RESONANCES

IF C HEAVY:

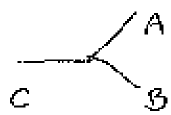
- DECAYS QUICKLY
- NOT DIRECTLY OBSERVED (ex Z or J/ψ)

BUT FROM AB → AB LEARN

(1) C EXISTS (FROM RES. BUMP)

(2) $m_C^2 = s = E_{CM}^2$ AT RES.

(3) C DECAYS TO A & B (FROM PRODUCTS)

⇒ $\mathcal{L}$ CONTAINS )

(4) DECAY RATE (FROM WIDTH) → WILL DISCUSS

(5) C IS SCALAR PARTICLE:

$\frac{d\sigma}{d\Omega}$ AT RES. IS INDEP. OF θ, φ

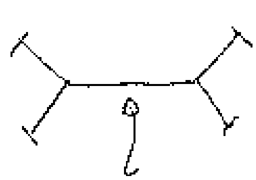
⇒ A & B IN J = 0 STATE (S STATE)

(ROTATIONALLY INVARIANT)

⇒ C BEFORE DECAY: J = 0 ⇒ NO SPIN

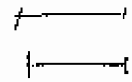
PROBLEM: σ = ∞ AT RES

FROM

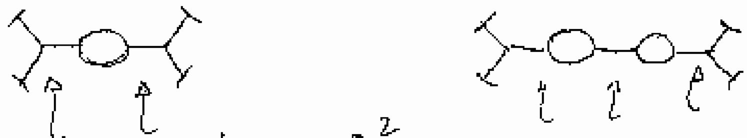


$$\frac{i}{\underbrace{(p_A^i + p_B^i)^2}_{s} - m_C^2 + i\epsilon} \quad \text{AT } s \sim m_C^2$$

FIX:

(a) NOT A SMALL CORRECTION TO 
DESPITE g^2

(b) HIGHER ORDERS: EVEN WORSE



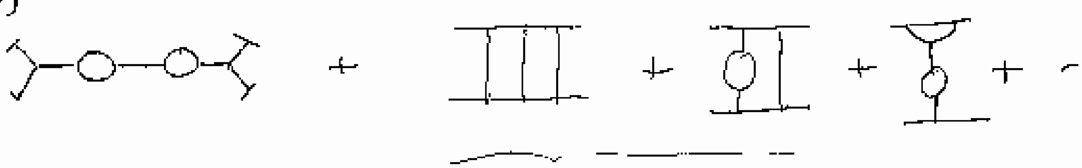
$$(-ig)^4\left[\frac{i}{(s-m_c^2+i\epsilon)}\right]^2 \qquad (-ig)^6\left[\frac{i}{(s-m_c^2+i\epsilon)}\right]^3$$

$\Rightarrow$ THESE CAN'T BE NEGLECTED: NOT SMALL CORRECTIONS

(c) LOOK AT ENTIRE SERIES (ie ALL ORDERS)

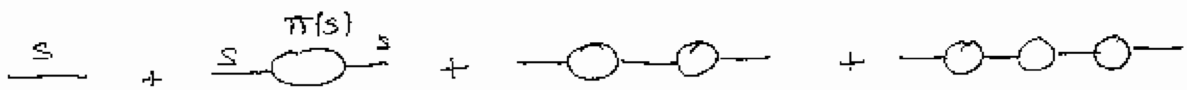
(i) NEAR $s \approx m_c^2$ IDENTIFY BIGGEST CONTRIBS:

ex $\mathcal{O}(g^6)$



IGNORE

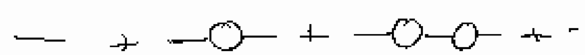
(ii) SUM UP: ("RESUMMATION" $\Rightarrow$ COMMON TOOL)



$$\frac{i}{s-m_c^2+i\epsilon} + \frac{i}{s-m_c^2+i\epsilon}\,\Pi(s)\,\frac{i}{s-m_c^2+i\epsilon} + \frac{i}{(\ )}\,\Pi\,\frac{i}{(\ )}\,\Pi\,\frac{i}{(\ )} + \cdots$$

