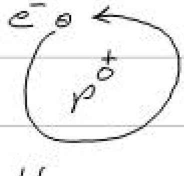
Consider an atom:

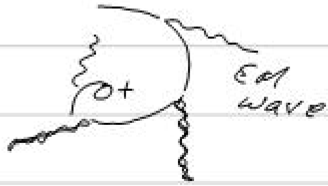


H atom

Classical idea that e^- revolve around positive nucleus: planetary model

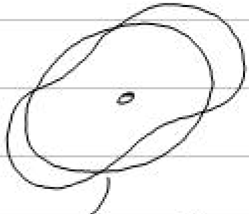
But accelerating e^- emits EM waves:

Collapse of atom!



Using de Broglie idea!

- have a standing wave in orbital around nucleus (only integer # waves in orbit)



$2\pi r = n\lambda$

$$2\pi r = n\lambda = n\frac{h}{p}$$

$$pr = n\frac{h}{2\pi}$$

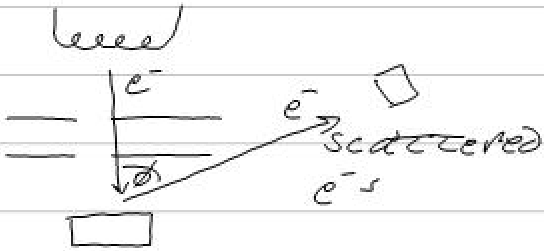
$$\boxed{L = n\hbar}$$

condition for stable atoms!

→ known experimentally to be case (Bohr model)

So how do we establish wave-like nature of e^-?

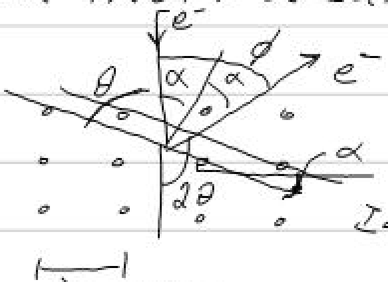
<u>Electron Scattering</u>



Ni crystal

Look @ intensity vs. scattering angle

In more detail:



$D = 0.215\,nm$ inter-atomic distance

d = lattice plane spacing

$$\underline{d = D \sin\alpha}$$

Condition for constructive interf.

$$\underline{\underline{n\lambda}} = 2d\sin\theta = 2d\cos\alpha$$
$$= 2D\sin\alpha\cos\alpha$$
$$= D\sin 2\alpha = \boxed{D\sin\phi}$$

Turning this into a measure of λ:

$$\lambda = (D \sin\phi)/n$$

$n = 1$ if 1st max. @ 50° ϕ =

$$\underline{\underline{\lambda_e}} = (0.215\,nm) \sin 50^\circ = \underline{\underline{0.165\,nm}}$$

→ uses directly wave-like picture describing classical EM-waves

How can we verify de Broglie?
- Experiment sets $\phi = 50^\circ$ + varies K.E. of e^-'s

I_{max} @ $\phi = 50^\circ$ when $\underline{\underline{K.E. = 54\,eV}}$

Calculate momentum

$V_0 = 54\,V$ to get to $K.E. = 54\,eV$

$$K.E. = eV_0 = \frac{p^2}{2m}$$

$$\underline{p = \sqrt{2meV_0}}$$

We can now calculate the de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{\sqrt{2 mc^2 eV_0}}$$

E_0 for $e^- = 0.511 MeV$

$$= \frac{1240 \text{ eV}\cdot\text{nm}}{\sqrt{2(0.511\times10^6 eV)(eV_0)}}$$

$$= \frac{1.226 \text{ nm } V^{1/2}}{V_0}$$

$$\lambda_e = \underline{\underline{0.167 \text{ nm}}}!!!$$

The agreement of the de Broglie derived result with the classical wave optics result confirms the wave-particle nature of e^-s.

The geometry of the crystals is of same order to λ_e and it causes interference of e^- consistent with Bragg's law.

