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Strong Nuclear Interactions

- Force binding nucleus together
  - interactions between hadrons
    - approximated by π exchange
  - governs binding of quarks & gluons into hadrons
    - → gluon messenger
  - very short range force

Become important when:

- reactions create hadrons
- large momentum trasfer
- interactions involving neutral hadrons

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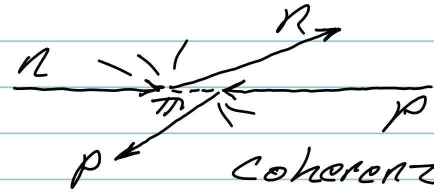
Different Types of Interactions

Elastic:

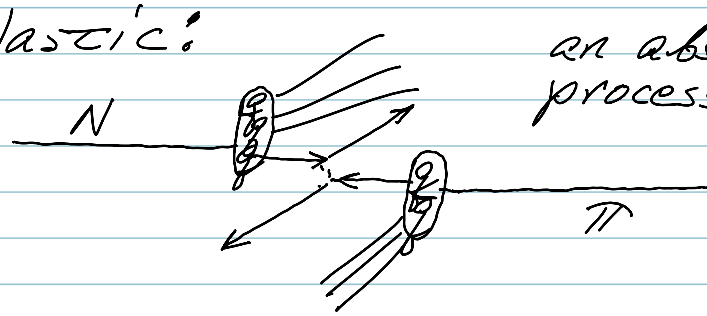


- secondary hadrons from collisions

coherent scattering

- cross section, σ_{el}, fairly constant for energies 2 GeV - 100 TeV

Inelastic:



an absorption process

- Initial hadrons disrupted
  - lose their identity as quark composition changes

- cross section, σ_{inel}, energy dependent @ low energy

Quasi-elastic:

- particle identity retained
- wave-like character important

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Absorption Cross Section

Total cross section

$$\sigma_{tot} = \sigma_{el} + \sigma_{q} + \sigma_{inel}$$

lose hadrons

particle identity retained
"scattering"

σ_{inel} can be established by
- consider σ_{tot}
- subtract known σ_q + σ_{el} contributions
- subtract Coulomb scattering

For interactions above 20 GeV
- roughly independent of momentum

- For $A \geq 9$, observe

$$\sigma_{inel} \text{ or } \boxed{\sigma_{abs} = \sigma_0 A^{\alpha}} \text{ empirical}$$

$\sigma_0 = 41.2 mb$
$\alpha = 0.711$

Simple Model

Assume uniform nuclear density

- also, all nuclei have same density

Volume $\propto A$ (sphere)

$\therefore r_{nucleus} = r_0 A^{1/3}$

observed nuclear sizes yield $r_0 \sim 1.4 \times 10^{-13}$ cm

Cross section of the sphere

$$\sigma_{abs} = \pi r_{nucleus}^2 = \boxed{\underbrace{\pi r_0^2}_{\sigma_0'} A^{\overbrace{2/3}^{\alpha'}}}$$

$$\sigma_0' = \pi r_0^2 = 62 \text{ mb} \sim \sigma_0$$

$$\alpha' = 0.\overline{666} \sim \alpha$$

Not bad for a simple model.

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Absorption Length

Number of particles surviving thru distance 'x' of material

$$\boxed{N(x) = N_0 e^{-x/\lambda}}$$

where λ is the 'absorption length'

Absorption Length $\boxed{\lambda = A/(N_A \rho \sigma_{inel})}$

and

A = atomic weight
ρ = density
N_A = Avogadro's #

N_0 = incident # of particles
x = thickness

Characteristic distance scale to have inelastic ('absorption') interaction

Note: approximation since cross section depends somewhat on particle type and energy

Particle Multiplicity

Particularly when we have inelastic processes

e.g. pp collisions

- new particles produced as originals disrupted
  - generated in a range of angles relative to beam
- particle multiplicity
  - increases with increasing energy
  - more energy → masses of particles

$$\boxed{\langle N \rangle = 1.8 \ln(s) - 2.8}$$

($s = W^2$, total CM energy squared)

e⁺e⁻ Collisions:

A virtual γ^* can look like vector meson & interact strongly

- somewhat faster increase in $\langle N \rangle$ with 's'

