Physics 7360
Homework 2 10 Feb 2006
Due Friday, Feb 17

1. Read Seiden Ch 2.1 - 2.6
   Read A&H Ch 4.1 and all of Ch 5

2. The Lagrangian density for the electromagnetic vector potential $A_\mu(x)$ is
$$\mathcal{L} = -(1/4)F_{\mu\nu}F^{\mu\nu} - eJ^\mu A_\mu \tag{1}$$
   where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and $J^\mu(x)$ is a fixed (or external) electromagnetic current. Derive Maxwell's equations from the Euler-Lagrange equations for this system. Give these first in terms of $A_\mu(x)$, then convert them to $\mathbf{E}$ and $\mathbf{B}$. (Note $E^i \equiv -F^{0i}$ and $\epsilon_{ijk}B^k \equiv -F_{ij}$.) Finally, simplify the equations by imposing Landau gauge $\partial_\mu A^\mu = 0$.

3. Give the most general Lagrangian for a vector field $V^\mu$ with coefficients of mass dimension no less than zero. To help fix conventions, take the first term to be
$$-\frac{1}{2}\partial_\mu V^\nu \partial^\mu V_\nu \,. \tag{2}$$

4. For the previous Lagrangian for the vector field, it is possible to construct a term which is a scalar under proper Lorentz transformations, but not under parity ($\mathbf{x} \to -\mathbf{x}$) or time-reversal ($x^0 \to -x^0$). State the term, and explain why it is a total four-divergence (and so a surface contribution to the action). (Note that this term could also appear in the action for electrodynamics.)

5. For the scalar field, show that the canonical commutation relations,
$$[\pi(t,\mathbf{x}), \phi(t,\mathbf{x}')] = -i\delta^3(\mathbf{x}-\mathbf{x}') \tag{3}$$
   combined with the Heisenberg equations,
$$\partial_t\phi(t,\mathbf{x}) = i\,[H, \phi(t,\mathbf{x})] \tag{4}$$
$$\partial_t\pi(t,\mathbf{x}) = i\,[H, \pi(t,\mathbf{x})] \tag{5}$$
   correctly reproduce the Euler-Lagrange equation for $\phi(x)$.