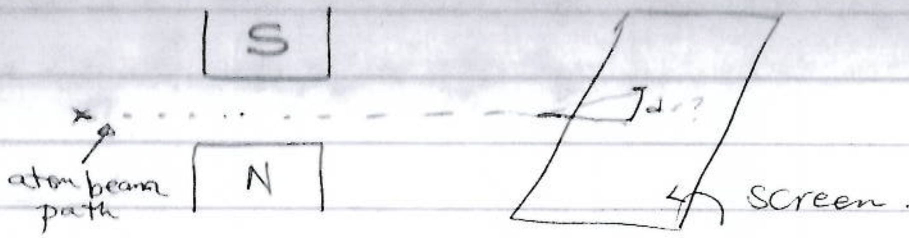
Chapter 8. Spin

Problem 28. Stern-Gerlach experiment



By how much, it would be deflected.

we have

$$KE = \frac{3}{2} k_B T = \frac{1}{2} m v^2 \Rightarrow v^2 = \frac{3 k_B T}{m}$$

$$v^2 = \frac{3(1.38\ 10^{-23}\ J/K)(500K)}{(1.67\ 10^{-27}\ Kg)}$$

$$\boxed{v = 3.52 \times 10^3\ m/s}$$

The time needed to get through 1m of magnetic field.

$$t = \frac{d}{v} = \frac{1m}{3.52\ 10^3\ m/s} = 2.84 \times 10^{-4}\ s$$

and the force experienced is $F_z = -\frac{e}{m_e}(m_s \hbar)\frac{\partial B_z}{\partial z}$

$$|F_z| = \frac{1.6 \times 10^{-19}\ C}{9.11 \times 10^{-31}\ kg}\left(\left(\frac{1}{2}\right) \times 1.055\ 10^{-34}\ J.s\right)\left(10\ T/m\right) = 9.26\ 10^{-23}\ N$$

The second law of Newton $F = ma$ $\quad a = \frac{F}{m}$

$$a = \frac{F}{m} = \frac{9.26 \times 10^{-23}\ N}{1.67\ 10^{-27}\ Kg} = 5.55 \times 10^4\ m/s^2$$

The transverse displacement is (in z-direction)

$$z = \frac{1}{2} a t^2 = \frac{1}{2}\left(5.55 \times 10^4\ m/s^2\right)\left(2.84 \times 10^{-4}\ s\right)^2$$

$$\boxed{z = 2.2\ mm}$$

Problem 8.30.

Hydrogen atom at ground state, , B = 1 T

We have : $U = -\vec{\mu} \cdot \vec{B}$

Assuming $\vec{B}$ is in the z direction, so $U = -\mu_z B_z$

with: $\vec{\mu} = -\frac{e}{m} \vec{S} \Rightarrow \mu_z = \frac{-e}{m} S_z$

Thus $U = -\left(\frac{-e}{m} S_z\right) B_z = \frac{e}{m} S_z B_z$

so for $S_z = +\frac{1}{2}\hbar \qquad U_+ = +\frac{e}{m} \frac{1}{2} \hbar B_z$

$S_z = -\frac{1}{2}\hbar \qquad U_- = -\frac{e}{m} \frac{1}{2} \hbar B_z$

$$\Delta U = U_+ - U_- = \frac{e}{m} \hbar B_z = \frac{1.6 \times 10^{-19} C}{9.11 \times 10^{-31} kg} \left(1.055 \times 10^{-34} J.s\right)(1\ T)$$

$$\boxed{\Delta U = 1.85 \times 10^{-23} J = 1.16 \times 10^{-4} eV}$$

Problem 8.31

Ω ~~[illegible]~~ particle has spin $s = \frac{3}{2}$

What angles might its intrinsic angular momentum vector make with the z-axis.

We have $S = \sqrt{s(s+1)}\,\hbar = \sqrt{\frac{3}{2}\left(\frac{3}{2}+1\right)}\,\hbar = \frac{\sqrt{15}}{2}\hbar$

with Components $S_z = m_s \hbar = -\frac{3}{2}\hbar, -\frac{1}{2}\hbar, \frac{1}{2}\hbar, \frac{3}{2}\hbar$

$S_z = S \cos\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{S_z}{S}\right)$

$\theta = 140.8°, 105°, 75°$ and $39.2°$

Problem 8.39

$$P = \int_0^{L/2} \frac{\sqrt{2}}{L}\left(\sin\left(\frac{1\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right) \pm \sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{1\pi x_2}{L}\right)\right)^2 dx_1\,dx_2$$

$$= \frac{2}{L^2}\int_0^{L/2}\sin^2\frac{1\pi x_1}{L}\,dx_1\int_0^{L/2}\sin^2\frac{2\pi x_2}{L}\,dx + \frac{2}{L^2}\int_0^{L/2}\sin^2\frac{2\pi x_1}{L}\,dx_1\int_0^{L/2}\sin^2\frac{1\pi x_2}{L}\,dx_2$$

$$\pm 2\frac{2}{L}\int_0^{L/2}\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_1}{L}\,dx\int_0^{L/2}\sin\frac{2\pi x_2}{L}\sin\frac{1\pi x_2}{L}\,dx_2$$

