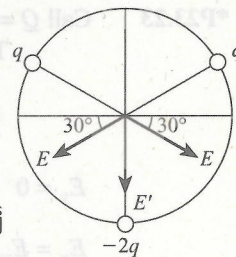


***P23.26** Call the fields $E = \frac{k_e q}{r^2}$ and $E' = \frac{k_e (2q)}{r^2} = 2E$.

The total field at the center of the circle has components

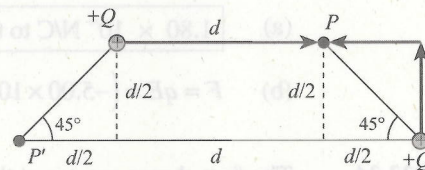
$$\begin{aligned}\vec{E} &= (E \cos 30.0^\circ - E \cos 30.0^\circ) \hat{i} - (E' + 2E \sin 30.0^\circ) \hat{j} \\ \vec{E} &= -(E' + 2E \sin 30.0^\circ) \hat{j} = -(2E + 2E \sin 30.0^\circ) \hat{j} = -2E(1 + \sin 30.0^\circ) \hat{j} \\ \vec{E} &= -2 \frac{k_e q}{r^2} (1 + \sin 30.0^\circ) \hat{j} = -2 \frac{k_e q}{r^2} (1.50) \hat{j} = \boxed{-k_e \frac{3q}{r^2} \hat{j}}\end{aligned}$$



ANS FIG. P23.26

***P23.27** (a) The distance from lower right $+Q$ to point P is $\sqrt{(d/2)^2 + (d/2)^2}$.

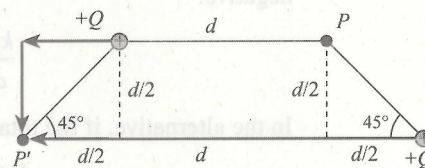
$$\begin{aligned}\vec{E}_P &= k_e \frac{Q}{d^2} \hat{i} + k_e \frac{Q}{[(d/2)^2 + (d/2)^2]} \left(\frac{-\hat{i} + \hat{j}}{\sqrt{2}} \right) \\ &= k_e \left[\frac{Q}{d^2} \hat{i} + \frac{Q}{d^2/2} \left(\frac{-\hat{i} + \hat{j}}{\sqrt{2}} \right) \right] \\ &= \boxed{k_e \frac{Q}{d^2} [(1 - \sqrt{2}) \hat{i} + \sqrt{2} \hat{j}]}\end{aligned}$$



ANS FIG. P23.27(a)

(b) The distance from upper left charge $+Q$ to point P' is $\sqrt{(d/2)^2 + (d/2)^2}$.

$$\begin{aligned}\vec{E}_{P'} &= k_e \frac{Q}{[(d/2)^2 + (d/2)^2]} \left(\frac{-\hat{i} - \hat{j}}{\sqrt{2}} \right) + k_e \frac{Q}{(2d)^2} (-\hat{i}) \\ \vec{E}_{P'} &= -k_e \left[\frac{Q}{d^2/2} \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right) + \frac{Q}{4d^2} (-\hat{i}) \right] \\ &= -k_e \frac{Q}{4d^2} \left[\frac{8}{\sqrt{2}} (\hat{i} + \hat{j}) + (\hat{i}) \right] \\ \vec{E}_{P'} &= \boxed{-k_e \frac{Q}{4d^2} [(1 + 4\sqrt{2}) \hat{i} + 4\sqrt{2} \hat{j}]}\end{aligned}$$



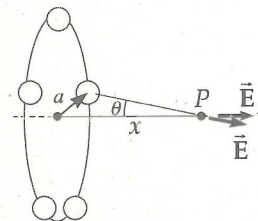
ANS FIG. P23.27(b)

P23.28 (a) One of the charges creates at P a field

$$\vec{E} = E_x \hat{i} = \frac{(k_e Q/n)}{a^2 + x^2} \hat{i}$$

at an angle θ to the x axis as shown.

When all the charges produce the field, for $n > 1$, by symmetry the components perpendicular to the x axis add to zero.



