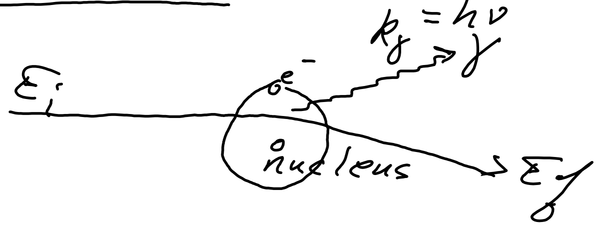
Bremsstrahlung

Energy loss from charged particle passing thru E-field of atoms in material

(Note: synchrotron radiation not in matter.)



$$\underline{\underline{E_i = E_f + \boxed{k_\gamma}}}$$ — EM radiation emitted

semiclassical cross-section

$$\boxed{\frac{d\sigma}{dk_\gamma} \simeq 5 \frac{e^2}{\hbar c} z^4 Z^2 \left(\frac{mc}{Mv}\right)^2 \frac{r_e^2}{k_\gamma} \ln \frac{Mv^2\gamma^2}{k_\gamma}}$$

Notes:

$$\frac{d\sigma}{dk_\gamma} \propto \frac{1}{M^2}$$ M = incident particle mass

→ mainly e's + high energy μ's

$\propto Z^2$ heavy elements most effective in causing bremsstrahlung

$\propto \frac{1}{k_\gamma}$ cross section decreases with increasing γ energy incident particle interacts with:

- electrons (σ goes as Z)
- nuclei (σ goes as Z^2)
  - ↳ contribution dominates for most elements

$\therefore$ $Z^2 \rightarrow Z^2 + Z = Z(Z+1)$ in $d\sigma/dk_\gamma$ expression

Electrons screen charge of the nucleus

classically: if impact parameter $>$ radius of atom

→ sharp drop-off in bremsstrahlung cross section

quantum mechanically:

→ analogous when distance greater than $\hbar/(\text{momentum transfer from } e^- \text{ to nucleus})$

Energy Loss

For an electron traversing some material

$$\left.\frac{dE}{dx}\right|_{rad.} = \int_0^{k_{max}} k_\gamma n_a \frac{d\sigma}{dk_\gamma}\, dk_\gamma$$

$$= n_a E_i \underbrace{\left[\frac{1}{E_i}\int_0^{k_{max}} k_\gamma \frac{d\sigma}{dk_\gamma}\, dk_\gamma\right]}_{\sigma_{rad} = 4\alpha Z^2 r_e^2 \left[\ln(183\, Z^{-1/3}) + \frac{1}{18}\right]}$$

$$\boxed{\left.\frac{dE}{dx}\right|_{rad.} = 4\alpha Z^2 r_e^2 n_a E_i \left[\ln(183\, Z^{-1/3}) + \frac{1}{18}\right]}$$

when $E_i \gg mc^2/(\alpha Z^{1/3})$: condition for complete screening of nucleus at large impact parameter (Q.M. → small momentum transfer.

$$\left.\frac{dE}{dx}\right|_{rad.} \propto E_i$$

- faster dependence than $\left.dE/dx\right|_{coll.} \propto \ln E$

∴ radiative energy loss dominates at high energy

$$\left.\frac{dE}{dx}\right|_{rad.} \propto Z^2$$

- stronger dependence than $\left.dE/dx\right|_{coll.} \propto Z$

∴ heavy elements cause radiation to dominate sooner

Radiation Length

Consider electron $\frac{dE}{dx}\Big|_{rad}$ expression

- Define characteristic length:

Radiation Length $$\boxed{X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln(183\, Z^{-1/3})}}$$

Then

$$\frac{-dE}{dx} \simeq \frac{E_i}{X_0}$$

Rearranging and integrating:

$$\int \frac{dE}{E} = -\int \frac{dx}{X_0}$$

$$\ln E = -x/X_0$$

$$\boxed{E = \exp(-x/X_0)}$$

$\therefore$ X_0 is the distance (or g/cm²) in which particle looses 1/e of its energy.

Atomic electron interactions mean

$$Z^2 \to Z(Z+1)$$

Critical Energy

For e^- (or e^+) in relativistic rise

Energy loss $\propto \ln(E)$

For Bremsstrahlung

Energy loss $\propto E$ (Dominates @ high E)

At what energy do the two lead to equal losses

$$\left.\frac{dE}{dx}\right|_{\text{ioniz.}} = \left.\frac{dE}{dx}\right|_{\text{rad.}}$$

For solids, a good rule is

$$E_{\text{crit.}} \approx \frac{610\text{ MeV}}{Z + 1.24}$$

For particles beyond $e^\pm$, mass^2 effect:

$$E^{\mu}_{\text{crit.}} \approx E^{e}_{\text{crit.}} \left(\frac{m_\mu}{m_e}\right)^2 = (20.7\text{ MeV})(4\times10^4)$$

