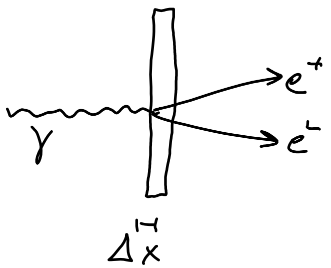
Photon Interactions w/Matter (66)

Photon interactions primarily destructive: disappears with resulting e^- (photoelectric effect) or e^+e^- pair



Loss photons in some depth of material Δx

$$dN = -\mu N dx$$

mass ~~attenuation~~ coefficient (per g/cm^2)

$$\frac{dN}{N} = -\mu dx$$

this gives after integration:

$$N(x) = N_0 e^{-\mu x}$$

intensity of 'beam' of photons decreases exponentially

Photoelectric Effect

$\gamma + \text{atom} \longrightarrow \text{atom}^+ + e^-$

$E_\gamma < 500\,\text{keV}$

Compton scattering

$\gamma + e^- \rightarrow \gamma + e^-$

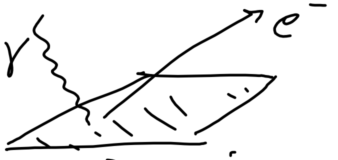
$100\,\text{keV} < E_\gamma < 10\,\text{MeV}$

Pair Production

$\gamma + \text{nucleus} \longrightarrow \text{nucleus} + e^+ e^-$

$E_\gamma \gtrsim \cancel{\text{few}}\ 1\ \text{MeV} \quad (2m_e)$

Photoelectric Effect



conducting plate

Must have $E_\gamma >$ binding energy ϕ

$E_e = E_\gamma - \phi$

↳ kinetic energy of e^-

→ for high energy, e^- roughly in direction of incident γ

Generally, cross section rises abruptly when each ionization threshold reached for individual atomic shells

Since destructive process
- γ absorbed
- $\therefore$ nucleus must permit conservation of momentum in the interaction
- inner (K-shell) electrons most important
  - nucleus closest

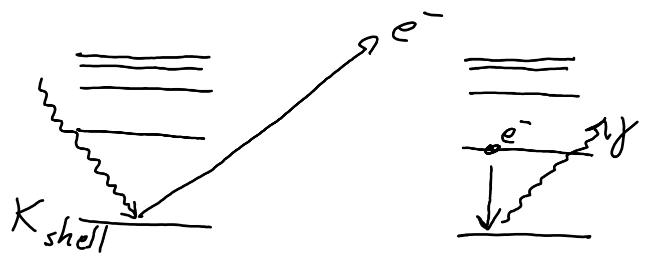
Cross section for K-shell electrons away from threshold

$$\boxed{\sigma^{K}_{photo} = 4\pi r_e^2 Z^5 \alpha^4 \left(\frac{m_e c^2}{E_\gamma}\right)}$$

$\propto E_\gamma^{-1}$: decreasing probability with energy

$\propto Z^5$: strong dependence on material

X-ray Emission



hole in K-shell filled

Energy level transitions

$$\boxed{E_\gamma = R\,(Z-1)^2\left(\frac{1}{n^2}-\frac{1}{m^2}\right)}$$

Moseley's Law

Rydberg constant (13.6eV)

Example: L→K transition

$m=2,\ n=1$

$$E_\gamma = \frac{3}{4} R\,(Z-1)^2$$

$\sim$ 7.5 keV (X-ray)

λ

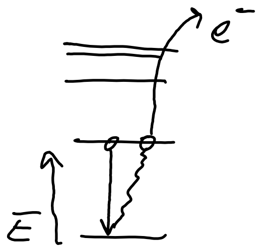
Auger Electrons

Energy transfer from
deexciting electron

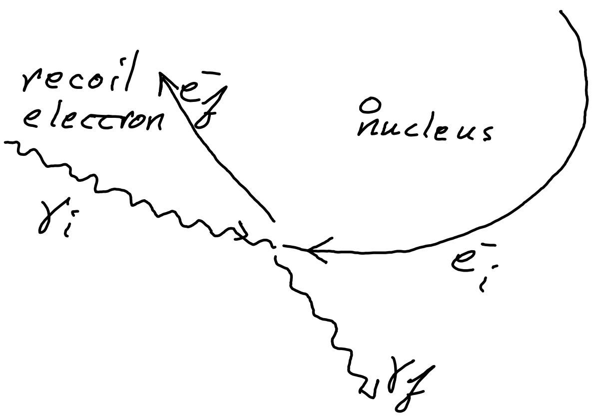
→ can interact with another
e^- in the same atom

