Maxwell's Equations & Motion

In electromagnetism theory, we see 2nd order partial differential eq.s'

$$\nabla^2 E = \epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} \qquad \left(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\nabla^2 B = \epsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

arise from Maxwell's Eq.s to describe E (electric) & B (magnetic) fields.

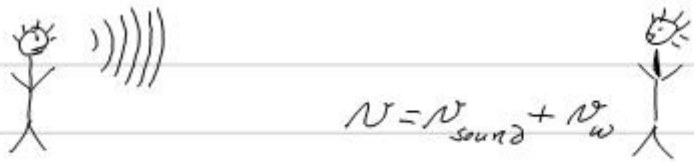
By analogy with mechanical wave eq.

$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$, suggest this a wave eq. with 'electromagnetic' wave speed = 'c' = $1/\sqrt{\epsilon_0 \mu_0} = 3\times 10^8$ m/s.

- what speed in reference to?
- what is mode of propagation of this wave?

Galilean Relativity mode of propagation first.

- mechanical waves propagated by medium (e.g. sound in air)
  - speed a feature of medium's properties
  - speed relative to medium, not source or observer



$v = v_{sound} + v_w$

wind → (v_w velocity relative to observer)

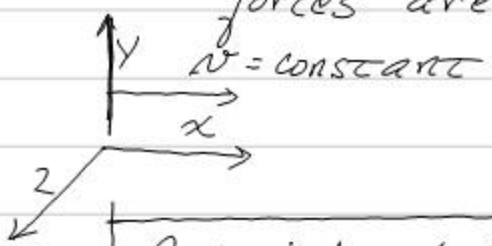
To understand better, let's consider Galilean relativity...

For light, this medium was postulated and called "luméniferous ether".

To quantify motion, we need to consider our coordinate sytems carefully.

<u>reference frame</u>: a coordinate system with respect to which motion is measured.

<u>inertial reference frame</u> (IRF): a frame in which no external forces are felt (i.e. no accelerations

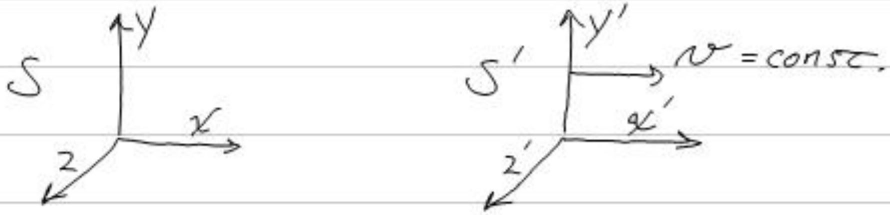


<u>Principle of Galilean Relativity</u>: Laws of mechanics (only physical laws known at time) are independent of IRF → all IRFs are equivalent

∴ no absolute frame in space

(Note: Newton (incorrectly) felt there was one. Spin a bucket with water + it knows to move up sides.)

Galilean Transformations



Two IRFs with relative velocity, v.

We need to be able to calculate coordinates in S' from v, + (x, y, z):

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

} motion only along x-axis

The inverse transformations are:

$$x = x' + vt$$
$$y = y'$$
$$z = z'$$

Note that in all cases, time is considered an absolute (i.e. $t' = t$).

Newton's Laws have same form (are invariant) after transformation

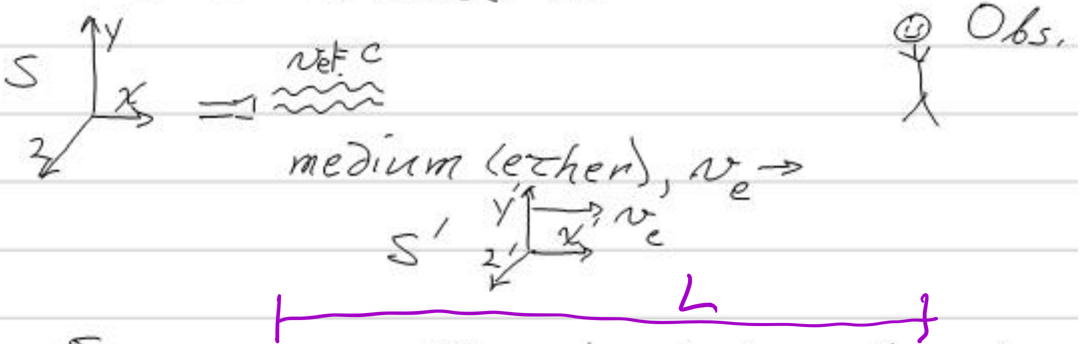
$$F_S = F_{S'} = ma$$

Velocities & Medium of Propagation

Observer in S' sees objects stationary in S moving with velocity $-v$:

$$v' = -v$$

Imagine a light emitting towards an observer



Expect to add velocities linearly, ($v = v_e + c$).