The first four integrals are $L/4$ and the later two, using the formulas from the example, are $2L/3\pi$ thus,

$$P = \frac{2}{L^2}\left(\left(\frac{1}{4}L\right)^2 + \left(\frac{1}{4}L\right)^2 \pm 2\left(\frac{2L}{3\pi}\right)^2\right) = \frac{1}{4} \pm \frac{16}{9\pi^2}$$

The 0.25 is the classical probability $\left(\frac{1}{2}\times\frac{1}{2}\right)$. The symmetric state tends to have particle closer together, so there is a greater than normal probability of finding them on the same side; ~~then~~ anti-symmetric state tends to seperate particles. Symmetric (+ sign) 0.43, Antisymmetric (− sign) 0.07

Problem 8.41 5 noninteracting spin $\frac{1}{2}$ particle in 1D box, L

1) There may be two in $n=1$ state, $E = 2\times\frac{1^2\pi^2\hbar^2}{2mL^2}$ two in $n=2$ state, $E = 2\times\frac{2^2\pi^2\hbar^2}{2mL^2}$, and the last one in $n=3$ state $E = \frac{3^2\hbar^2\pi^2}{2mL^2}$

2) Bosons do not obey an exclusion principle, All may be in the same state, eg $n=1$, $E = 5\times\frac{\pi^2\hbar^2}{2mL^2}$

3) If $s=3/2 \Rightarrow$ 4 different possible value of $m_s = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, 3/2$ Thus without violation of the exclusion principle, 4 particles could have $n=1$, with the fifth in the $n=2$,

$$E_{Tot} = 4\cdot\frac{1^2\pi^2\hbar^2}{2mL^2} + 1\times\frac{2^2\pi^2\hbar^2}{2mL^2} = 8\,\pi^2\hbar^2/2mL^2$$

Problem 8.49

Electronic Configuration

Phosphorus ($Z = 15$) : $1s^2 2s^2 2p^6 3s^2 3p^3$

Germanium ($Z = 32$) : $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^2$

Cesium ($Z = 55$) : $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2$ -
- $5p^6 6s^1$

Problem 8.50.

According to the periodic table, the element 117 would be directly under fluorine, in the valence negative one (-1) or 7 column, with $7s^2 7p^5$

Problem 8-62

Total angular momentum $\vec{J} = \vec{L} + \vec{S}$

$|l - s| \leq j \leq |l + s|$ and $-j \leq m_j \leq +j$

3d hydrogen. $l = 2$, $s = 1/2$

$$J = \begin{cases} 5/2 & \text{L and S are aligned} \\ 3/2 & \text{L and S are antialigned} \end{cases}$$

So m_j

$$m_j = \begin{cases} \pm 5/2, \pm 3/2, \pm 1/2 \\ \pm 3/2, \pm 1/2 \end{cases}$$

So

$$(j, m_j) = \begin{cases} (5/2, +5/2)\,(5/2, +3/2)\,(5/2, +1/2)\,(5/2, -5/2)\,(\frac{5}{2}, -\frac{3}{2})\,(\frac{5}{2}, -\frac{1}{2} \\ (\frac{3}{2}, 3/2)\,(\frac{3}{2}, \frac{1}{2})\,(\frac{3}{2}, -3/2)\,(\frac{3}{2}, -1/2) \end{cases}$$

Problem # [illegible]

Here j_T is $= 1$ (to be split into 3 lines)

s_T is 0 or 1 ignore 2 or higher.

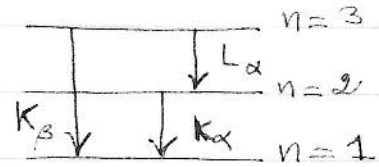
$j_T = l + s$ to $j_T = |l - s| \Rightarrow |l_T - s_T| \leq j_T \leq |l_T + s_T|$

that implies:

$$\{s_T, l_T\} = \{0,1\}, \{1,0\}, \{1,1\}, \{1,2\}$$

Problem 8.56.

$E_\gamma = \dfrac{hc}{\lambda} \Rightarrow$ { higher energy is equivalent to low λ.



Despite $E \propto n^2$, energy levels tend to get closer as n increases, So, the 2 to 1 jump is bigger than the 3 to 2.

Therefore, the highest energy (/shortest wave length photon) is the K_β, next is the K_α, then the L_α is the lowest.

in K_β : Energy of photon is $\Delta E = -13.6\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = 12$ eV

K_α $\qquad \Delta E = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2$ eV

L_α $\qquad \Delta E = -13.6\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.88$ eV