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Particle Spectrum

Produced particles in range of species

- π's up to 90%
  - roughly $\frac{1}{3}$ per π specie:

    π^- less than π^0 or π^+

- protons (p) + $K^\pm$
  - 10% or more

- K^0, Λ, $\bar{p}$
  - reduced by another order of magnitude

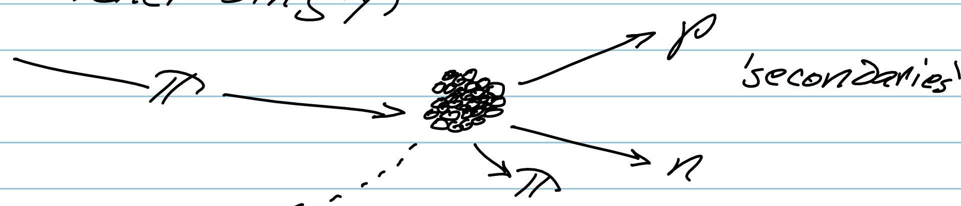
e^+e^- collisions:

- produce somewhat fewer π's

Spallation

Nuclear processes where nucleons knocked out of nucleus:

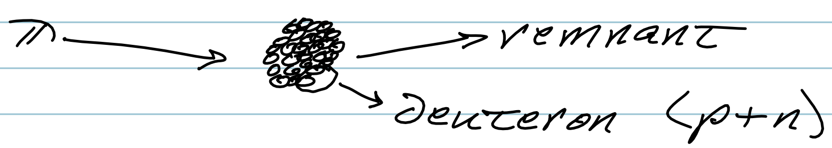
Either singly,



- internally, interactions among nucleons

  - as momentum transferred causes further internal collisions

Or as an aggregate,



Participating particles can escape nucleus if sufficient E

- composition of emitted particles

$$\frac{N_p}{N_n} \sim \frac{Z}{A-Z} \text{ (of initial nucleus)}$$

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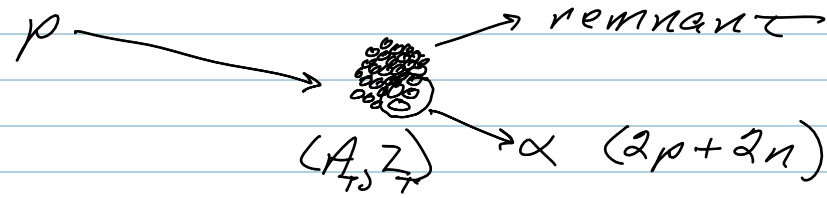
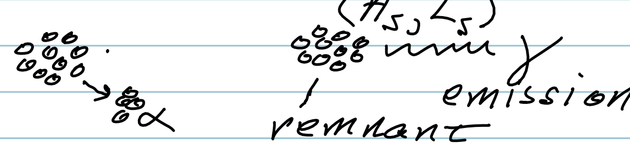
Nuclear Remnant



Remaining nucleus may be unstable

= excited state (A_S, Z_S)