Time to O: $\tau_a = L/(c - v_e)$

Time to Source: $\tau_b = L/(c + v_e)$

Sum in times an indicator of v_e.

$$\tau_a + \tau_b = \frac{L}{c+v_e} + \frac{L}{c-v_e}$$

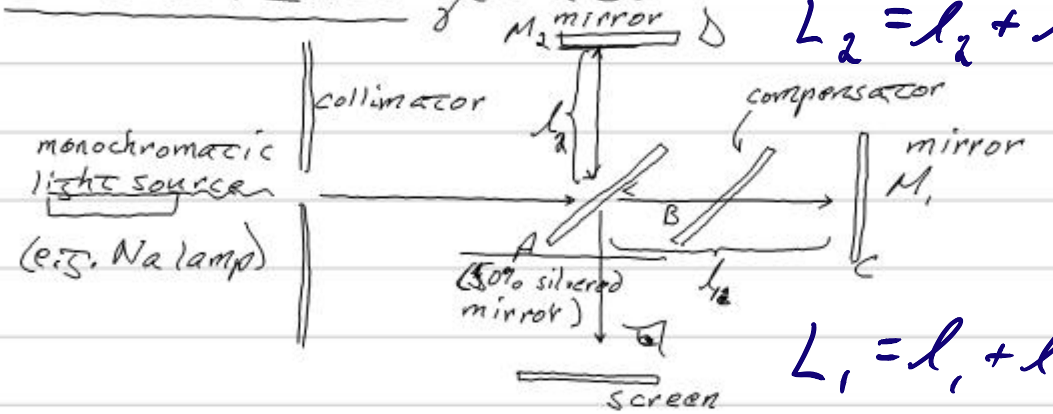
$$= \frac{L(c-v_e) + L(c+v_e)}{(c+v_e)(c-v_e)} = \boxed{2cL/(c^2 - v_e^2)}$$

Michelson-Morley Experiment

→ 1800's, Case

- require largest range of velocities relative to ether: Earth's orbital motion

Michelson Interferometer



$L_2 = l_2 + l_2'$

$L_1 = l_1 + l_1'$

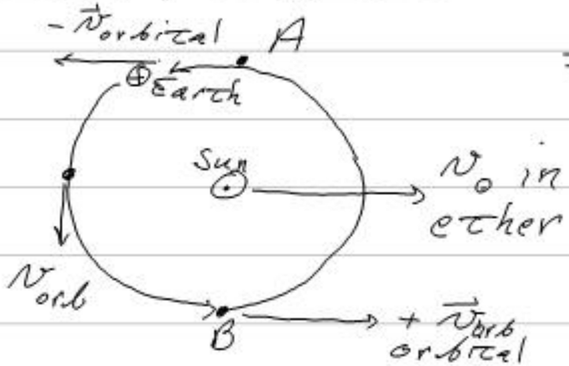
Idea is to obtain sensitivity by comparing travel time in 2 ⊥ directions (generally have different orientation to $\vec{v}_e$).

- see interference fringe @ eyepiece

$$(L_1 - L_2) = n\lambda$$

So difference in path amounts to some # of wavelengths

Velocity Comparison



$\Rightarrow v_{e1} = v_0 - v_{orb}$

$\Rightarrow v_{e2} = v_0 + v_{orb}$

Have a range of velocities $|v_{e1} - v_{e2}|$ wide $= 2v_{orb}$.

For a given position (e.g. A above), round-trip time to M_1

$$\underline{\underline{\tau_{\parallel}}} = 2cL/(c^2 - v_e^2) = \boxed{\frac{2L}{c}\frac{1}{(1 - v_e^2/c^2)}}$$

to M_2:

$$\tau_{\perp} = \frac{2\sqrt{l_2^2 + v_e^2\tau_{\perp}^2}}{c}$$

$$c^2\tau_{\perp}^2 = 4(l_2^2 + v_e^2\tau_{\perp}^2)$$

rearranging: $\underline{\underline{\tau_{\perp}}} = 2l_2/(c^2 - v_e^2)^{1/2} = \boxed{\frac{2l}{c}\frac{1}{\sqrt{1 - v_e^2/c^2}}}$

The time difference is then:

$$\underline{\underline{\Delta\tau}} = \tau_{\perp} - \tau_{\parallel} = \boxed{\frac{2}{c}\left[\frac{l_2}{1 - v_e^2/c^2} - \frac{l_1}{\sqrt{1 - v_e^2/c^2}}\right]}$$

Eq. 1

It's difficult to know l_1 + l_2 well enough to extract a measure of v_e from this time difference.