ANS FIG. P23.28

$$\text{The total field is } \vec{E} = nE_x \hat{i} = n \left(\frac{k_e (Q/n)}{a^2 + x^2} \cos \theta \right) = \boxed{\frac{k_e Qx \hat{i}}{(a^2 + x^2)^{3/2}}}$$

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n . Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

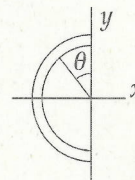
But for $x \gg R$, $\frac{1}{x^2 + R^2/2} \approx \frac{1}{x^2}$, so

$$E_x \approx \frac{k_e Q}{x^2} \text{ for a disk at large distances}$$

P23.35

Due to symmetry $E_y = \int dE_y = 0$, and $E_x = -\int dE \sin \theta = -k_e \int \frac{dq \sin \theta}{r^2}$

where $dq = \lambda ds = \lambda r d\theta$; the component E_x is negative because charge $q = -7.50 \mu\text{C}$, causing the net electric field to be directed to the left.



ANS FIG. P23.35

$$E_x = -\frac{k_e \lambda}{r} \int_0^\pi \sin \theta d\theta = -\frac{k_e \lambda}{r} (-\cos \theta) \Big|_0^\pi = -\frac{2k_e \lambda}{r}$$

where $\lambda = \frac{|q|}{L}$ and $r = \frac{L}{\pi}$.

Thus,

$$E_x = -\frac{2k_e |q| \pi}{L^2} = -\frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(7.50 \times 10^{-6} \text{ C})\pi}{(0.140 \text{ m})^2}$$

$$E_x = -2.16 \times 10^7 \text{ N/C}$$

(a) magnitude $E = \boxed{2.16 \times 10^7 \text{ N/C}}$

(b) to the left

***P23.36**

(a) The electric field at point P due to each element of length dx is $dE = \frac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point P . The charge element $dq = Qdx/L$. The x and y components are

$$E = E_x = \int dE_x = \int dE \sin \theta \quad \text{where} \quad \sin \theta = \frac{x}{\sqrt{d^2 + x^2}}$$

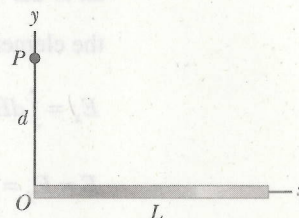
and

$$E = E_y = \int dE_y = \int dE \cos \theta \quad \text{where} \quad \cos \theta = \frac{d}{\sqrt{d^2 + x^2}}$$

Therefore,

$$E_x = -k_e \frac{Q}{L} \int_0^L \frac{x dx}{(d^2 + x^2)^{3/2}} = -k_e \frac{Q}{L} \left[\frac{-1}{(d^2 + x^2)^{1/2}} \right]_0^L$$

$$E_x = -k_e \frac{Q}{L} \left[\frac{-1}{(d^2 + L^2)^{1/2}} - \frac{-1}{(d^2 + 0)^{1/2}} \right] \rightarrow E_x = -k_e \frac{Q}{L} \left[\frac{1}{d} - \frac{1}{(d^2 + L^2)^{1/2}} \right]$$



ANS FIG. P23.36

and

$$E_y = k_e \frac{Qd}{L} \int_0^L \frac{dx}{(d^2 + x^2)^{3/2}} = k_e \frac{Qd}{L} \left[\frac{x}{d^2 (d^2 + x^2)^{1/2}} \right]_0^L$$

$$E_y = k_e \frac{Q}{Ld} \left[\frac{L}{(d^2 + L^2)^{1/2}} - 0 \right] \rightarrow E_y = \boxed{k_e \frac{Q}{d (d^2 + L^2)^{1/2}}}$$

(b) When $d \gg L$,

$$E_x = -k_e \frac{Q}{L} \left[\frac{1}{d} - \frac{1}{(d^2 + L^2)^{1/2}} \right] \rightarrow -k_e \frac{Q}{L} \left[\frac{1}{d} - \frac{1}{(d^2)^{1/2}} \right] \rightarrow \boxed{E_x \approx 0}$$

and

$$E_y = k_e \frac{Q}{d} \frac{1}{(d^2 + L^2)^{1/2}} \rightarrow k_e \frac{Q}{d} \frac{1}{(d^2)^{1/2}} \rightarrow \boxed{E_y \approx k_e \frac{Q}{d^2}}$$

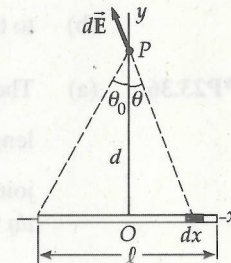
which is the field of a point charge Q at a distance d along the y axis above the charge.