Problem

What is λ_e when $E = 50\ \text{GeV}$?

$$\lambda_e = h/p = h/(E/c) \qquad (p \sim E/c \text{ @ high energy})$$

$$= \frac{4.14\times10^{-15}\ \cancel{\text{eV}}\cdot\text{s}}{50\times10^{9}\ \cancel{\text{eV}} / 3\times10^{8}\ \text{m/s}}$$

$$= \boxed{2.5\times10^{-17}\ \text{m}}$$

this is 1.2% of proton's diameter

Problem

thermal neutrons $K.E. = 0.025\,eV$

interatomic spacing $\delta = 0.45\,nm$

Where is the 1st order ($n=1$) Bragg peak?

$$\lambda_n = \frac{h}{p} = \frac{h}{\sqrt{2m(K.E.)}} = \frac{4.14\times10^{-15}\,eV\cdot s}{\sqrt{2\,(939\times10^{6}\,eV)/c^2\,(0.025\,eV)}}$$

de Broglie

$$= 1.8\times10^{-10}\,m = \underline{0.18\,nm}$$

$$\lambda_n = \frac{2d\sin\theta}{n} = \delta\sin\phi$$

Bragg

$$\phi = \sin^{-1}\left(\frac{\lambda}{\delta}\right) = \sin^{-1}\left(\frac{0.18\,nm}{0.45\,nm}\right)$$

$$\boxed{\phi = 23.6^\circ}$$

Wave - Particle Duality

So we have something which behaves as both a wave & a particle
- in fact, exhibits facets of one or the other in different situations
  - → when 'd' $\gg \lambda$: particle
  - → when $d \approx \lambda$: wave-like

depends on measurement making!

d | Double slit Exp.

Consider "slowing down" ⇒ very low intensity

Individual 'hits' still "build-up" to interference Intensity distribution ⇒ particles interfere w/self!

the wave-like behavior: probability to get γ with $E = h\nu$

⇒ interference of Probability waves ⇒ Intensity distribution

$$\boxed{N_\gamma \propto \text{Prob}_\gamma \propto \langle E^2 \rangle}$$

→ E-field

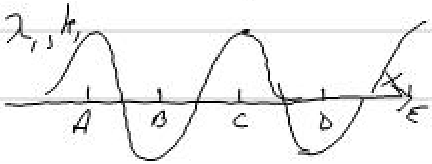
Wave Packets

When multiple waves in same (x,t), they behave independently

principle of superposition can add waves linearly to get net amplitude

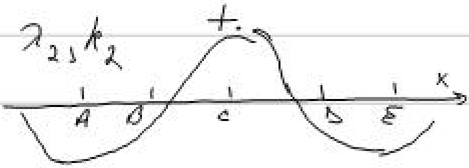
It is possible, by summing a large # of waves to get a localized (i.e. particle-like) amplitude.

λ_1, k_1



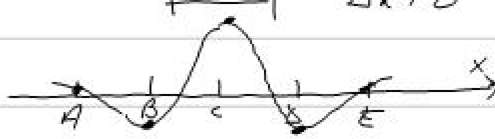
$\Delta k = 0, \ \Delta x = \infty$

λ_2, k_2



=

$\Delta x \neq \infty$ $\quad$ $\Delta k \neq 0$



+

a wave-packet is where a large amplitude exists

many wave #':

$\Delta k \neq 0$

$\Delta x \neq 0$

Wave Motion

Classical wave Equation

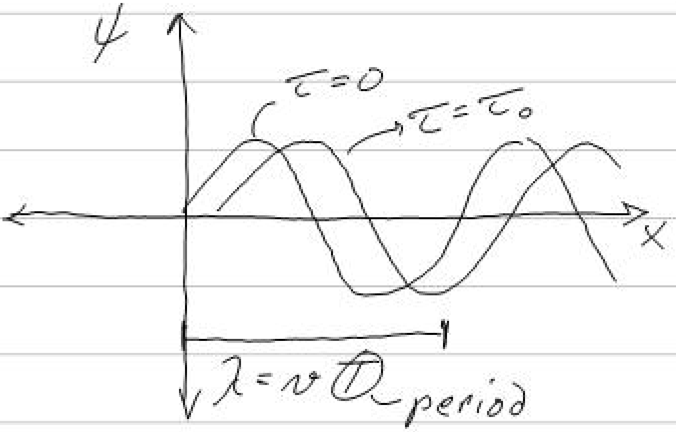
$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

v → phase velocity of wave

A general solution is

$$\boxed{\psi(x,t) = A \sin\left(\frac{2\pi}{\lambda}(x - vt)\right)}$$

$\psi(x,t)$ → instantaneous amplitude (could be vertical displacement of string)