Solution: change orientation of device relative to v_e

- rotate by 90°

$$\Delta t' = t_{\parallel}' - t_{\perp}' = \frac{2}{c}\left[\frac{l_1}{1-v_e^2/c^2} - \frac{l_2}{\sqrt{1-v_e^2/c^2}}\right]$$

The interference pattern shift

$$\propto \Delta t' - \Delta t = \frac{2}{c}\left\{\frac{l_2}{1-v_e^2/c^2} - \frac{l_1}{\sqrt{1-v_e^2/c^2}} - \left(\frac{l_2}{\sqrt{1-v_e^2/c^2}} - \frac{l_1}{1-v_e^2/c^2}\right)\right\}$$

"proportional to"

$$\Delta t' - \Delta t = \frac{2(l_1+l_2)}{c}\left[\frac{1}{1-v_e^2/c^2} - \frac{1}{\sqrt{1-v_e^2/c^2}}\right]$$

expand

Useful Binomial Expansions

$$\frac{1}{1 \pm x} = 1 \mp x + x^2 \mp x^3 + \ldots$$

$$\frac{1}{\sqrt{1 \pm x}} = 1 \mp \frac{1}{2}x - \frac{1}{8}x^2 \pm \frac{1}{16}x^3 + \ldots$$

$$\rightarrow \underline{\underline{\Delta t' - \Delta t}} = \frac{2(l_1+l_2)}{c}\left[\left(1+\frac{v_e^2}{c^2} \ldots\right) - \left(1 + \frac{v_e^2}{2c^2} \ldots\right)\right]$$

$$\Longrightarrow \approx \boxed{v_e^2(l_1+l_2)/c^3}$$

If $v_e \sim v_{\oplus} \sim 3\times10^4$ m/s

$$\Delta\tau' - \Delta\tau \sim 10^{-16}\,s$$

this is ~1/20 of λ of Na (sodium)
- apparatus needs to yield
sensitivity at level of $\frac{1}{20}\lambda_{Na}$
→ @ Case sensitivity 10x better

Measurement: $\Delta\tau' - \Delta\tau = 0s$!!
→ tried many orientations &
seasons → same result

So motion of observer does not
effect ~~measurement~~ measured velocity of
light by observer

In other words, Maxwell's Eq.s
are the same form in all ~~inertial~~ IRF's → speed of EM waves
verified invariant

→ conclusions drawn by Einstein
→ Newtonian Mechanics flawed

Einsteinian Relativity:

Two postulates motivated by the apparent behavior of Maxwell's Eq.s

1) Laws of physics same in all inertial systems. There is no way to detect absolute motion; no preferred inertial system exists.

2) Observers in all inertial systems measure same value for speed of light in a vacuum.
(→ follows from 1) since has to be true for Maxwell's Eq.s to be invariant.)

One way to satisfy 2) in context of Michelson-Morley:

- when l_1 or l_2 in direction of $\vec{v}_e$, they are "contracted" by

$$l' = \sqrt{1 - \frac{v^2}{c^2}}\, l$$

→ will cause $\Delta t' - \Delta t = 0$
(suggested earlier by FitzGerald + Lorentz)

Relative Simultaneity

Generally, we assume time is absolute
- in particular, synchronization of clocks for different observers means we expect all agree on if events simultaneous

Event: anything with a location in space + time.

We need to keep track of time in seperate reference frames.

Frank in S frame

Mary in S'

→ Frank synchronizes lights so see as simultaneous ($l_1 = l_2$)

→ Mary observes $l_1' > l_1$, $l_2' < l_2$ because train moving to right while light travels

→ if Frank sees simultaneous signals, t_1' must be seen by him first.

Because 'c' not dependent on frame of observer.

Loss of Simultaneity

"Two events simultaneous in one IRF are not necessarily simultaneous in another IRF moving with respect to the first reference frame."

- A consequence of 'c' being independent of motion of observer or source.

- events specified in each frame by (x, t) or (x', t')
  - → each frame has own space AND time coordinates

- cannot ensure always stay "synchronized"
  - can bring everybody together @ $x=0, v=0$
  - call some moment $t=0$
  - but motion of individuals will result in new differences

this is not due to position of observer.

Lorentz Transformations

- speed of light, c, for all observers
  ∴ from a point of emission, light wave-front must be spherical in S + S' frames!

$$x^2 + y^2 + z^2 = c^2 t^2$$
$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

→ not consistent w/ Galilean relativity
→ i.e. $t \neq t'$

Take motion along x only
- need a linear transformation between (x', t') and (x, t)

$$x' = \gamma(x - vt)$$
$$t' = \gamma t\left(1 - \frac{v}{c}\right) = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$y' = y;\ z' = z$$

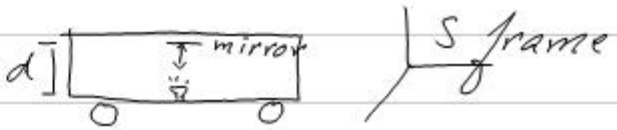
- Simplest if $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ See handout

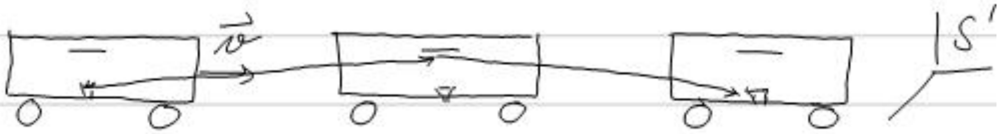
$\gamma \sim 1$ when $v \ll c$ (Galilean case)
- not dependent on x or t directly

Note: if $v > c$, (x', t') become imaginary.