- fission possible for heavier nuclei

- cross section $\sigma(Z, A_S)$ highest when

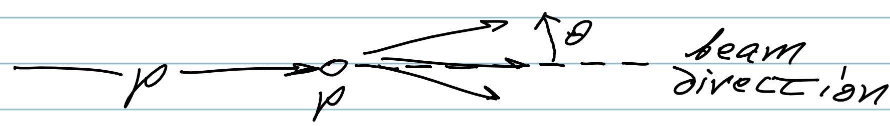
$$\langle A_S \rangle \sim A_T - 20$$

A_S – spallation product atomic weight

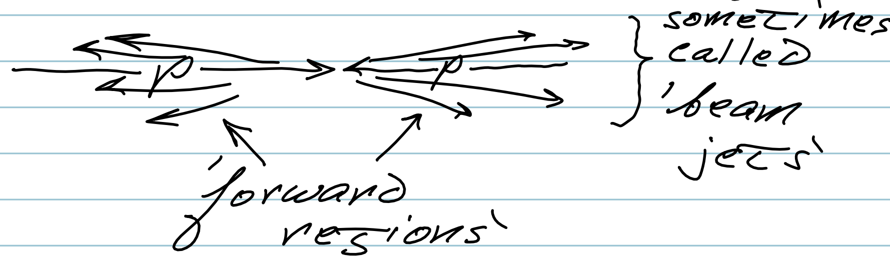
A_T – target nucleus atomic weight

Angular Distributions of Secondaries

Consider the interaction

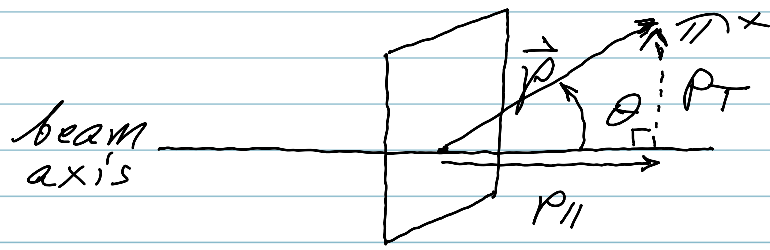


- produced secondary particles
  - tend strongly to go in beam direction in CM frame



  - forward particles carry away most of incident hadron momentum

Transverse Momentum



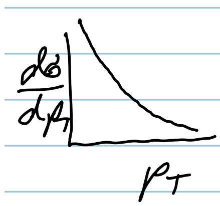
$$p_T^{\pi^+} \left(= p_\perp^{\pi^+}\right) = \sqrt{p_{x,\pi}^{2} + p_{y,\pi}^{2}}$$

Typically, only around Fermi momentum (~350 MeV)

$\langle p_T \rangle$ grows slowly with $\sqrt{s}$

Differential cross section

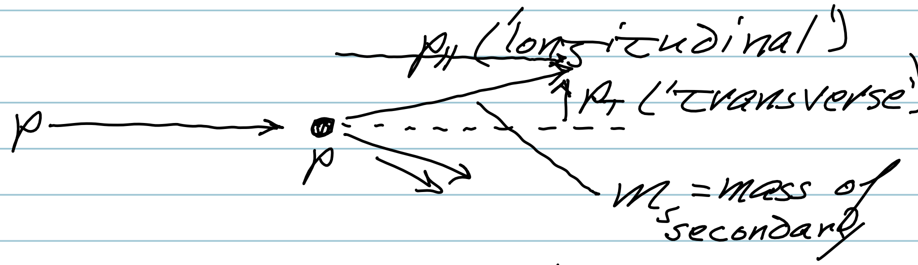
- $d\sigma/dp_T$



- exponential fall-off for small p_T
- But, larger $\sqrt{s} \to$ massive virtual states
  $\to$ slower decline in $d\sigma/dp_T$
- general behavior in pp, $p\bar{p}$ and e^+e^- collions

Rapidity

A useful way to consider $p_\parallel$ of secondaries
- e.g. pp, pN or $p\bar{p}$ collisions



$$y = \frac{1}{2}\ln\left(\frac{E + p_\parallel c}{E - p_\parallel c}\right)$$

$$= \frac{1}{2}\ln\left(\frac{E+p_\parallel c}{E-p_\parallel c} \cdot \frac{E+p_\parallel c}{E+p_\parallel c}\right)$$

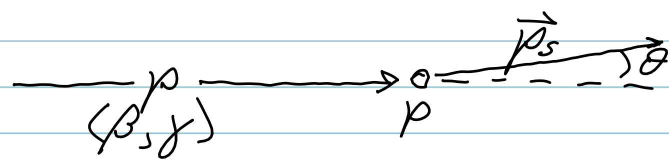
$$= \ln\left((E+p_\parallel c)\Big/\sqrt{E^2 - p_\parallel^2 c^2}\right)$$

Using $E^2 = p_\parallel^2 c^2 + p_\perp^2 c^2 + m_s^2 c^4$

$$\boxed{y = \ln\left(\frac{E + p_\parallel c}{\sqrt{p_\perp^2 c^2 + m_s^2 c^4}}\right)}$$

Forward Rapidity Case

Secondary is scattered in direction of incident hadron



$$\theta \sim 0, \quad p_T \sim 0, \quad p_{\parallel} \sim |\vec{p}_s|$$

$$y_{max} = \ln\left((E + p_{\parallel} c)/m_s c^2\right)$$
$$= \ln(\gamma + \gamma\beta)$$

