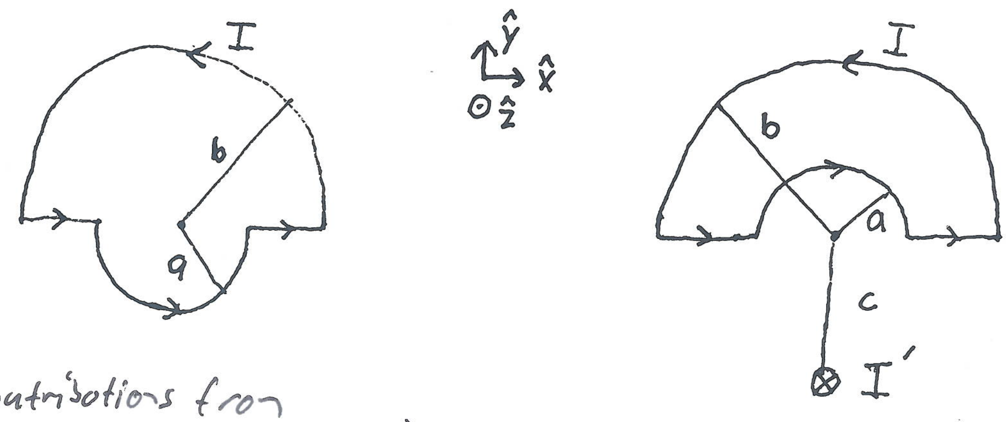
SMU Physics 1304 : Spring 2011

Exam 2

Problem 1 : The figure at left below shows a loop of wire made up of two half-circles of radii $a = 0.1\,\mathrm{m}$ and $b = 0.2\,\mathrm{m}$ and two line segments of length $b - a$. The wire carries carrying a current $I = 2\,\mathrm{A}$ in the direction indicated. Find the magnetic field vector $\vec{B}$ at the common center of the half-circles using the axes indicated. In the figure at right the half circle of radius a has been flipped and there is now another wire with current $I' = 3\,\mathrm{A}$ going into the page which is located a distance $c = 0.15\,\mathrm{m}$ below the center of the half-circles. Again, find the magnetic field vector $\vec{B}$ at the common center of the half-circles using the axes indicated.



No contributions from straight sections: $I d\vec{r} \times \vec{R} = 0$

Left picture:

top: $\vec{B}_T = \frac{\mu_0 I}{4\pi} \int_0^{\pi} d\theta\, b\hat{\theta} \times \frac{(-b\hat{r})}{b^3}$

$= -\frac{\mu_0 I}{4\pi b} \pi\, \hat{\theta} \times \hat{r} = \frac{\mu_0 I}{4b} \hat{z}$

bot: $\vec{B}_B = \frac{\mu_0 I}{4\pi} \int_{-\pi}^{0} d\theta\, a\hat{\theta} \times \frac{(-a\hat{r})}{a^3}$

$= \frac{\mu_0 I}{4a} \hat{z}$

so

$\vec{B}_L = \vec{B}_T + \vec{B}_B = \frac{\mu_0 I}{4} \hat{z} \left(1/b + 1/a\right)$

Right picture:

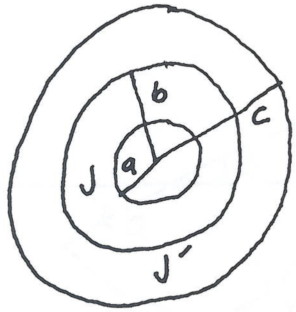
$\vec{B}' = \frac{\mu_0 I'}{2\pi c} \hat{x}$

$\vec{B}_R = \vec{B}_T - \vec{B}_B + \vec{B}'$

$= \frac{\mu_0 I}{4} \hat{z} \left(1/b - 1/a\right)$

$+ \frac{\mu_0 I'}{2\pi c} \hat{x}$

Problem 2 : The figure below shows the cross section of an infinite wire which has a cavity of radius $a = 0.02\,\mathrm{m}$ surrounded by a constant current density $J = 2\,\mathrm{A/m^2}$ with outer radius $b = 0.03\,\mathrm{m}$, with positive J coming out of the page. Surrounding this is a constant current density $J' = -3\,\mathrm{A/m^2}$ with outer radius $c = 0.05\,\mathrm{m}$. Find an expression for $\vec{B}$ for $r < a$, $a < r < b$, $b < r < c$, and $r > c$. Find what the value of c would have to be to impose $\vec{B} = 0$ for $r > c$.



$r<a$

$\vec{B} = 0$

$b>r>a$

$$2\pi r B = \mu_0 J \pi (r^2 - a^2)$$

$$\vec{B} = \frac{\mu_0 J}{2r}(r^2 - a^2)\hat{\theta}$$

$c>r>b$

$$2\pi r B = \mu_0 J \pi (b^2 - a^2) + \mu_0 J' \pi (r^2 - b^2)$$

$$\vec{B} = \frac{\mu_0}{2r}\left(J(b^2 - a^2) + J'(r^2 - b^2)\right)\hat{\theta}$$