Time Dilation

Imagine a train traveling @ velocity, v, with a flashlight + mirror:





For observer in S (in train), light travels a distance $\delta = 2d$

round trip time $$\boxed{\Delta t_p = \frac{2d}{c}}$$

→ termed "proper time":

"Time difference between two events ocurring @

this same position in a system as measured by a clock at rest in that system."

Time Dilation

For observer in S',

- light travels > 2d
- speed of light still observed to be c

$\therefore \; \Delta t > \Delta t_p$

Specifically, extra distance traveled means

$$(2D)^2 = (2d)^2 + (v\Delta t)^2$$

multiplying both sides by $1/c^2$ gives

$$\Delta t^2 = \frac{4D^2}{c^2} = \underbrace{\frac{4d^2}{c^2}}_{\Delta t_p^2} + \frac{v^2 \Delta t^2}{c^2}$$

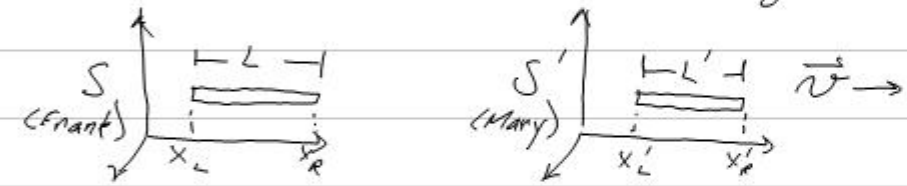
$$\left(1 - \frac{v^2}{c^2}\right)\Delta t^2 = \Delta t_p^2$$

$$\boxed{\Delta t = \gamma \, \Delta t_p}$$

So the observer in S' sees the round trip take <u>longer</u> since $\gamma > 1$.

Length Contraction Revisited

A metal rod is measured in two frames:



$L = x_R - x_L$

$L' = x'_R - x'_L$
(rod @ rest in S')

<u>proper length</u>: length measured in frame at rest with respect to rod. (here, it's $L' = L_p$)

Frank measures x_R & x_L <u>at same time</u>, t ($= t_R = t_L$), so that

(needed because of motion)

$$x'_R - x'_L = \frac{(x_R - x_L) - v\,(t_R - t_L)}{\sqrt{1 - v^2/c^2}}$$

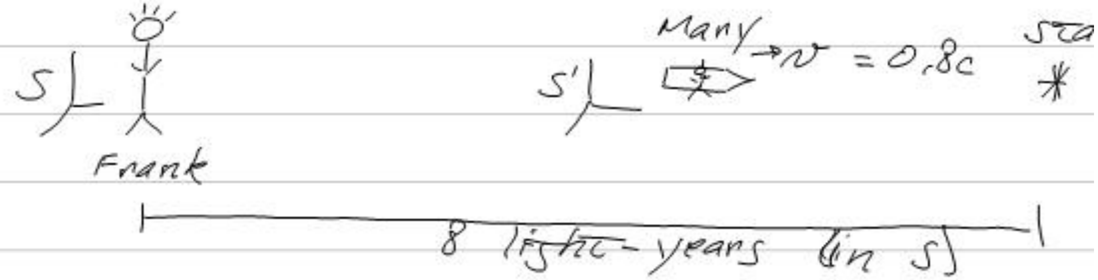
(Note: primed times $\neq$.)