$v = \lambda / T = \frac{\omega}{k}$

$k \equiv 2\pi/\lambda$ (wave #)

$\omega = 2\pi/T$ (angular freq.)

$$\psi(x,t) = A \sin(kx - \omega t + \phi)$$ (most general)

Quantum Wave Eq.

Since particles are de Broglie waves:

- adhere to wave Equation
  1) a probabilistic intepretation
  2) depends on potential field a particle experiences
  3) obtain a wave function telling probable values of: x, E, p ("observables")

$$\boxed{i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V \psi(x,t)}$$

→ potential

<u>One Dimensional</u>
<u>Time-dependent</u>
<u>Schrödinger Equation</u>

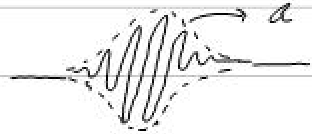
→ obtained because it describes observed results

<u>Not</u> same form as classical wave equation

→ so this is something new, physically

Uncertainty:

Gaussian wave packet:



→ a Gaussian envelope around wave packet

It can be shown that such a wave satisfies

$$\underline{\Delta k \, \Delta x = 1/2}$$

since $k = 2\pi/\lambda$, this means λ and x are uncertain as given in this relation

- Consider a de Broglie wave,

$$k = 2\pi/\lambda = 2\pi/(h/p) = p/\hbar$$

- then,

$$\Delta k \, \Delta x = \frac{\Delta p}{\hbar} \Delta x = 1/2$$

or,

$$\boxed{\Delta p \, \Delta x = \hbar/2}$$

this is smallest this product can be. Written

(1) $$\boxed{\Delta p \, \Delta x \geq \hbar/2}$$ Heisenberg Uncertainty Principle

A similar relation for Gaussian wave packets,

$$\Delta\omega \, \Delta t = 1/2$$

gives us Energy-time uncertainty

(2) $$\boxed{\Delta E \, \Delta t \geq \hbar/2}$$

(1) + (2) are intrinsic uncertainties,

→ not measurement errors

→ reflect degree to which p_x, x, E + t are definable for wave-particles

∴ provide fundamental limit to conservation of Energy + momentum

Note: if $\hbar \to 0$, live in completely classical universe

Problem

Consider π^0 decaying to 2γ's

$$\tau_{1/2} = 10^{-16}\,s$$

$$m_{\pi^0} = 136\ MeV/c^2$$

What is the "Energy width"?

This asks what is the range of masses for particle decaying this way.

Use Uncertainty Principle:

$$\Delta E \Delta \tau \geq \hbar/2$$

In this context, width sometimes labelled Γ (capital γ).

$$\frac{\Gamma}{2} \Delta\tau \geq \hbar/2$$

$$\underline{\Gamma} = \hbar/\Delta\tau = \frac{6.6\times\cancel{10^{-16}}\,eV\cdot\cancel{s}}{\cancel{10^{-16}}\,\cancel{s}}$$

$$= \underline{\underline{6.6\,eV}}$$

Bohr Model of Hydrogen Atom

Classical description plus some key assumptions

Stable atoms: "stationary states" exist where e^- don't radiate γ

Energy states:

- emission or absorption of EM radiation happens in transition between two "stationary states"

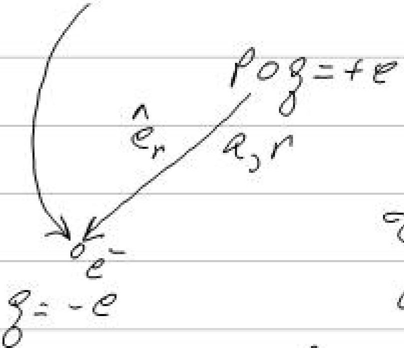
$$\underline{E = E_1 - E_2 = h\nu}$$

most difficult to accept

- K.E. of e-nucleus system $= nh\nu$ — frequency of revolution
  - ↳ equivalent to angular momentum

$$\boxed{L = n\hbar}$$ (de Broglie condition)