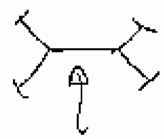
$i\Pi \equiv$ 
"SELF-ENERGY"

RECALL $\dfrac{1}{x-\alpha} \sim \dfrac{1}{x} + \dfrac{\alpha}{x^2} + \dfrac{\alpha^2}{x^3} + \cdots$

$$\Rightarrow \frac{i}{s-m_c^2-i\Pi(s)+i\epsilon} =$$ 

(iii) EQUIV. TO ASKING FOR $\mathcal{O}(g^2)$ CORRECTION TO INVERSE PROP.

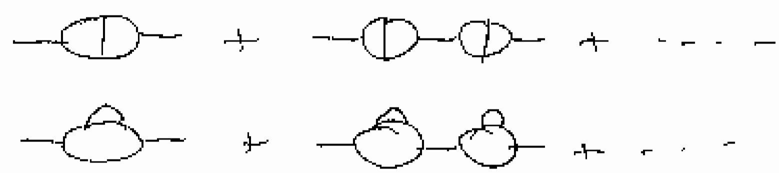
(d) SHOULD USE FOR σ:

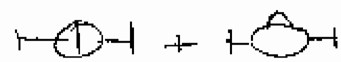


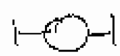
$$\frac{i}{s - m_c^2 - i\Pi(s) + i\epsilon}$$

$s = p^2$ w/ p_μ THE MOM. CARRIED BY C PROP.

(e) HIGHER ORDERS.

ex 

$\Rightarrow$ $\mathcal{O}(g^4)$ CORRECTION TO Π = 

(f) FOR $s \sim m_c^2$, PROPS INSIDE  CAN GO ON-SHELL (WILL SEE) MEANS $i\Pi$ GETS IMAG PART $\Rightarrow$ DON'T HIT POLE IN σ SO WELL-DEF'D $\Rightarrow$ DROP $i\epsilon$

(g) Π CORRECTS MASS:

$m^2_{c,\text{PHYS}}$ $\equiv$ LOCATION OF RES. IN s

(OR "m^2_{cR}" = "RENORMALIZED") $\equiv$ " " POLE IN PROP

NEAR $s = m^2_{c,\text{PHYS}}$

$$\text{Re}\{i\Pi(s)\} \approx \text{Re}[i\Pi(m^2_{c,\text{PHYS}})]$$

$$s - m_c^2 - \text{Re}[i\Pi(s)] \approx m^2_{c,\text{PHYS}} - m_c^2 - \underbrace{\text{Re}[i\Pi(m^2_{c,\text{PHYS}})]}_{\equiv\, \delta m_c^2} \equiv$$

$$\boxed{m^2_{c,\text{PHYS}} = m_c^2 + \delta m_c^2}$$ { PHYS MASS, LOCATION OF RES $\Rightarrow$ CORRECTED

(h) $\text{Im}[i\Pi]$ ALSO HAS PHYS. INTERP:

OPTICAL THM: ALT METHOD FOR σ; APPEARS IN QM, E&M

RECALL S MATRIX OPERATOR

$$S = \mathcal{U}_I(\infty, -\infty) = T e^{-i\int_{-\infty}^{\infty} \mathcal{H}_I(x)\, d^4x}$$

$\Rightarrow$ UNITARY

FOR CROSS SECTIONS, DROP LEADING 1 FOR S (NOTHING HAPPENS)

DEF. T MATRIX: (not time ordering, just part of S)

$$S \equiv \underbrace{1}_{\text{NOTHING HAPPENS}} + \underbrace{iT}_{\text{SCATTERING OCCURS}}$$