$$L_p = \frac{L - 0}{\sqrt{1 - v^2/c^2}} = \gamma L$$

$$\therefore \boxed{L = L_p/\gamma}$$

- length look smaller to S because of relative motion

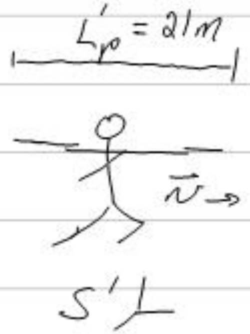
Twin Paradox

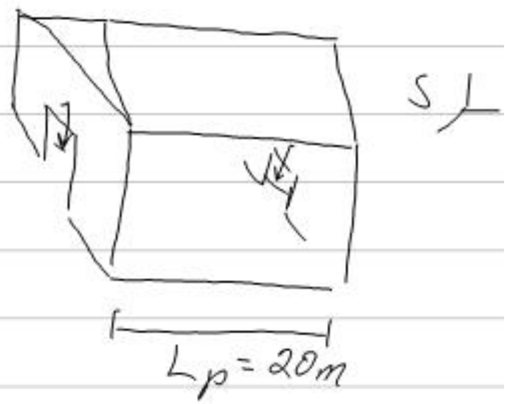


- to Frank, Mary travels + appears to age more slowly
  - he watches her clock
  - he is in an approximate IRF on Earth
  - Mary's trip takes $2 \times 8\,\text{l-y}/0.8c$
    $\Delta\tau = 20\,\text{yrs}$
  - Mary ages $20\,\text{yrs} \times \sqrt{1-0.8^2} = 12\,\text{yrs} = \Delta\tau_p$
- But Mary can seemingly say she sees same motion by Frank
  - so her trip takes 12yrs and Frank should look to age less than this
  - but she isn't in an IRF
    - needs to turn around (accelerate)

$\therefore$ we can only use Frank's obs.

Pole-in-the-Barn Paradox





Is it possible for pole to fit in barn with both doors closed & opened simultaneously such that the do not hit the pole?

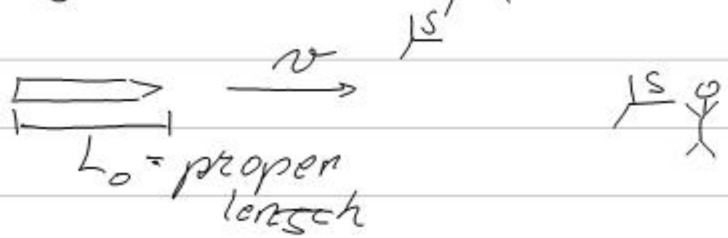
In frame of barn
- see length $L = L_p'/\gamma$
  - if $v = \frac{1}{3}c$ then $\gamma = 1.06$ and $L = 19.7m$!

Fits!

Major problem: in runner's IRF,
- pole won't shrink, but barn shrinks by 6%!!!

Simultaneity the key here (i.e. doors in S doin close @ same time.)
1 - far door closes + then opens
2 - leading end of pole moves out of barn
3 - trailing end enters barn
4 - 'front' door closes

Example: Length Contraction



rocket travels velocity, v

what velocity gives 1% smaller length in stationary frame

$$L = L_0 / \gamma = \sqrt{1 - \frac{v^2}{c^2}}\, L_0$$

$$\frac{L}{L_0} = 0.99 = \sqrt{1 - \frac{v^2}{c^2}}$$

$$0.99^2 = 1 - \frac{v^2}{c^2} = 1 - \beta^2$$

$\beta = v/c$

$$-\beta^2 = -1 + 0.98$$

$$\beta = \sqrt{1 - 0.98} = \sqrt{0.02}$$

$$= \boxed{0.142}$$

Experimental Verification: Muon Decay

- protons, etc. from astrophysical events (e.g. SNe)
  - collide with atmospheric atoms @ relativistic velocities
    - nuclei destroyed ⇒ "cosmic ray shower"

In the showers, $\pi^{\pm}$ produced & decay quickly to unstable μ's.

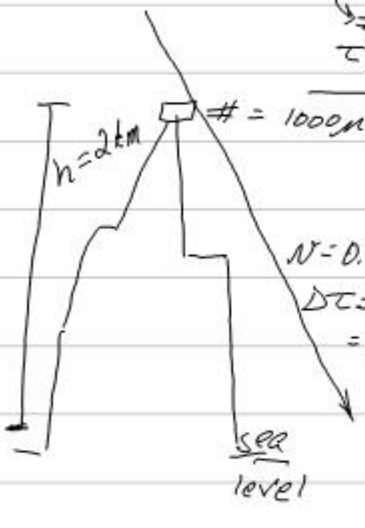
half-life: $\tau^{\mu}_{1/2} = 1.52\,\mu s$

Exponential Decay Law

$$\boxed{N(\tau) = N_0 \exp\left(-\frac{\overbrace{\ln(2)}^{0.693}\,\tau}{\tau^{\mu}_{1/2}}\right)}$$

$N(\tau)$: # @ time, τ

N_0: # @ $\tau = 0$



At sea level: (in our frame)

$\tau' = \gamma \tau_p \; (= \gamma \tau_{1/2})$

$\gamma \to 5$ for $v = 0.98c$

$\underline{\tau' = 7.5\,\mu s}$

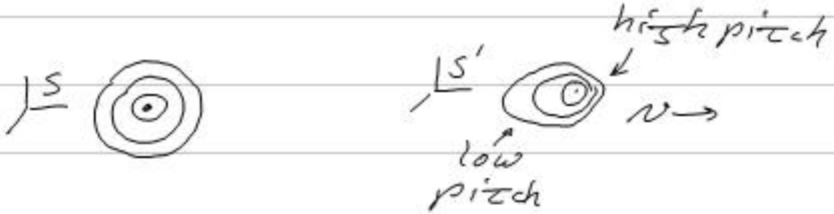
$\therefore N = 1000 \exp\left(\frac{-0.693\,(6.8 \times 10^{-6}\,s)}{7.5 \times 10^{-6}\,s}\right)$

$= \boxed{533\ \mu\text{'s}}$

542 observed & only 45 expected classically.