- P23.37** (a) The electric field at point P due to each element of length dx is $dE = \frac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point P . By symmetry,

$$E_x = \int dE_x = 0 \quad \text{and since} \quad dq = \lambda dx,$$

$$E = E_y = \int dE_y = \int dE \cos \theta \quad \text{where} \quad \cos \theta = \frac{d}{\sqrt{x^2 + d^2}}$$

$$\text{Therefore,} \quad E = 2k_e \lambda d \int_0^{\ell/2} \frac{dx}{(x^2 + d^2)^{3/2}} = \boxed{\frac{2k_e \lambda \sin \theta_0}{d}}$$



ANS FIG. P23.37

- (b) For a bar of infinite length, $\theta_0 = 90^\circ$ and $E_y = \boxed{\frac{2k_e \lambda}{d}}$

- P23.38** (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx , has charge $\frac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\vec{E} = \frac{k_e x}{(x^2 + R^2)^{3/2}} \frac{Qdx}{h} \hat{i}$$

The total field is

$$\vec{E} = \int_{\text{all charge}} d\vec{E} = \int_d^{d+h} \frac{k_e Q dx}{h(x^2 + R^2)^{3/2}} \hat{i} = \frac{k_e Q \hat{i}}{2h} \int_{x=d}^{d+h} (x^2 + R^2)^{-3/2} 2x dx$$

$$\vec{E} = \frac{k_e Q \hat{i}}{2h} \frac{(x^2 + R^2)^{-1/2}}{(-1/2)} \bigg|_{x=d}^{d+h} = \frac{k_e Q \hat{i}}{h} \left[\frac{1}{(d^2 + R^2)^{1/2}} - \frac{1}{((d+h)^2 + R^2)^{1/2}} \right]$$

- (b) Think of the cylinder as a stack of disks, each with thickness dx , charge $\frac{Qdx}{h}$, and charge-per-area $\sigma = \frac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\vec{E} = \frac{2\pi k_e Q dx}{\pi R^2 h} \left(1 - \frac{x}{(x^2 + R^2)^{1/2}} \right) \hat{i}$$

$$\text{So, } \vec{E} = \int_{\text{all charge}} d\vec{E} = \int_{x=d}^{d+h} \frac{2k_e Q dx}{R^2 h} \left(1 - \frac{x}{(x^2 + R^2)^{1/2}} \right) \hat{i}$$

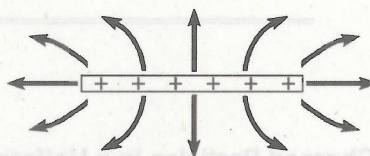
$$\vec{E} = \frac{2k_e Q \hat{i}}{R^2 h} \left[\int_d^{d+h} dx - \frac{1}{2} \int_d^{d+h} (x^2 + R^2)^{-1/2} 2x dx \right] = \frac{2k_e Q \hat{i}}{R^2 h} \left[x \bigg|_d^{d+h} - \frac{1}{2} \frac{(x^2 + R^2)^{1/2}}{1/2} \bigg|_d^{d+h} \right]$$

$$\vec{E} = \frac{2k_e Q \hat{i}}{R^2 h} \left[d+h-d - \left((d+h)^2 + R^2 \right)^{1/2} + (d^2 + R^2)^{1/2} \right]$$

$$\vec{E} = \frac{2k_e Q \hat{i}}{R^2 h} \left[h + (d^2 + R^2)^{1/2} - ((d+h)^2 + R^2)^{1/2} \right]$$