- a large value

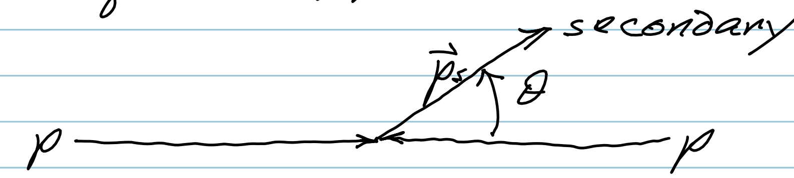
Example: A 100 GeV π from pp collisions

$$\beta \sim 1, \quad \gamma \sim 700$$

$$y_{max} = \ln(700 + 700)$$
$$= 7.2$$

CM System

Both colliding particles have equal + opposite momentum



If secondary has no $p_{\parallel}$

$$\theta = 90^\circ, \quad |\vec{p}_s| = p_T$$

$$\underline{y_{\perp, CM}} = \ln\left(\frac{E}{E}\right) = \underline{\underline{0}}$$

Have cases of secondaries from left-moving proton (above)

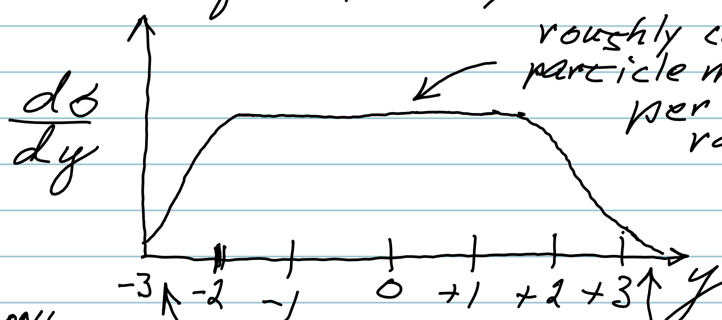
- a minimum rapidity occurs for such scattered secondaries

$$\underline{y_{min, CM} = - y_{max, CM}}$$

- also called 'forward' direction

Shape of Rapidity Distribution

- Observed in experiments
  - roughly 'flat' for small values of rapidity



roughly constant particle multiplicity per unit rapidity

falls rapidly for rapidities near y_{max} or y_{min}

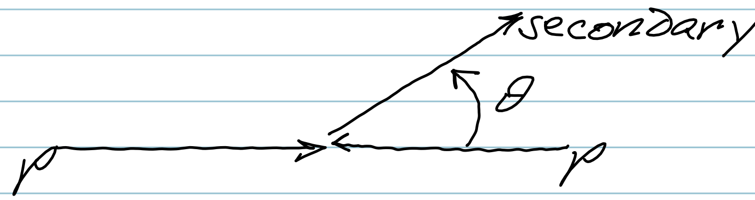
CAUTION! y-axis is not energy density or jet multiplicity per y

  - this distribution is result of strong interaction
  - low momentum, low mass case

- Production of massive states
  - more isotropic decay products
  - ∴ more 'central'

Pseudorapidity

Expand $E + p_{\parallel}c$ in terms of scattering angle, θ



$$y = \frac{1}{2}\ln\left[\frac{\cos^2\frac{\theta}{2} + \frac{m_s^2}{4p^2} + \ldots}{\sin^2\frac{\theta}{2} + \frac{m_s^2}{4p^2} + \ldots}\right]$$

In the case where m_s is negligible (i.e. $E \gg m_s c^2$)

$$y = \ln\left[\left(\frac{\cos\theta/2}{\sin\theta/2}\right)^2\right]^{1/2}$$

$$= \ln\left(\cot(\theta/2)\right)$$

$$= \boxed{-\ln(\tan\theta/2) \equiv \eta}$$

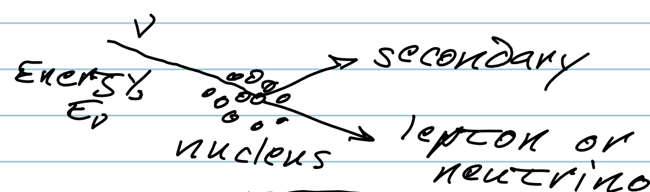
Definition of "pseudorapidity"