SCATTERING OCCURS $\Rightarrow$ GOES INTO $\mathcal{M}$

UNITARY:

$$S^\dagger S = 1 = (1 - iT^\dagger)(1 + iT)$$

$$\Rightarrow \boxed{T - T^\dagger = i T^\dagger T}$$

replace $\bar p_i'$ w/ $\bar p_i$

CONSIDER MAT EL w/ STATES $|\bar p_1, \bar p_2, \ldots\rangle \equiv |\bar p_i\rangle$ & $\langle \bar p_i'|$:

$$\langle \bar p_i'| T |\bar p_i\rangle - \underbrace{\langle \bar p_i'| T^\dagger |\bar p_i\rangle}_{\langle \bar p_i | T | \bar p_i'\rangle^*} = i \langle \bar p_i' | T^\dagger T |\bar p_i\rangle$$

LHS: $$\langle \bar p_i' | T | \bar p_i\rangle \equiv \underbrace{(2\pi)^4 \delta^4(\Sigma \bar p_i' - \Sigma \bar p_i)}_{\text{UNIVERSAL FACTOR}} \underbrace{\mathcal{M}}_{\text{INVAR. AMPLITUDE}}(\bar p_i', \bar p_i)$$

$$\text{LHS} = (2\pi)^4 \delta^4(\quad)\left[\mathcal{M}(\bar p_i', \bar p_i) - \mathcal{M}^*(\bar p_i, \bar p_i')\right]$$

RHS: INSERT COMPLETE SET $\sum_N \int [dq_1] \cdots [dq_N] \, |\bar{q}_1 \cdots \bar{q}_N\rangle\langle \bar{q}_1 \cdots \bar{q}_N|$

w/ $[dq_1] \equiv \dfrac{d^3 q_1}{(2\pi)^3 \, 2E_{q_1}}$ $\qquad |\bar{q}_j\rangle \equiv |\bar{q}_1 \cdots \bar{q}_r\rangle$

$$\text{RHS} = i \sum_N \int \Pi_j [dq_j] \, \underbrace{\langle \bar{p}_i' | T | \bar{q}_j\rangle}_{(2\pi)^4 \delta^4(\Sigma p_i' - \Sigma q_j) \mathcal{M}(\bar{p}_i', \bar{q}_j)} \underbrace{\langle \bar{q}_j | T^\dagger | \bar{p}_i\rangle}_{(2\pi)^4 \delta^4(\Sigma p_i - \Sigma q_j) \mathcal{M}^*(\bar{p}_i, \bar{q}_j)}$$

$\equiv \delta^4(\Sigma p_i' - \Sigma p_i)$ BY

SET LHS = RHS:

(1) CANCEL $(2\pi)^4 \delta^4(\Sigma p_i' - \Sigma p_i)$

(2) SET $\bar{p}_i' = \bar{p}_i$

OPTICAL THM

$$\Rightarrow \boxed{2 \, \mathrm{Im} \, \mathcal{M}(\bar{p}_i, \bar{p}_i) = \sum_N \Pi_j [dq_j] \, (2\pi)^4 \delta^4(\Sigma p_i - \Sigma q_j) |\mathcal{M}(\bar{q}_j, \bar{p}_i)|^2}$$

SPECIAL CASE:

$|p_i\rangle = |\bar{p}_1, \bar{p}_2\rangle$ $\qquad \xrightarrow{\;} \bar{p}_1 \quad \xleftarrow{\;} \bar{p}_2$

{IN OUR ABC THY CALC, $\bar{p}_1 \equiv \bar{p}_i^A$, $\bar{p}_2 \equiv \bar{p}_i^B$ }

$$\text{RHS} = 4 \underbrace{E_{cm} \, p_{cm}}_{[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{\frac{1}{2}} \text{ IN GEN. FRAME}} \sigma_{\text{TOTAL}}(p_1, p_2 \to \text{ANYTHING})$$

OPTICAL THM FOR THIS CASE:

$$\boxed{\mathrm{Im} \, \mathcal{M}((\bar{p}_1, \bar{p}_2), (\bar{p}_1, \bar{p}_2)) = 2[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{\frac{1}{2}} \sigma_{\text{TOTAL}}(\bar{p}_1, \bar{p}_2 \to \text{ANYTHING})}$$