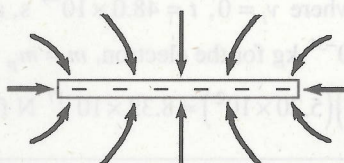
Section 23.6 Electric Field Lines

P23.39



ANS FIG. P23.39

P23.40



ANS FIG. P23.40

$$(b) \quad \vec{E}_1 = \frac{k_e q}{r^2} \hat{r} = (-8.46 \text{ N/C})(0.243\hat{i} + 0.970\hat{j})$$

$$\vec{E}_2 = \frac{k_e q}{r^2} \hat{r} = (11.2 \text{ N/C})(+\hat{j})$$

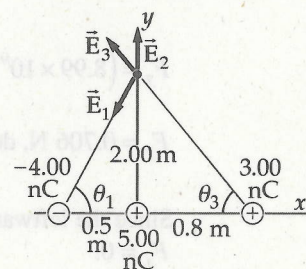
$$\vec{E}_3 = \frac{k_e q}{r^2} \hat{r} = (5.81 \text{ N/C})(-0.371\hat{i} + 0.928\hat{j})$$

$$E_x = E_{1x} + E_{3x} = -4.21 \hat{i} \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} = 8.43 \hat{j} \text{ N/C}$$

$$E_R = \boxed{9.42 \text{ N/C}}$$

$$\theta = \boxed{63.4^\circ \text{ above } -x \text{ axis}}$$



ANS FIG. P23.61(b)

P23.62

$$F = \frac{k_e q_1 q_2}{r^2} : \quad \tan \theta = \frac{15.0}{60.0}$$

$$\theta = 14.0^\circ$$

$$F_1 = \frac{(8.99 \times 10^9)(10.0 \times 10^{-6})^2}{(0.150)^2} = 40.0 \text{ N}$$

$$F_3 = \frac{(8.99 \times 10^9)(10.0 \times 10^{-6})^2}{(0.600)^2} = 2.50 \text{ N}$$

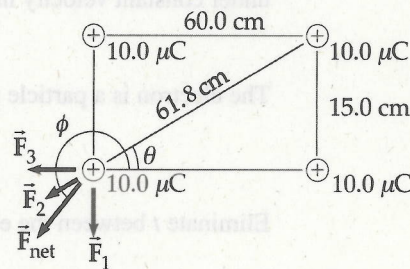
$$F_2 = \frac{(8.99 \times 10^9)(10.0 \times 10^{-6})^2}{(0.618)^2} = 2.35 \text{ N}$$

$$F_x = -F_3 - F_2 \cos 14.0^\circ = -2.50 - 2.35 \cos 14.0^\circ = -4.78 \text{ N}$$

$$F_y = -F_1 - F_2 \sin 14.0^\circ = -40.0 - 2.35 \sin 14.0^\circ = -40.5 \text{ N}$$

$$(a) \quad F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(4.78)^2 + (40.5)^2} = \boxed{40.8 \text{ N}}$$

$$(b) \quad \tan \phi = \frac{F_y}{F_x} = \frac{-40.5}{-4.78} \rightarrow \phi = \boxed{263^\circ}$$



ANS FIG. P23.62

P23.63

$$Q = \int \lambda dl = \int_{-90.0^\circ}^{90.0^\circ} \lambda_0 \cos \theta R d\theta = \lambda_0 R \sin \theta \Big|_{-90.0^\circ}^{90.0^\circ} = \lambda_0 R [1 - (-1)] = 2\lambda_0 R$$

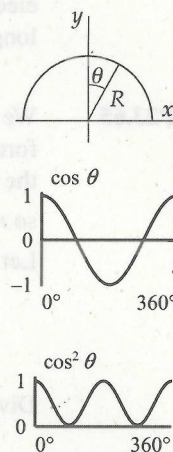
$$Q = 12.0 \mu\text{C} = (2\lambda_0)(0.600) \text{ m} = 12.0 \mu\text{C} \quad \text{so} \quad \lambda_0 = 10.0 \mu\text{C/m}$$

$$dF_y = \frac{1}{4\pi\epsilon_0} \left(\frac{q(\lambda dl)}{R^2} \right) \cos \theta = \frac{1}{4\pi\epsilon_0} \left(\frac{q(\lambda_0 \cos^2 \theta R d\theta)}{R^2} \right)$$

$$F_y = \int_{-90.0^\circ}^{90.0^\circ} \frac{1}{4\pi\epsilon_0} \frac{q\lambda_0}{R} \cos^2 \theta d\theta = \frac{1}{4\pi\epsilon_0} \frac{q\lambda_0}{R} \int_{-\pi/2}^{\pi/2} \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) d\theta$$

$$F_y = \frac{1}{4\pi\epsilon_0} \frac{q\lambda_0}{R} \left(\frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right) \Big|_{-\pi/2}^{\pi/2} = \frac{1}{4\pi\epsilon_0} \frac{q\lambda_0}{R} \left[\left(\frac{\pi}{4} + 0 \right) - \left(-\frac{\pi}{4} + 0 \right) \right]$$

$$F_y = \frac{1}{4\pi\epsilon_0} \frac{q\lambda_0}{R} \left(\frac{\pi}{2} \right)$$