Weak Interactions

Much weaker than other interactions

- only interaction by which can probe
  - neutrinos, ν's
  - weakly interacting massive particles ("WIMPs")

Example of neutrino-nucleon (νN) ~~interactions~~ scattering



At $E_\nu = 10$ GeV, $\boxed{\sigma_{\nu N} \sim 7\times10^{-38}\ \text{cm}^2/\text{nucleon}}$

- varies approximately liearly with E_ν

- compare $\sigma_{\text{strong}} \sim 4\times10^{-26}\ \text{cm}^2$

Probability of Interactions

Probability of interaction in some amount of matter

$$R = \sigma N_A (/\text{grams})\, d\, \rho$$

thickness

density

- a per-nucleon probability

In 10m of iron (Fe)

$$R = 7\times10^{-38}\,\text{cm}^2 \left(\frac{6.023\times10^{23}}{g}\right)(10^3\,\text{cm})\left(7.6\,\frac{g}{\text{cm}^3}\right)$$

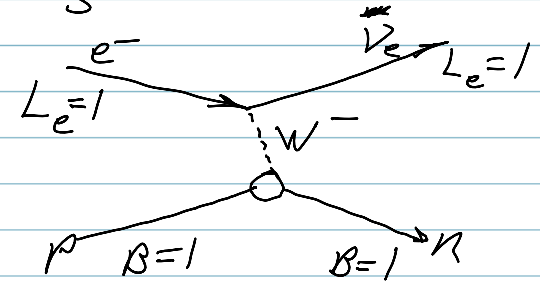
$$= 3\times10^{-10}!!$$

Only 3 out of 10 billion interact

Extremely unlikely for interaction - even in 10m of iron

Neutrino Production

Charged current:

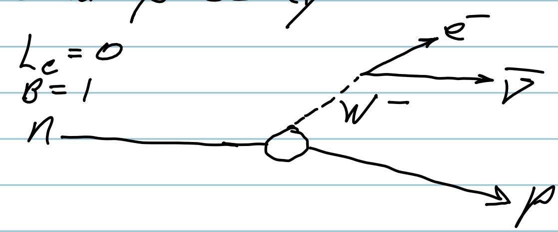


"electron capture"

Lepton + Baryon #s conserved

$$p + e^- \rightarrow n + \nu_e$$

Nuclear β decay:



	L_e	B
e^-	+1	0
$\bar{\nu}$	−1	0
p	0	1

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Hadron decay:

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$K^- \rightarrow \mu^- \bar{\nu}_\mu$$

$$\hookrightarrow \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

Sources of Neutrinos

Solar Neutrinos

→ "pp chain" of reactions
- 1st step

$$\cancel{p + p \rightarrow d} \quad {}^1_1H + {}^1_1H \rightarrow {}^2_2He + \gamma$$

$$ {}^2_2He \rightarrow {}^2_1D + e^+ + \nu_e$$

- also
  - electron capture

$$ {}^7Be + e^- \rightarrow {}^7Li + \nu_e$$

  - 'boron-8'

$$ {}^8B \rightarrow {}^8Be + e^+ + \nu_e$$

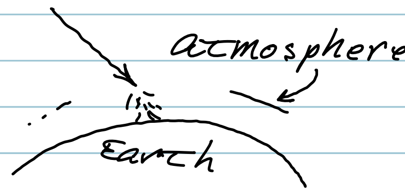
  - maximum $E_\nu \sim 15\,MeV$

- energies from keV to 15 MeV

Atmospheric Neutrinos

Cosmic ray nuclei
- mostly protons

incident on atmosphere;
→ "shower" of secondaries



Mostly π's produced

- can produce neutrinos via decay chain

$$\pi^{\pm} \rightarrow \mu^{\pm} \overset{(-)}{\nu}_{\mu}$$
$$\quad \hookrightarrow e^{\pm} \overset{(-)}{\nu}_{e} \overset{(-)}{\nu}_{\mu}$$