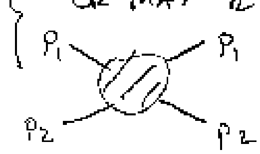
NOTE:

- SIMPLE ALT. METHOD TO GET σ_{TOT}:

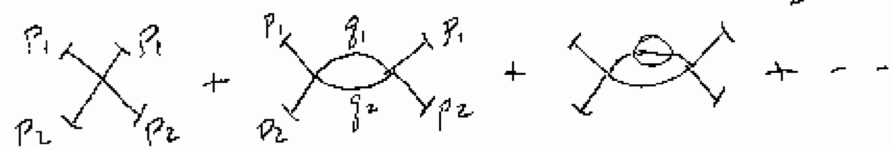
  COMPUTE $\mathcal{M}((\bar{p}_1, \bar{p}_2), (\bar{p}_1, \bar{p}_2))$ TO SOME ORDER

  TAKE Im PART

{ "FORWARD AMPL OR MAT EL"



ex LHS $\lambda \phi^4$:

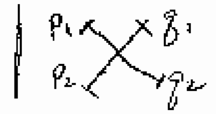


GIVES

^{2} ON RHS

NO IM. PART

IM. PART GIVES $\int_q$



^{2} ON RHS

ETC

$\Rightarrow$ DIAGRAMS ACQUIRE IM. PARTS WHEN INTERNAL LINES CAN GO ON-SHELL (SO COULD BE PRODUCED); ie INSIDE INTEGRALS FOR q_1 & q_2 (IN THIS CASE) IT'S POSSIBLE TO HAVE BOTH $q_1^2 = m^2$ AND $q_2^2 = m^2$ SIMULTANEOUSLY (THIS DEP. ON p_1 & p_2, SINCE $q_1 + q_2 = p_1 + p_2$ ALWAYS)

$\Rightarrow$ CF PESKIN FOR "CUTTING RULES":

SIMPLE RULES FOR GETTING IM PARTS OF DIAGS

$\Rightarrow$ MORE GENERAL:

NOTE LHS OF OPTICAL THM HAS $\mathcal{M}$ w/ p_1, p_2 FOR ON-SHELL PARTICLES

BUT IF COMPUTED ANY AMPUTATED FEYN. DIAGRAM (ie ) EVEN w/ OFF-SHELL LEGS

(ex — PART OF LARGER DIAGRAM)

$\Rightarrow$ GET IM. PART IF INTERNAL LINES CAN GO ON SHELL; GET BY CUTTING RULES

RESONANCES & DECAY RATES:

SPECIAL CASE: SINGLE PARTICLE IN $|p_i\rangle \equiv |\bar{p}\rangle$

OPTICAL THM:

$$2\,\mathrm{Im}\,\mathcal{M}(\bar{p},\bar{p}) = \sum_n \prod_j [dq_j]\,(2\pi)^4\delta^4(p-\sum q_i)\,|\mathcal{M}(\bar{p},\bar{q}_j)|^2$$

RHS: $2E_p\Gamma(p)$ ($= 2m\Gamma$ IN REST FRAME)

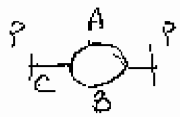
$\uparrow$ DECAY RATE

$$\Rightarrow \quad \boxed{\mathrm{Im}\,\mathcal{M}(\bar{p},\bar{p}) = E_p\Gamma(p)}$$

NOTE HERE p^μ SATISFIES $p^2 = m^2$

NOTE: FOR ABC

LHS: AMPUTATED 1 PART $\rightarrow$ 1 PART DIAG:

$=$ 

$+ \ldots$ FOR OUR MODEL

$\equiv \Pi(p_c^2)$ (SELF ENERGY)