Classically,



$$\vec{F}_e = \frac{-1}{4\pi\epsilon_0}\,\frac{e^2}{r^2}\,\hat{e}_r$$
$$= -m\vec{a}$$

There is radial _____ acceleration, $a_r = \cancel{v^2/r}$

$$\frac{e^2}{4\pi r^2 \epsilon_0} = m\left(\frac{v^2}{r}\right)$$

Which gives,

mass of e^-

$$\boxed{v = e\Big/\sqrt{4\pi\epsilon_0\, m_e\, r}}$$

radius of atom

$\ll c$ $\qquad \sim 5\times 10^{-11}\,m$

While from de Broglie, we have

$$L = |\vec{r}\times\vec{p}| = mvr = n\hbar$$

$$\hookrightarrow \boxed{v = n\hbar/mr}$$

corresponds to

$$K.E. = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 r^2}$$

We can equate these

$$v^2 = \frac{n^2 \hbar^2}{m^2 r^2} = \frac{e^2}{4\pi\epsilon_0 m r}$$

Rearranging in terms of 'r' gives

$$\underline{r_n} = n^2 \underbrace{\frac{\hbar^2 4\pi\epsilon_0}{me^2}}_{a_0 \text{ (Bohr radius)}}$$

$$= \boxed{n^2 a_0}$$

So r_n is quantized! Lowest 'n' gives

$$\underline{a_0 = 0.53 \times 10^{-10}\, m}$$ (very close to observation)

<u>Energy levels</u>

→ using classical total energy

$$E_n = \underbrace{\frac{e^2}{8\pi\epsilon_0 r_n}}_{K.E.} + \underbrace{\frac{-e^2}{4\pi\epsilon_0 r_n}}_{V} = \underline{\frac{-e^2}{8\pi\epsilon_0 r_n}}$$

$$= \underline{-E_0/n^2} \Longrightarrow$$ quantized since r_n is quantized

Lowest energy

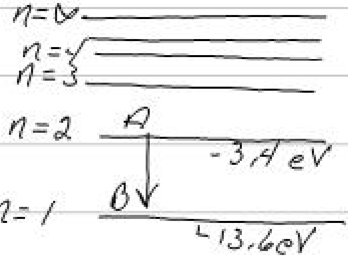
$$\underline{E_1 (n=1)} = \frac{-e^2}{8\pi\epsilon_0}\left(\frac{me^2}{\hbar^2 4\pi\epsilon_0}\right)$$

$$= \boxed{\frac{-me^4}{2\hbar^2 (4\pi\epsilon_0)^2}} = -E_0 = \underline{-13.6eV}$$

Need > 13.6eV to ionize H

Derivation of Rydberg Equation

Emission of a γ occurs when an "excited" atom "decays" or transitions to a lower energy level.



$$E_\gamma = h\nu = E_A - E_B$$

Since $\lambda\nu = c$,

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{(E_A - E_B)/h}{c} = \frac{\left(-E_0/n_A^2 - \left(-E_0/n_B^2\right)\right)}{hc}$$

$$\boxed{\frac{1}{\lambda} = \frac{E_0}{hc}\left[\frac{1}{n_B^2} - \frac{1}{n_A^2}\right]}$$

= Rydberg constant, R_∞

$$= \frac{me^4}{2\hbar^2(4\pi\epsilon_0)^2}\frac{1}{hc} = \boxed{\frac{me^4}{4\pi\hbar^3 c(4\pi\epsilon_0)^2}}$$

R_∞ all in terms of known fundamental constants

→ corresponds to ∞ nuclear mass

Balmer => $n_B = 1$

Lyman => $n_B = 2$

Paschen => $n_B = 3$

Notes on Bohr Model

- can only describe atoms with one e^-
- intensities of spectral lines not understandable
- fine structure of some lines not predictable
- no explanation of binding of atoms to molecules

Not a coherent Quantum model

→ many classical ideas retained

e.g. idea of rotating e^- is fundamentally wrong

- Need wave description with probability distribution