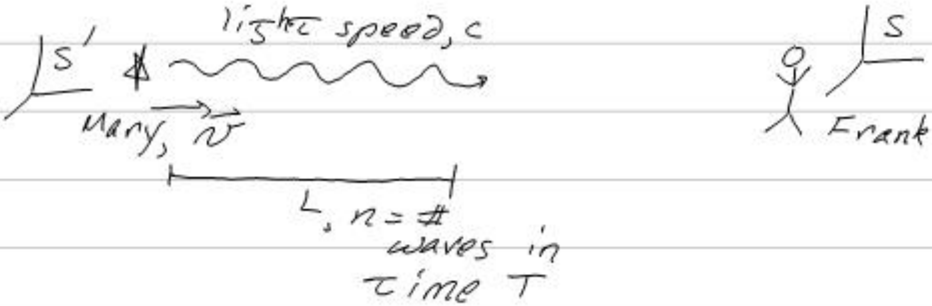
Doppler Effect

Consider sound waves



sound propagates in air (its medium)
→ so velocity same in medium
→ bunches up wave @ leading side of source

Consider light by analogy



Both observers agree on c & n,
but not on ν, λ

L = length of wave train during interval T

$= \underline{cT - vT}$ (In Frank's frame)

For the light:

$$\lambda = (cT - vT)/n$$

$$f = c/\lambda = cn/(cT - vT)$$

we don't want to measure n, but rather find f as it relates to f_0 (frequency seen by Mary)

The # of waves in S' frame calculated as

$$n = f_0 T_0'$$ (proper time is @ rest in S')

The relationship between proper time and time seen by Frank (observer)

$$T_0' = T/\gamma$$

$$\therefore \underline{\underline{n = f_0 T/\gamma}}$$

By substitution

$$f = \left(c f_0 \cancel{t}/\gamma\right) / \left(c\cancel{t} - v\cancel{t}\right)$$

$$= \cancel{c} f_0 / \gamma\left(\cancel{c} - v/c\right) = \frac{f_0}{\gamma}\,\frac{1}{1 - v/c}$$

$$= f_0\,\frac{\sqrt{1 - v^2/c^2}}{1 - v/c} = f_0\,\frac{\sqrt{(1 - v/c)(1 + v/c)}}{1 - v/c}$$

$$\boxed{f = f_0\,\frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}}}$$ Relativistic Doppler Effect

$\beta = v/c$ → Note: convention here has $\beta < 0$ when source & receiver receding

Example: what velocity (β) blueshifts light from 670nm to 540nm?

$$f/f_0 = \lambda_0/\lambda = \sqrt{1+\beta}/\sqrt{1-\beta}$$

$$\frac{670}{540} = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \Rightarrow 1.24\sqrt{1-\beta} = \sqrt{1+\beta}$$

square both sides:

$$1.54(1-\beta) = 1 + \beta$$

$$1.54 - 1 = \beta + 1.54\beta$$

$$0.54 = 2.54\beta$$

$$\boxed{\beta = 0.22}$$

(66,000 km/s!)

Application of Doppler

When $\beta < 0$, then $f < f_0$ (shift to red, or 'redshift')

→ analogous for light to case with sound of a siren receding
→ pitch gets lower
↓
longer λ!

It's been observed that galaxies that are further away are receding from us

Expanding universe, Big Bang

Let's imagine a galaxy with a recessional velocity 6000 km/s.

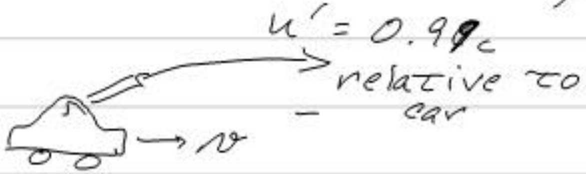
What is redshift?

$$\frac{f}{f_0} = \frac{\sqrt{1-0.02}}{\sqrt{1+0.02}} = \frac{\sqrt{0.98}}{\sqrt{1.02}}$$

$= \boxed{0.98}$ so redder light, longer λ

Addition of Velocities

Consider how a moving object, with velocity v, which sends out a projectile at relative velocity, u, looks to a stationary observer.



sees u velocity of projectile. What is u?

