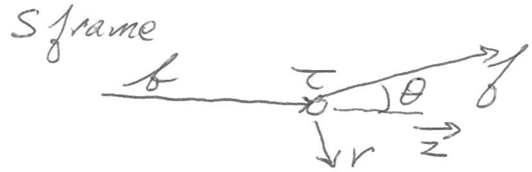
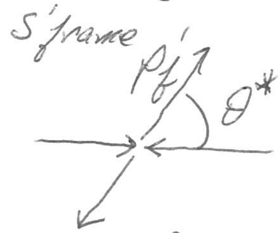
Momentum transfer to forward scattered particle

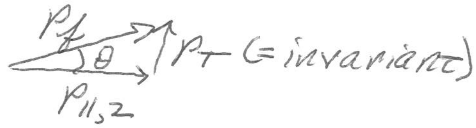
$$\tau = (\tilde{p}_b - \tilde{p}_f)^2$$

$$= m_b^2 + m_f^2 - 2(E_b E_f - \vec{p}_b \cdot \vec{p}_f)$$

Relation between polar scattering angles in CM and LAB frames:







β_0 is velocity of S relative to S'.

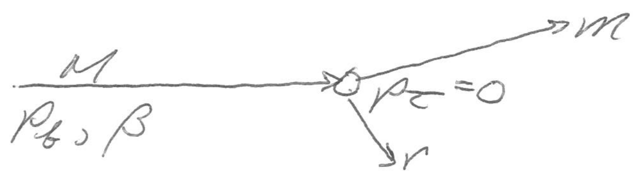
$$\tan\theta_{LAB} = \frac{p_T}{p_z}$$

$$= \frac{p'_T}{\gamma_0(p'_z + \beta_0 E'/c)} \left(\frac{1/p'_f}{1/p'_f}\right)$$

$$= \boxed{\frac{\sin\theta^*}{\gamma_0(\cos\theta^* + \beta_0/\beta^*)}}$$

Maximum Energy Transfer

Consider beam particle with mass $M >> m$, where m is target mass.



In CM frame

$$p_r^* = mc\beta_0\gamma_0$$
$$E_r^* = mc^2\gamma_0$$

β_0 is target velocity ~~LAB~~ CM

Since m is very small

(Be careful! of approximations)

$$\beta_0 \simeq \frac{p_b c}{(p_b^2 c^2 + M^2 c^4)^{1/2}} = \beta$$

Transforming from CM → LAB

$$E_r \simeq \gamma c\left(\frac{E_r^*}{c} + \beta p_r^*\right)$$
$$= \gamma c\,(mc\gamma_0 + \beta mc\beta_0\gamma_0)$$

E_t was just mc^2, so

$$\boxed{\Delta E_{max} \simeq 2mc^2\beta^2\gamma^2}$$

Particle Physics Experiments

Extreme extensions of our senses

- → eyes: intensity, λ
- → ears: heat, acoustic

This information arises in these organs & must be mapped and calibrated to world observed

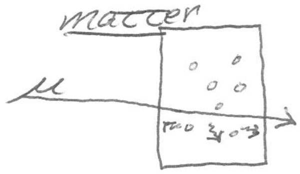
No different with our detectors

But first there are some earlier steps:

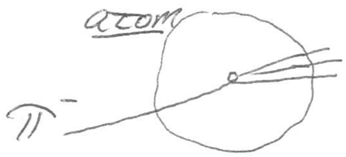
- generally, we must extract relatively low energy particles of mundane type, such as e^-, p, π
- → accelerate them
  - astrophysical means (eg cosmic rays)
  - particle accelerator ⟹ manipulation of electric & magnetic fields
- → collide them (an 'event')
  - produce interaction with some probability of exhibiting process for study

$\boxed{\text{Fixed target}}$ $\boxed{\text{collider}}$

Detect resulting particles



Electromagnetic interactions



Strong nuclear Interactions

Weak interactions

Detectors designed and built by precise knowledge of these

Immediate result: charge, light, phonons

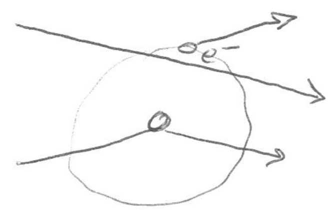
From these we obtain measures of

- kinematics $(\vec{p}, E)$
- particle properties $(q, m, \gamma, \text{spin}, \tau)$

Electromagnetic Interactions of Charged Particles w/Matter

- inelastic collisions with atomic electrons
- elastic scattering from nuclei

These processes give



- loss of incident particle energy
- change in particle trajectory

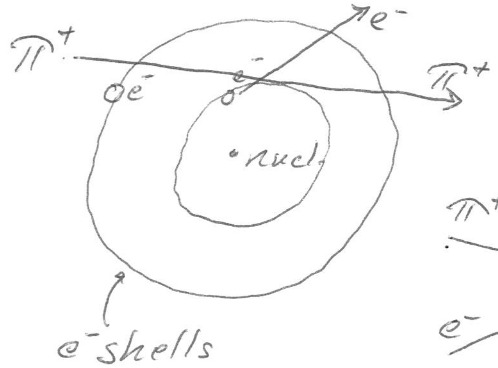
Additional reactions

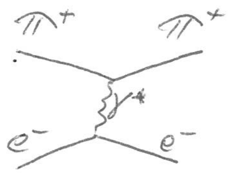
- Cerenkov emission
- transition radiation
- bremsstrahlung
- scintillation