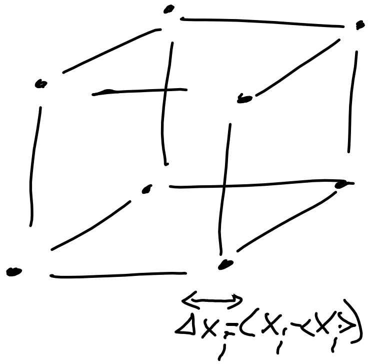
in iron ↗

$$= \underline{\underline{800\text{ GeV}}}\,!$$

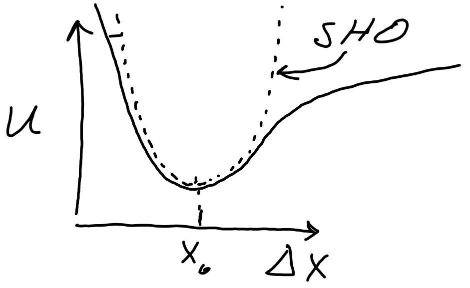
Energy Deposition to Phonons

A crystal lattice

a restoring force holds atoms/molecules in place



Potential Energy



In region around x_0 → approximately like simple harmonic oscillator (SHO)

Energy in vibrational modes
→ classically we speak of normal modes of vibration in crystal lattice

Quantum mechanically we have wave-particle phenomenon

<u>phonon</u>: quantum of vibrational energy in crystal lattice

Can correspond to energies much lower than for atomic transitions (ie. $\ll 1$ eV)

→ for condensed matter

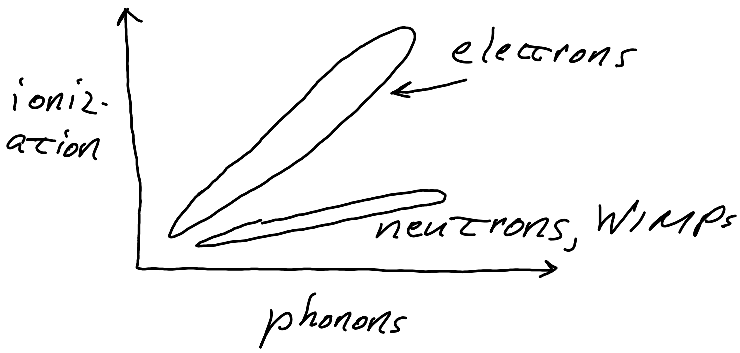
→ if $T \sim 0.1$ K $\Rightarrow$ phonon energy $\sim 10^{-5}$ eV

Phonons can be excited by energy depositions, ΔE, $\Delta T \propto \Delta E$

Example Particle Interactions

Electron interacting with lattice

- ionization energy deposition
- phonon deposition



Neutron interaction

→ mainly deposit energy to phonons → not much ionization

Weakly interacting massive particle (WIMP)

- conjectured to account for "dark matter" signatures in astrophysics

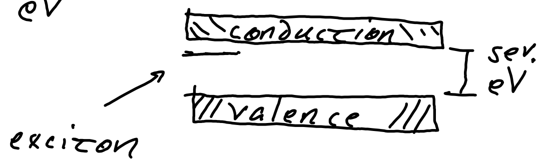
Scintillation

Production of a pulse of light promptly after passage of charged particle

- → related to fluorescence
- → two mechanisms: organic & inorganic scintillators
- → generally not linearly related to deposited energy
  - → when have alot of ionization

Inorganic Scintillator

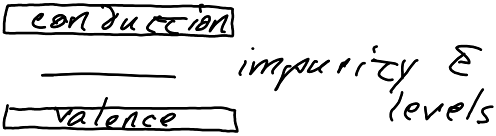
Band structure of material
- → electronic energy levels dispersed into discrete 'bands' of allowed energy
- → inorganic scintillator: an insulator
  - → conduction and valence band several eV apart



Passage of incident particle
- → frees e⁻'s to conduction band
  - → moves some distance
    - → forms e⁻-hole pair: "exciton"
      - slightly reduced energy level
- → exciton can interact with a phonon
  - → deexcitation + emission of a scintillation γ

Role of Dopants

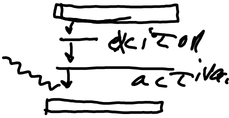
Impurities added as "activator centers" (eg. Thallium to NaI crystal)



Exciton can collide with activator center

→ Either:

- phonon production
- light: scintillation



Inorganic characteristics

→ large light output

→ long duration of pulse (~ ~~1.5 μs~~) $\tau \sim 100$'s ns

NaI: absorbs H_2O (hygroscopic)

BGO: non hygroscopic, but less light

material $X_0 \sim 1-3$cm

Organic Scintillator

Organic crystals, liquid scintillators, plastic scintillators

Incident particle

- Deposits energy to vibrational and electronic modes
-

Transmission of some energy

- by dipole couplings in material ("Forster mechanism")
- generally first step of deexcitation

Remaining deexcitation

- electron transition : scintillation
- this transition gives λ longer than initial absorption ∴ transparent to own scintillation