We use Lorentz Transformations to calculate "u".

$$dx = \gamma(dx' + v\,dt')$$

$$dy = dy', \quad dz = dz'$$

$$dt = \gamma\left(dt' + (v/c^2)\,dx'\right)$$

Recall that $u_x = dx/dt$

$$\underline{\underline{u_x}} = \frac{\gamma(dx' + v\,dt')}{\gamma(dt' + (v/c^2)\,dx')} \times \frac{1/dt'}{1/dt'}$$

$$= \boxed{\frac{u_x' + v}{1 + \frac{v}{c^2} u_x'}}$$

Also, for $\perp$ directions,

$$\boxed{\begin{aligned} u_y &= u_y' \Big/ \left(\gamma\left(1 + \left(\tfrac{v}{c^2}\right) u_x'\right)\right) \\ u_z &= u_z' \Big/ \left(\gamma\left(1 + \tfrac{v}{c^2} u_x'\right)\right) \end{aligned}}$$

Try the calculation,

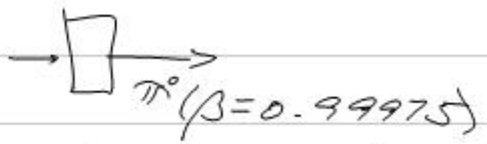
$u_x' = 0.99c$ $\qquad u_y = 0.1c$

$v = 0.6c$ $\qquad u_z = 0$

$$u_x = \frac{0.99c + 0.6c}{1 + \frac{(0.6c)(0.99c)}{c^2}} = \underline{\underline{0.997c}}$$

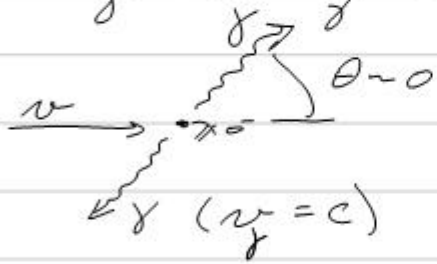
Test of Velocity Addition

Imagine a proton incident on a target



A short $\tau_{1/2}$

In the "rest frame" of the π°:



What does the observer in frame where π° has velocity, v, see?

$$u = \frac{c + 0.99975c}{1 + \frac{(0.99975c)c}{c^2}} = c\,!$$

Galilean Relativity would give $u = 1.99975c$. Experiment performed by measuring time-of-flight over 30m

$$\Longrightarrow \boxed{v_\gamma^{exp.} = c}$$

Relativistic Momentum

Consider Newton's 2nd Law,

$$\vec{F} \equiv \frac{d\vec{p}}{dt} = m\vec{a}$$

How does this behave under Lorentz transformation?

$u_{M_x} = v$

$u_{M_y} = -u\sqrt{1 - v^2/c^2}$

$u_{F_x} = 0$

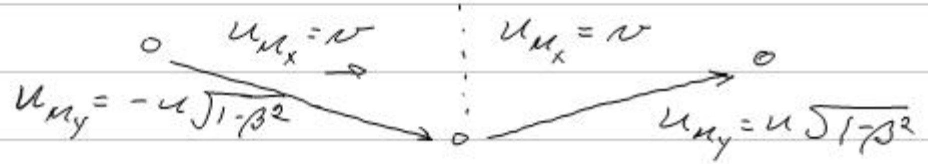
$u_{F_y} = u_0$

- Two balls in two frames with 'transverse' velocity relative to ea. other
- elastic collision: measure velocity before + after → gives Δp

For Frank, $p_{F_y} = mu_0$ and $\vec{p}$ only in y-direction.

$$\underline{\underline{\Delta p_y}} = \Delta p_{F_y} = \boxed{-2mu_0}$$

But look @ Mary's ball as measured by Frank (same units)



	Before	After	Δp
p_{M_x}	mv	mv	0
p_{M_y}	$-mu_0\sqrt{1-\beta^2}$	$+mu_0\sqrt{1-\beta^2}$	$+2mu_0\sqrt{1-\beta^2}$

$\neq \Delta p_{F_y}$!

Do we violate momentum conservation? $(\Delta p_{M_y} \neq \Delta p_{F_y})$

$\rightarrow$ can be resolved if $\boxed{\vec{p} = \gamma m \vec{v}}$ which allows $\vec{F} = d\vec{p}/d\tau$ to be Lorentz invariant.

So it takes enormous force to produce small change in velocity when $v \sim c$.

- cannot reach $v = c$ for massive particles (would require ∞ force)

Example

$$\overset{v = 0.5c}{o \longrightarrow}$$

$p = \gamma m v$

What velocity will give a Δp of 1%, 10%, 100%?