$\therefore$ 2 ν_μ to each 1 ν_e

- ν's can have GeV energy

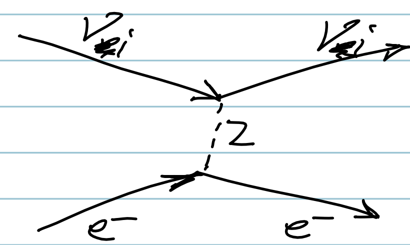
- some K's also produced in shower

Neutrino Interactions

Neutral Current:

$\nu_i + e^- \rightarrow \nu_i + e^- \quad (i = e, \mu, \tau)$

↑
atomic
electron



- Can measure presence of ν_i
  - some of its energy transferred to e^-
- Can also have $\nu p \rightarrow \nu p$ or $\nu n \rightarrow \nu n$ interactions
  - Here, may actually have ν_i interactions with a quark

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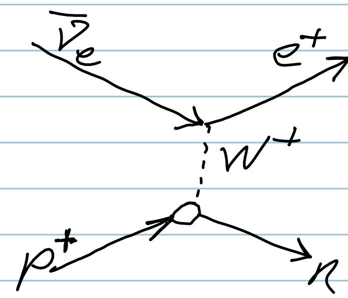
Charged Current:

Corresponds to change of charge of incident particle

$\bar{\nu}_e + p \to n + e^+$
(inverse β decay)
- has 1.8 MeV



Also

$\bar{\nu}_\mu + p \to \mu^+ + n$

$\bar{\nu}_\tau + n \to \tau^- + p$

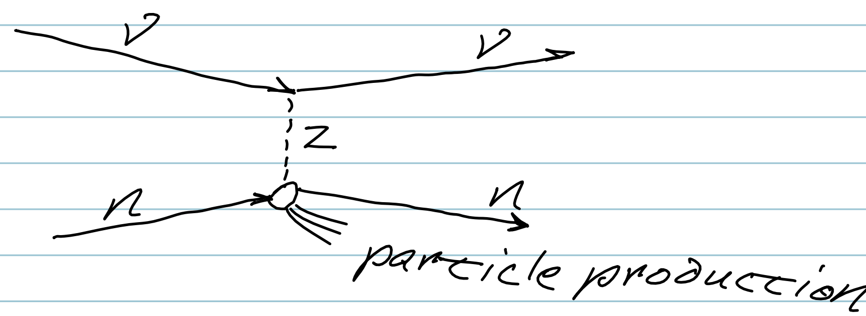
- and all "charge conjugate" states (eg. $\tau^+ + \bar{p}$) as in $\nu_\tau + \bar{n} \to \tau^+ + \bar{p}$

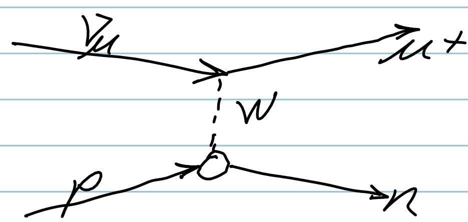
Lepton appearance facilitates inference of neutrino

Particle Production

Consider again νN scattering

- important, because this is one way to understand nucleons



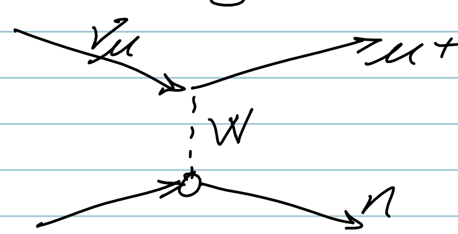


As energy transfer W increases

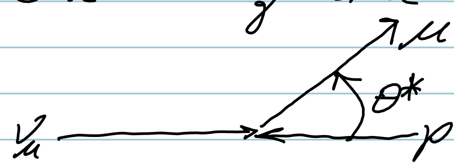
$$\boxed{\langle N_{charged} \rangle \propto \ln(W)}$$

Angular Dependence

Consider again charged lepton



In the CM frame



- in a quark model of nucleon; taking $(1-y) = \frac{1}{2}(1+\cos\theta^*)$

$$\left.\frac{d\sigma}{dy}\right|_{\bar{\nu}N\,scatt.} \propto (1+\cos\theta^*)^2 \propto (1-y)^2$$

$$\left.\frac{d\sigma}{dy}\right|_{\nu N\,scatt.} \sim \text{constant}$$