Physics 3344
Homework 6 26 Feb 2008
Due Tues, Mar 4

1. Read Marion through the remainder of Ch 3

   True/False: I read this material.

2. Marion Ch 3, Problems 1, 2, 11, 22(a)

3. (a) State the kinetic energy and potential $U(\theta)$ due to gravity for a pendulum of length ℓ as a function of its angle θ with respect to the vertical, and $\dot{\theta}$.

   (b) Sketch $U(\theta)$.

   (c) Derive an approximation to the potential (up to second order) good for small oscillations around the stable equilibrium point.

   (d) Give the component of the force in the direction of increasing θ (perpendicular to the pendulum rod) both for the exact and the approximate cases. You may either simply use a free-body diagram, or compute the component of the gradient of U in that direction. (I'd recommend using the gradient in polar coordinates if you do the latter; see the appendix.) Also, give the torque for both cases.

   Based on the approximation and our discussion of the mass on a spring:

   (e) What is the period for small oscillations?

   (f) Give $\theta(t)$, the angle as a function of t.

   (You should be able to answer these last two by referring to our solution to the mass on a spring, though you're welcome to re-solve the appropriate differential equation if you'd like.)

4. To solve a first-order equation numerically, we approximated the derivative dx/dt by $\Delta x/\Delta t$, with $\Delta x \equiv x(t+\Delta t)-x(t)$. If you wish to solve a second-order equation numerically, how could you approximate the second derivative d^2x/dt^2? Can you find a version that only depends on x at neighboring times; that is, on $x(t-\Delta t)$, $x(t)$, and $x(t+\Delta t)$? Based on this, how many initial conditions will you need to solve a second-order equation numerically?