Define $$\rho = \frac{p(v)}{p_0} = \frac{\gamma(v)\, m v}{\gamma(0.5c)\, m (0.5c)}$$

$$= \frac{\gamma v}{\frac{1}{\sqrt{1-0.25}}(0.5c)} = \frac{\gamma v}{0.58c}$$

Rearranging gives

$$\rho(0.58)\sqrt{1-\beta^2} = v/c$$

Squaring both sides =>

$$\rho^2 (0.58)^2 (1-\beta^2) = \beta^2$$

$$(0.58)^2 \rho^2 = \beta^2 \left(1 + (0.58\rho)^2\right)$$

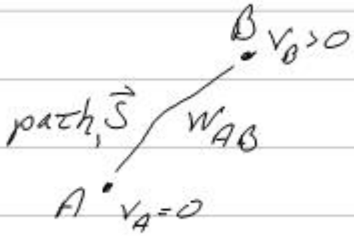
$$\beta = 0.58\rho\left(1 + (0.58\rho)^2\right)^{-1/2}$$

a) $\rho = 1.01$ => $\boxed{\beta = 0.508}$

b) $\rho = 1.10$ => $\boxed{\beta = 0.538}$

c) $\rho = 2.00$ => $\boxed{\beta = 0.757}$

Relativistic Energy



Kinetic energy: work done on a particle by a net force

$$W_{AB} = \int_A^B \vec{F}\cdot d\vec{s} \; \left(= KE_A - KE_B\right)$$

$$= \int \frac{d\vec{p}}{d\tau}\cdot d\vec{s}$$

$$= \int \frac{d}{d\tau}(\gamma m\vec{u})\cdot \overbrace{(\vec{u}\,d\tau)}^{\text{path, } \vec{s}}$$

Since $\gamma = f(u)$, we have the following,

$$W = KE = m\int_0^{\gamma u} u\, d(\gamma u) = m\underbrace{\int_0^{v} (v^2 d\gamma + v\gamma\, dv}$$

$\left(= m\int_0^{v} v\, dv = \frac{1}{2} m v^2 \text{ when } v \ll c\right)$

Using integration by parts,

$$\underline{\underline{KE}} = m(\gamma c^2 - c^2)$$

$$= \boxed{mc^2(\gamma - 1)}$$

Mass, Energy & Momentum

The KE expression suggests the following one:

$$\boxed{\gamma mc^2 = KE + mc^2}$$

$\underbrace{\gamma mc^2}_{\text{Total Energy, } \mathcal{E}_{TOT}}$ $\underbrace{mc^2}_{\text{"Rest" Energy, } \mathcal{E}_0}$

- a massive particle @ rest still has an associated energy, $\mathcal{E}_0 = mc^2$

Mass $\Longleftrightarrow$ Energy

We can also express energy in terms of momentum,

$$p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \beta^2$$

$$= \gamma^2 m^2 c^4 (1 - 1/\gamma^2) = \gamma^2 m^2 c^4 - m^2 c^4$$

$$p^2c^2 = \underbrace{\gamma^2 m^2 c^4}_{\mathcal{E}_{TOT}^2} - \underbrace{m^2 c^4}_{\mathcal{E}_0^2}$$

$$\therefore \boxed{\mathcal{E}_{TOT}^2 = p^2c^2 + m^2c^4}$$

If converted 4 kg H (2 kg H^+, 2 H^-) to energy $\Rightarrow$ same $\mathcal{E}$ as all gasoline burned in U.S. per year!

Units:

SI units can be very inconvenient

Charge: of electron, q_e, is tiny $= 1.6 \times 10^{-19}$ C

$\therefore$ use 1 "e" as a unit

Energy:

W = work to accelerate 'e' across 1 V of potential

$= q_e (1V) = \underline{1.6 \times 10^{-19} J}$

= 1 "electron-volt" (eV)

Mass:

m_p = mass of proton $= 1.7 \times 10^{-27}$ kg

- instead consider as rest energy $\div c^2$

$\therefore m_p = E_0/c^2 = 1.5 \times 10^{-10}$ J/c^2

$= \underline{938 \text{ MeV}/c^2}$

So mass & energy units same to within const. of $1/c^2$.

(Note: in particle physics, we sometimes even define units so $c = 1$!)

Binding Energy

By Bringing mutually attractive particles-

→ there is energy in the force that binds

→ on the macroscopic scale, chemical (i.e. electromagnetic) ~~reaction~~ bonds too little energy to notice.

Atomic nuclei different (strong force)

$$\boxed{E_{Binding} = \sum_i m_i c^2 - M_{\substack{Bound \\ System}} c^2}$$

o→←o

$M < 2m_p$

γ $\Delta M = E_\gamma$

Hydrogen: 1 proton 938.27 MeV

1 neutron 939.57 MeV

Deuteron (pn): 1875.6 MeV

$< m_p + m_n$ by 2.23 MeV

$E_{Binding} = 2.23$ MeV is 2×10^5 times the binding energy of an electron in a Hydrogen atom (13.6 eV)

strong nuclear force