ANS FIG. P23.63

$$F_y = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(3.00 \times 10^{-6} \text{ C})(10.0 \times 10^{-6} \text{ C/m})}{(0.600 \text{ m})} \left(\frac{\pi}{2} \right)$$

$$F_y = 0.706 \text{ N, downward} = \boxed{-0.706 \hat{i} \text{ N}}$$

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

***P23.64** Use Active Figure 23.24 for guidance on the physical setup of this problem. Let the electron enter at the origin of coordinates at the left end and just under the upper plate, which we choose to be negative so that the electron accelerates downward. The electron is a particle under constant velocity in the horizontal direction:

$$x_f = v_{xi} t$$

The electron is a particle under constant acceleration in the vertical direction:

$$y_f = \frac{1}{2} a_y t^2$$

Eliminate t between the equations:

$$y_f = \frac{1}{2} a_y \left(\frac{x_f}{v_{xi}} \right)^2 \rightarrow y_f = \left(\frac{a_y}{2v_{xi}^2} \right) x_f^2$$

Substitute for the acceleration of the particle in terms of the electric force:

$$y_f = \left(\frac{-eE}{2v_{xi}^2 m_e} \right) x_f^2$$

Substitute numerical values, letting the final horizontal position be at the right end of the plates:

$$y_f = \left(\frac{-(1.60 \times 10^{-19} \text{ C})(200 \text{ N/C})}{2(3.00 \times 10^6 \text{ m/s})^2 (9.11 \times 10^{-31} \text{ kg})} \right) (0.200 \text{ m})^2 = -0.078 \text{ m}$$

Therefore, when the electron leaves the plates, its final position is well below that of the lower plate, which is at position $y = -1.50 \text{ cm} = -0.015 \text{ m}$. Consequently, because we have let the electron enter the field as at high a position as possible, the electron will strike the lower plate long before it reaches the end, regardless of where it enters the field.

P23.65 We model the spheres as particles. They have different charges. They exert on each other forces of equal magnitude. They have equal masses, so their strings make equal angles θ with the vertical. The distance r between them is described by $\sin \theta = (r/2)/40.0 \text{ cm}$, so $r = 80.0 \text{ cm} \sin \theta$

Let T represent the string tension. We have

$$\Sigma F_x = 0: k_e q_1 q_2 / r^2 = T \sin \theta$$

$$\Sigma F_y = 0: mg = T \cos \theta$$

Divide to eliminate T : $\frac{k_e q_1 q_2}{r^2 mg} = \tan \theta = \frac{r/2}{\sqrt{(40.0 \text{ cm})^2 - r^2}/4}$

Cleared of fractions, $k_e q_1 q_2 \sqrt{(80.0 \text{ cm})^2 - r^2} = mgr^3$

$$[8.99 \times 10^9 (300 \times 10^{-9}) (200 \times 10^{-9})] \sqrt{(0.800)^2 - r^2} = (2.40 \times 10^{-3}) (9.80) r^3$$

$$(0.800)^2 - r^2 = 1901 r^6$$

We home in on a solution by trying values.

r	$0.640 - r^2 - 1901 r^6$
0	+0.64
0.5	-29.3
0.2	+0.48
0.3	-0.84
0.24	+0.22
0.27	-0.17
0.258	+0.013
0.259	-0.001

Thus the distance to three digits is $0.259 \text{ m} = \boxed{2.59 \text{ cm.}}$

***P23.66** Consider the free-body diagram of the rightmost charge given below.

$$\Sigma F_y = 0 \Rightarrow T \cos \theta = mg \text{ or } T = mg / \cos \theta$$

and

$$\Sigma F_x = 0 \Rightarrow F_e = T \sin \theta = (mg / \cos \theta) \sin \theta = mg \tan \theta$$