$r>c$

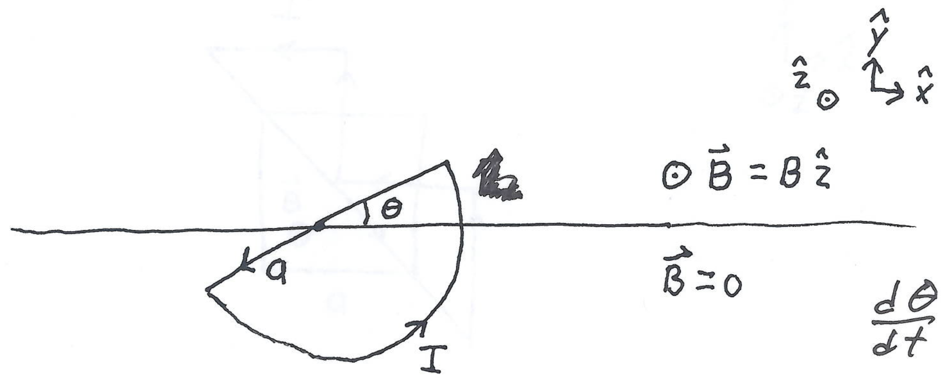
$$\vec{B} = \frac{\mu_0}{2r}\left(J(b^2 - a^2) + J'(c^2 - b^2)\right)\hat{\theta}$$

if we fix $\vec{B} = 0$:
for $r>c$

$$J(b^2 - a^2) = -J'(c^2 - b^2)$$

$$c^2 = b^2 - (J/J')(b^2 - a^2)$$

Problem 3 : The figure below shows a half-circle of wire of radius $a = 0.1\,\mathrm{m}$ which is rotating counter-clockwise into a constant magnetic field $\vec{B} = B\hat{z}$ which has $B = 1\,\mathrm{T}$ for $y > 0$ and $B = 0$ for $y < 0$. The angle θ in the figure is given (in radians) by $\theta = \omega t$ where $\omega = 4\,\mathrm{s}^{-1}$. Find the current I induced in the wire as a function of t, with positive I taken to be counter-clockwise as indicated. Also find the magnetic force $\vec{F}_B$ on the wire as a function of t, in terms of the axes indicated. To find $\vec{F}_B$ you will need $\int \cos\theta = \sin\theta$ and $\int \sin\theta = -\cos\theta$.



$$\frac{d\theta}{dt} = \omega$$

$$IR = -\frac{d}{dt}\int dA\, \vec{B}\cdot\hat{z} = -\frac{d}{dt}\left(B\pi a^2 (\theta/2\pi)\right)$$

$$IR = -\frac{Ba^2}{2}\omega \qquad 0 < \theta < \pi$$

$$= +\frac{Ba^2}{2}\omega \qquad \pi < \theta < 2\pi$$

assume:

$0 < \theta < \pi$

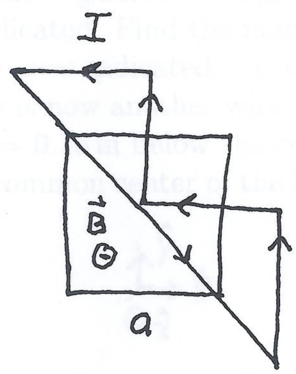
$$\vec{F}_L = I(-a\hat{r}) \times B\hat{z}$$
$$= IaB\hat{\theta}$$
$$= IaB(\cos\theta\,\hat{y} - \sin\theta\,\hat{x})$$

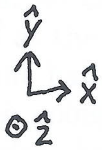
$$\vec{F}_c = I\int_0^\theta d\vec{r} \times \vec{B}$$
$$= I\int_0^\theta d\theta'\, a\,\hat{\theta} \times B\hat{z}$$
$$= IaB\int_0^\theta d\theta'\,\hat{r}(\theta')$$
$$= IaB\int_0^\theta d\theta'(\cos\theta'\,\hat{x} + \sin\theta'\,\hat{y})$$
$$= IaB(\sin\theta\,\hat{x} - (\cos\theta - 1)\hat{y})$$

$$\vec{F}_B = \vec{F}_L + \vec{F}_c = IaB\hat{y}$$

$$= -\frac{B^2a^3}{2}\omega\,\hat{y}$$

Problem 4 : The figure below shows a magnet of square cross section of width $a = 0.1\,\mathrm{m}$ which has a field of the form $\vec{B} = ct^2\hat{z}$, where $c = 3\,\mathrm{T/s^2}$. There is a loop of wire which traces out the figure shown. Find the current I in the wire as a function of t, with positive I taken to be counter-clockwise as shown. Also find the force $\vec{F}_B$ on the wire as a function of t, using the axes indicated.





$$IR = -\frac{d}{dt}\int dA\,\vec{B}\cdot\hat{z}$$

$$= -\frac{d}{dt}\left(\frac{a^2}{4}ct^2\right)$$

$$= -\frac{c}{2}a^2 t$$

$$\underline{I = -\frac{ca^2 t}{2R}}$$

$$\vec{F}_B = I\left(-(a/2)\hat{x} + (a/2)\hat{y} + a\hat{x} - a\hat{y}\right)\times\vec{B}$$

$$= I(a/2)(\hat{x}-\hat{y})\times(ct^2\hat{z})$$

$$= I(a/2)(ct^2)(-\hat{y}-\hat{x})$$

$$= \frac{(ca^2t)}{2R}(a/2)(ct^2)(\hat{x}+\hat{y})$$

$$= \underline{\frac{c^2a^3t^3}{4R}(\hat{x}+\hat{y})}$$