Physics 7360
Homework 4 3 Mar 2006
Due Fri, March 10

1. I'll probably add some reading assignments for next week and break shortly.

2. The destruction and creation operators $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$ are time-independent Schrödinger picture operators. For free fields ($\lambda = 0$), find the Heisenberg picture versions $a(t,\mathbf{k})$ and $a^\dagger(t,\mathbf{k})$ by solving their Heisenberg equations. As usual, choose initial conditions so that they equal the Schrödinger operators when $t = 0$. Use this result to show that $\phi(t,\mathbf{x})$ satisfies the (free) Klein-Gordon equation, with initial condition

$$\phi(0,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \left[a(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\right] \tag{1}$$

(This execise, especially the last step, should be simple.)

3. For small displacements, the Lagrangian for a vibrating string is (cf any text on classical mechanics)

$$L = \frac{1}{2}\int_0^L dx \left[\sigma(\partial_t\phi(t,x))^2 - \tau(\partial_x\phi(t,x))^2\right] \tag{2}$$

where $\phi(t,x)$ gives the displacement in the transverse direction, $\sigma$ is the string's (constant) linear mass density, and $\tau$ is the (constant) string tension.

For this system, rather than assuming that the string ends are fixed at $x = 0$ and $x = L$, assume that the ends are tied together but otherwise free to move, so that the string forms a loop. (These are called periodic boundary conditions. It makes things easier and closer to what we did in class – trust me. It also allows for momentum conservation; why?)

(a) Give the Euler-Lagrange equation for $\phi$. (That is, derive the wave equation.) At what speed do the waves move?

(b) Repeat our discussion of the scalar field to quantize this system and determine the energies and states of the string. (Almost everything is the same except that space is one-dimensional, and the string is tied together at the ends. This puts a condition on the momenta, so that only certain values are possible and the expansion in momentum space is a discrete sum. Note that A&H discusses this system briefly in Ch 5, though with different boundary conditions.)

4. A&H 5.6(b) (Don't worry about the discussion at the end; we'll talk briefly about this in class.) First do the exercise at the end of Appx E.