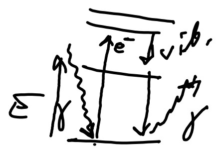
Wave Shifting

It's important for organic scintillator light λ to match photodetector

e.g. Photomultiplier tube ~400nm

Secondary material that

- absorbs scintillation
- reemits at longer λ
- non-radiative losses to vibrational modes



Organic scintillator properties:

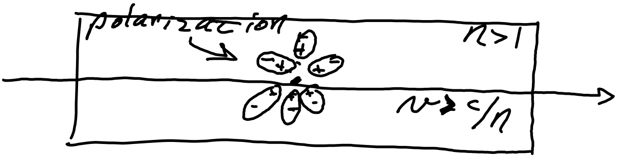
$\tau \lessapprox 2\,\text{ns}$

peak $\lambda \sim 400\,\text{nm}$

- less light generally than inorganics

Cerenkov Effect

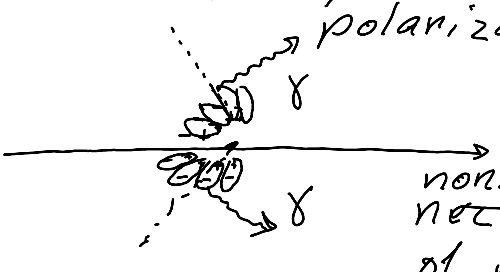
Charged particle passing thru a medium



- index of refraction n
  ∴ speed of light in medium $= c/n < c$
- as passes, dipoles align → change in electric field with time
  ∴ EM radiation emitted

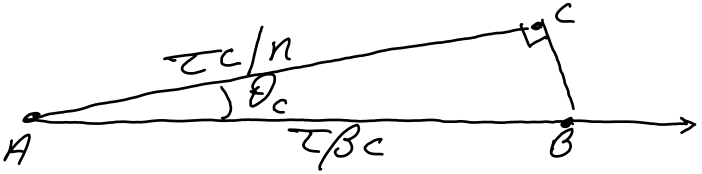
for $v << c/n$: symmetric emission + no net radiation

for $v > c/n$: asymmetry in polarization



nonzero coherent net radiation of photons

Cerenkov angle



Distance AB travelled by particle
$= \tau\beta c$

Distance AC travelled by photon
$= \tau c/n$

$$\therefore \underline{\underline{\cos\theta_c}} = \frac{\tau c/n}{\tau\beta c} = \boxed{\frac{1}{n\beta}}$$

Since $\cos\theta \leq 1$, there is threshold
→ light when $\underline{\beta \geq 1/n}$

Maximal condition: $\beta = 1$

$$\boxed{\theta_c^{max} = \cos^{-1}\left(\frac{1}{n}\right)}$$

→ angle of emission for ultra-relativistic particles

θ_c^{max} small in gases ($n = 1 + \epsilon$)
→ large for dense media
~45° for glass

Energy loss

→ when $Z > 7$: $< 1\%$ of $\frac{dE}{dx}\Big|_{\text{ioniz.}}$

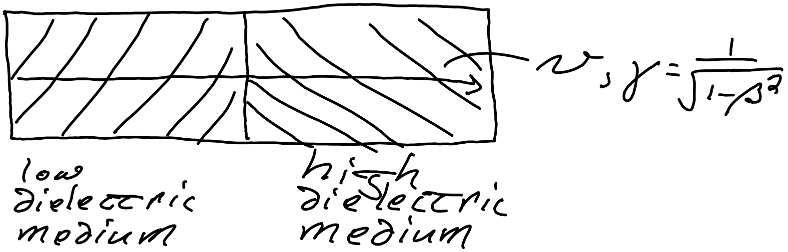
– for H/He $\sim 5\%$ of dE/dx

Emission properties

Since $n = n(\lambda)$, have an increase in θ_c at longer λ

- more γ's emitted in UV ($1/\lambda^2$ dependence)
  - cutoff by X-rays ($n = 1$)
- intensity $\sim 1\%$ of scintillation

Transition Radiation



Charged particle crosses dielectric boundary

<u>Low dielectric</u>: electric field extends over wide area

<u>High dielectric</u>: restriction in electric field extent. (polarization → screening)

As particle passes: sudden change in electric field

∴ EM radiation emitted: "Transition Radiation"

→ primarily X-rays

Light yield

$\langle N_\gamma \rangle \sim \frac{1}{2}\alpha$ (very low)

$\therefore$ need many dielectric transitions to get usable photon yield

Emission in cone of radius

$\theta = 1/\gamma$ in direction of particle

Interference means need

$\gamma \geq 1000$ to get radiation from many foils

Energy radiated at one boundary

$$S = \frac{1}{3}\alpha Z^2 \hbar \frac{\sqrt{4\pi N_e r_e^3}\, m_e c^2}{\alpha}\gamma$$

$$\underline{\underline{\propto \gamma}}$$

Important: when $\beta \sim 1$, Cerenkov no longer able to distinguish

10 GeV π^- : $\boxed{\gamma = 70}$, $\beta = 1$

10 GeV e^- : $\boxed{\gamma = 20{,}000}$, $\beta = 1$

→ photons for e^-, not for π^-