→ maybe enough for ionization

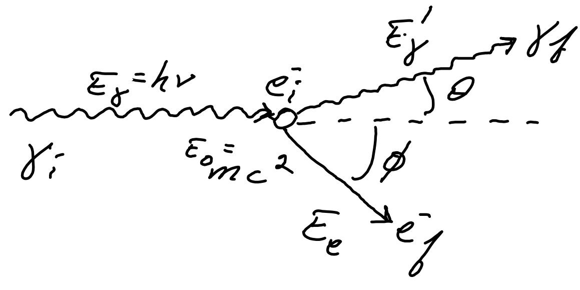
$E_{e2} << E_e$ (original e^-
more energetic)



Compton Effect



Photon scattering off of an atomic electron



LAB frame

Conservation of energy + momentum give

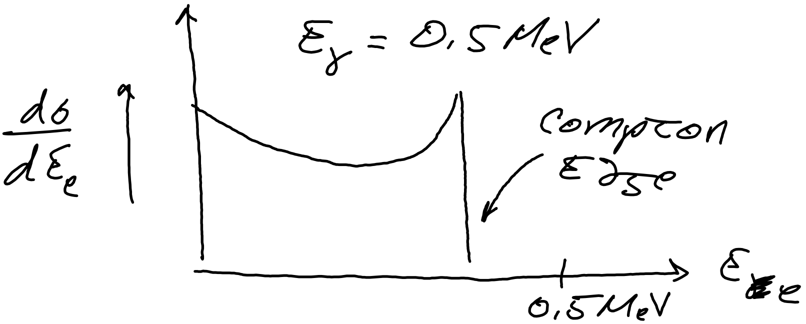
$$\Delta\lambda = \lambda_f' - \lambda_i = \frac{h}{mc}(1-\cos\theta) \quad \left(\Delta\lambda \text{ of photon}\right)$$

$$\cos\phi = \left(1 + \frac{E_\gamma}{E_0}\right)\left[\frac{1-\cos\theta}{2 + \frac{E_\gamma}{E_0}\left(\frac{E_\gamma}{E_0}+2\right)(1-\cos\theta)}\right]^{1/2}$$

↳ scattering angle of e^-

Recoil Electron Energy

Energy spectrum



there is a maximum recoil energy allowed by kinematics

Consider $\Delta\lambda$ expression and divide by hc

$$\frac{1}{E_\gamma'} - \frac{1}{E_\gamma} = \frac{1}{mc^2}(1-\cos\theta)$$

$$E_\gamma' = \left[\frac{E_\gamma(1-\cos\theta) + mc^2}{E_\gamma\, mc^2}\right]^{-1}$$

scattered photon

$$= \boxed{\frac{E_\gamma}{\frac{E_\gamma}{mc^2}(1-\cos\theta)+1}}$$

Photon energy, ε_γ', is minimized when $\theta = 180°$ ($\cos\theta = -1$)

$$\varepsilon_\gamma' = \frac{\varepsilon_\gamma}{\frac{2\varepsilon_\gamma}{mc^2} + 1}$$

the electron energy $\varepsilon_e = \varepsilon_\gamma - \varepsilon_\gamma'$ is then maximum

$$\varepsilon_e = \varepsilon_\gamma \left(1 - \frac{1}{\frac{2\varepsilon_\gamma}{mc^2} + 1}\right)$$

$$= \boxed{\frac{2\varepsilon_\gamma^2}{2\varepsilon_\gamma + mc^2}}$$

Angular Distribution of Compton Photons

Klein-Nishina

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \frac{(1+\cos^2\theta)}{[1+\epsilon(1-\cos\theta)]^2}$$

$$\times \left\{ 1 + \frac{\epsilon^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\epsilon(1-\cos\theta)]} \right\}$$

where $\epsilon = E_\gamma / E_0 = h\nu/mc^2$

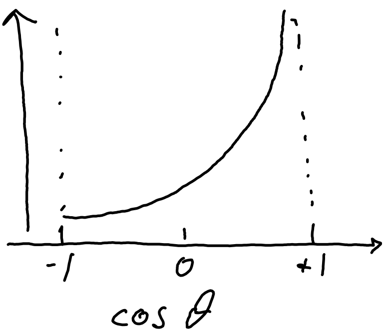
When $h \to 0$, then $\epsilon \to 0$ (Classical limit)

$$\frac{d\sigma}{d\Omega} \to I = I_0(1+\cos^2\theta)$$

– no dependence on frequency.