HAVE $p_c = q_A + q_B$

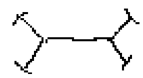
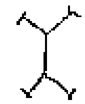
GET IMAG PART WHEN CAN ALSO HAVE INSIDE LOOP

$$q_A^2 = m_A^2 \qquad q_B^2 = m_B^2$$

(i.e. CAN MAKE REAL A & B)

RECALL IN $\sigma_{2\to2}$:

$$i\mathcal{M}_{2\to2} = (-ig)^2\left[\frac{i}{\underbrace{(p_A^i+p_B^i)^2}_{s} - m_c^2 + i\epsilon} + \frac{i}{\underbrace{(p_A^i-p_B^{A})^2}_{u} - m_c^2 + i\epsilon}\right]$$

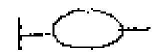
NEAR RES: $s \sim m_c^2$

- NEGLECT 2^{ND} TERM
- MUST INCL 

1^{ST} TERM $\rightarrow$ $\dfrac{i}{s - m_c^2 \underbrace{- i\Pi(s)}}$

$-\left[\mathrm{Re}(i\Pi(s)) + i\,\mathrm{Im}(i\Pi(s))\right]$

- FOR $s \equiv p_c^2 \sim m_{c,PHYS}^2$, $\mathrm{Re}\, i\Pi(m_{c,PHYS}^2) \equiv \delta m_c^2$

BUT ALSO $i\Pi(p_c^2 \sim m_{c,PHYS}^2) =$  $= \mathcal{M}_{1\to1}(\bar{p}_c, \bar{p}_c)$

$p_c \quad p_c$

ie CAN INTERPRET IT AS THE ON-SHELL MAT. EL. FOR 1 C-PART $\rightarrow$ 1 C-PART SCATT.

THEN

$$i\,\mathrm{Im}(i\Pi(p_c^2 \sim m_{c,PHYS}^2)) = i\,\mathrm{Im}\,\mathcal{M}_{1\to1} = i E_{p_c}\Gamma(p_c)$$

IN CM: $\quad i m_{c,PHYS}\Gamma \qquad \Big\}$ WE CALCULATED $\Gamma(p_c)$ EARLIER

$$\text{PROP} \sim \frac{i}{\underset{\uparrow \text{ INCL } \delta m_c^2}{s - m_{c,PHYS}^2} - iE_{p_c}\Gamma(p_c)} \underset{CM}{=} \frac{i}{s - m_{c,PHYS}^2 - i m_{c,P}\Gamma}$$

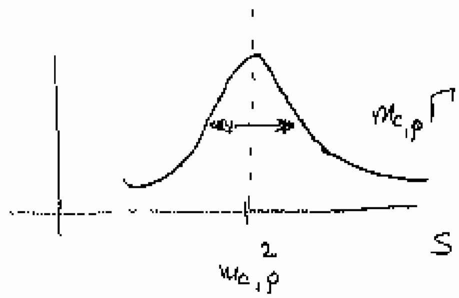
NEAR $s \sim m^2_{c,\,\text{PHYS}}$:

$$\frac{d\sigma}{d\Omega} \sim \frac{g^4}{(8\pi)^2 E_{CM}^2} \underbrace{\left| \frac{i}{s - m^2_{c,\rho} - i E_{p_c} \Gamma(p_c)} \right|^2}_{}$$

$$\frac{1}{(s - m^2_{c,\rho})^2 + E_\rho^2 \Gamma^2(p_c)}$$

$$\left\{ \text{IN CM} \Rightarrow \frac{1}{(s - m^2_{c,\rho})^2 + m^2_{c,\rho} \Gamma^2} \right\}$$

= BREIT-WIGNER RESONANCE



PEAK AT $m^2_{c,\,\text{PHYS}}$

WIDTH OF $m_{c,\,\text{PHYS}} \cdot \Gamma$ $\Rightarrow$ NARROW $\Leftrightarrow$ SLOW DECAY

BROAD $\Leftrightarrow$ RAPID DECAY