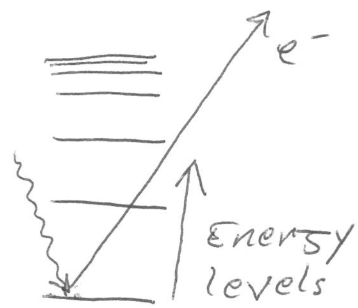
Aside from high energy electrons and positrons

- inelastic collisions dominate electromagnetic interactions

Ionization and Inelastic Collisions







charged particle interacts with atomic electrons

ionization or excitation result

Only few eV lost: but many interactions in material

Remember σ_{atom}!

Example: 10 MeV proton incident on copper plate

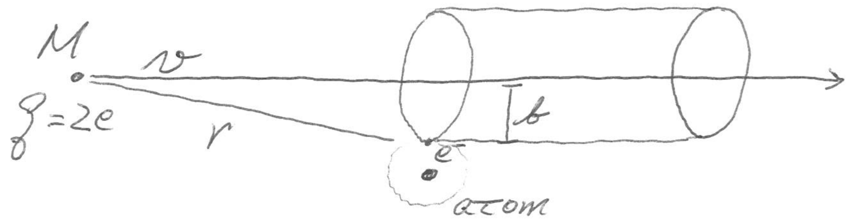
→ total energy loss in $\frac{1}{4}$ mm

Some collisions provide sufficient energy to ionization electrons

→ cause ionization themselves

"δ rays"

Classical Calculation of Energy Loss



Coulomb interaction with e^-, nucleus

Assumptions

- $M \gg m_e$
- electron free and initially at rest
- electron only moves slightly during interaction
  - E-field remains as in initial position
- incident particle undeviated

Momentum gained by electron

$$\Delta p = \int F\,dt = e\int E_\perp\,dt = e\int E_\perp \frac{dt}{dx}\,dx$$

$$= \frac{e}{v}\int E_\perp\,dx$$

(only care about $E_\perp$ due to symmetry.)

Momentum Transfer

Use Gauss's Law for the integral

$$\int \vec{E}_\perp \cdot d\vec{A} = 4\pi z e$$
$$= \int E_\perp \, 2\pi b \, dx$$

$$\int E_\perp dx = 2ze/b$$

this gives

$$\Delta p = \frac{e}{v}\int E_\perp dx = \frac{2ze^2}{bv}$$

$$\Delta E = \frac{\Delta p^2}{2m_e} = \frac{2z^2e^4}{m_e b^2 v^2} \propto \frac{1}{b^2 v^2}$$

$\therefore$ most of energy transfer due to close collisions

where is energy transfer happening?

atomic electrons: $m = m_e$; $e^2 \to e^2$

nuclei: $m = Am_p$; $e^2 \to Z^2 e^2$

$$\frac{\Delta E(e)}{\Delta E(\text{nucl.})} = \frac{2z^2e^4}{m_e b^2 v^2} \cdot \frac{b^2 v^2 A m_p}{2z^2 Z^2 e^4} \sim \frac{2Z m_p}{Z^2 m_e}$$
$$= 2m_p / Z m_e \gg 1$$

$\therefore$ atomic electrons mostly where happening

Energy Loss per Unit Length

If N_e is density of electrons

$$dE(b) = \Delta E\, N_e\, dV = \frac{2z^2e^4}{m_e b^2 v^2} N_e (2\pi b\, db\, dx)$$

$$= \frac{4\pi z^2 e^4}{m v^2} N_e \frac{db}{b} dx$$

energy lost to all electrons between b and $b+db$ in dx

Need to integrate over 'b' to get

$$\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

Problem: 1) a large b takes a long time: e^- moves

2) as $b \to 0$, $\Delta p \to \infty$

$$\therefore \frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{b_{max}}{b_{min}}$$

Minimum impact parameter:

→ maximum ΔE when a head-on collision

$$\Delta E_{max} = \frac{(2p)^2}{2m} = 2\gamma^2 m_e v^2$$

$$\text{also} = \frac{2z^2e^4}{m_e v^2 b_{min}^2}$$

$$\therefore \underline{b_{min} = ze^2/\gamma m_e v^2}$$

Maximum impact parameter

e^- has orbital frequency, ν

period $\tau = 1/\nu$

Interaction with passing particle short compared to τ

$$t/\gamma = (b/v)/\gamma = b/v\gamma \le \tau = 1/\nu$$

$$\therefore \underline{b_{max} = \gamma v/\nu}$$ → typical (or average) frequency

Substitution gives:

$$\boxed{\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln\left(\frac{\gamma^2 m_e v^2}{2e^2\nu}\right)}$$

$\frac{1}{v^2}$ @ low energy

$\ln\gamma^2$ @ high energy