At high energy

$$\frac{d\sigma}{d\Omega} \simeq \frac{r_e^2}{2\epsilon} \frac{1}{(1-\cos\theta)}$$



γ's mainly scattered in forward direction

Total Cross Section

Integration yields

$$\sigma_c^e = 2\pi r_e^2 \left[\left(\frac{1+\epsilon}{\epsilon^2}\right)\left(\frac{2(1+\epsilon)}{1+2\epsilon} - \frac{1}{\epsilon}\ln(1+2\epsilon)\right) + \frac{1}{2\epsilon}\ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2}\right]^2$$

At high energy $(\epsilon \gg 1)$

$$\boxed{\sigma_c^e \simeq \pi r_e^2 \frac{\ln(1+2\epsilon)}{\epsilon}} \quad \left(\propto \frac{\ln \epsilon}{\epsilon}\right)$$

$\rightarrow$ roughly $\propto \mathcal{E}_\gamma^{-1}$

For an atom with Z electrons

$$\sigma_{ac}^{atom} = Z \sigma_c^e$$

Cross section has absorption + scattering elements:

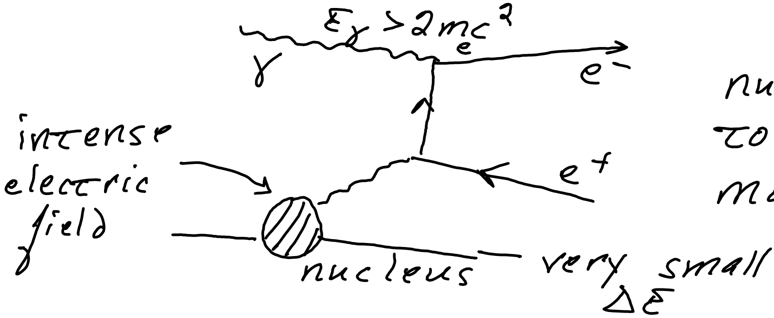
$$\sigma_{c,abs}^e = \sigma_c^e - \frac{\mathcal{E}_\gamma'}{\mathcal{E}_\gamma}\sigma_c^e$$

$$= \sigma_c^e \left[1 - \frac{1}{1+\epsilon(1-\cos\theta)}\right]$$

$$\sigma_c^e = \sigma_{c,abs}^e + \sigma_{c,scat}^e$$

Pair Production

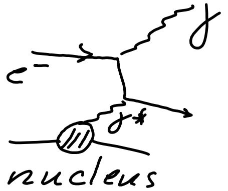
A photon traversing matter can interact with atomic nucleus



nucleus allows to conserve momentum

Once $E_\gamma > 2m_ec^2$ crossed, cross section quickly dominates γ-matter interactions

Theoretically related to bremsstrahlung



In Dirac theory, can use matrix elements from bremsstrahlung

~~Total cross section~~;

Opening angle of e^+e^- small

$$\theta \sim mc^2/E_\gamma$$

Total Cross Section

No screening case: $2mc^2 < h\nu < \frac{mc^2}{\alpha Z^{1/3}}$

$$\sigma_{e^+e^-} = 4 Z^2 \alpha r_e^2 \left[\frac{7}{9}\left(\ln\frac{2h\nu}{m_e c^2}\right) - \frac{109}{54}\right]$$

$\underline{\underline{\propto Z^2}}$

slow increase with E_γ

For complete screening case
$h\nu \gg 137 mc^2/Z^{1/3}$

→ energy sufficiently high that high impact parameter interactions unimportant

$$\sigma_{e^+e^-}^{\text{screen}} = 4 Z^2 \alpha r_e^2 \left[\frac{7}{9}\left(\ln(183/Z^{1/3})\right) - \frac{1}{54}\right]$$

$\underline{\propto Z^2}$

now independent of $E_\gamma = h\nu$

Interestingly $\sigma_{e^+e^-}^{\text{screen}} \sim \frac{7}{9}\,\sigma_{\text{brem}}$!

Mean Free Path

$$\frac{1}{\lambda_{pair}} = n_a \, \sigma_{e^+e^-}$$

n_a: $\frac{\#\text{ atoms}}{\text{volume}}$

$$\simeq \frac{7}{9}\left(4 Z^2 n_a r_e^2 \alpha \ln\left(183/Z^{1/3}\right)\right)$$

Distance over which pair production occurs

$$\boxed{\lambda_{pair} \simeq \frac{9}{7} X_0}$$

Directly related to bremsstrahlung characteristic distance

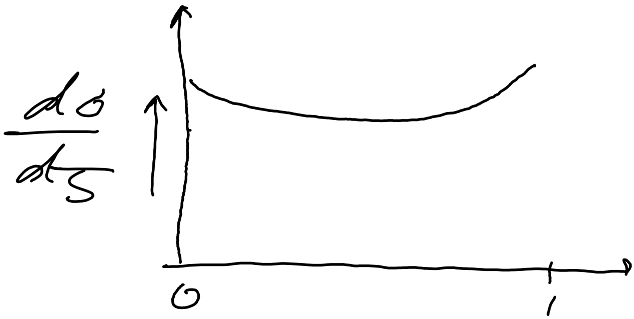
As with bremsstrahlung, to include interactions w/atomic e^-s

$$Z^2 \rightarrow Z(Z+1)$$

Energy Partition

Define kinetic energy fraction of positron

$$\zeta = \frac{E_+ - mc^2}{E_\gamma - 2mc^2}$$



Only 25% variation over most of allowed range for ζ

$\therefore$ e^+ and e^- rarely have same momentum