But,

$$F_e = \frac{k_e q^2}{r_1^2} + \frac{k_e q^2}{r_2^2} = \frac{k_e q^2}{(L \sin \theta)^2} + \frac{k_e q^2}{(2L \sin \theta)^2} = \frac{5k_e q^2}{4L^2 \sin^2 \theta}$$

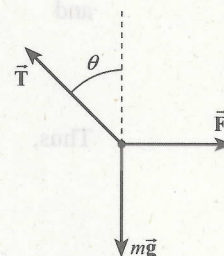
Thus,

$$\frac{5k_e q^2}{4L^2 \sin^2 \theta} = mg \tan \theta \text{ or } q = \sqrt{\frac{4L^2 mg \sin^2 \theta \tan \theta}{5k_e}}$$

If $\theta = 45^\circ$, $m = 0.10 \text{ kg}$, and $L = 0.300 \text{ m}$ then

$$q = \sqrt{\frac{4(0.300 \text{ m})^2 (0.10 \text{ kg}) (9.80 \text{ m/s}^2) \sin^2(45^\circ) \tan(45^\circ)}{5(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)}}$$

$$\text{or } q = 1.98 \times 10^{-6} \text{ C} = \boxed{1.98 \mu\text{C}}$$



ANS FIG. P23.66

***P23.67** Charge Q resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e Q^2}{L^2} = k(L - L_i)$$

Solving for Q , we find

$$Q = L \sqrt{\frac{k(L - L_i)}{k_e}} = (0.500) \sqrt{\frac{100(0.500 - 0.400)}{8.99 \times 10^9}} = \boxed{1.67 \times 10^{-5} \text{ C}}$$

P23.68 Charge Q resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e Q^2}{L^2} = k(L - L_i)$$

- P23.73** (a) The total non-contact force on the cork ball is: $F = qE + mg = m\left(g + \frac{qE}{m}\right)$, which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T = 2\pi \sqrt{\frac{L}{g + qE/m}} = 2\pi \sqrt{\frac{0.500 \text{ m}}{9.80 \text{ m/s}^2 + \left[(2.00 \times 10^{-6} \text{ C})(1.00 \times 10^5 \text{ N/C}) / 1.00 \times 10^{-3} \text{ kg}\right]}}$$

$$= \boxed{0.307 \text{ s}}$$

- (b) **Yes**. Without gravity in part (a), we get

$$T = 2\pi \sqrt{\frac{L}{qE/m}}$$

$$T = 2\pi \sqrt{\frac{0.500 \text{ m}}{(2.00 \times 10^{-6} \text{ C})(1.00 \times 10^5 \text{ N/C}) / 1.00 \times 10^{-3} \text{ kg}}} = 0.314 \text{ s (a 2.28% difference)}.$$

- P23.74** The field on the axis of the ring is calculated in an Example in the chapter text as

$$E = E_x = \frac{k_e x Q}{(x^2 + a^2)^{3/2}}$$

The force experienced by a charge $-q$ placed along the axis of the ring is

$$F = -k_e Q q \left[\frac{x}{(x^2 + a^2)^{3/2}} \right]$$

and when $x \ll a$, this becomes

$$F = -\left(\frac{k_e Q q}{a^3}\right)x$$

This expression for the force is in the form of Hooke's law, with an effective spring constant of

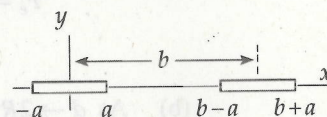
$$k = \frac{k_e Q q}{a^3}$$

Since $\omega = 2\pi f = \sqrt{\frac{k}{m}}$, we have

$$f = \frac{1}{2\pi} \sqrt{\frac{k_e Q q}{ma^3}}$$

CHALLENGE PROBLEMS

- P23.75** According to the result of an Example in the chapter text, the lefthand rod creates this field at a distance d from its right-hand end:



ANS FIG. P23.75

$$E = \frac{k_e Q}{d(2a+d)}$$

$$dF = \frac{k_e Q Q}{2a} \frac{dx}{d(d+2a)}$$

$$F = \frac{k_e Q^2}{2a} \int_{x=b-2a}^b \frac{dx}{x(x+2a)} = \frac{k_e Q^2}{2a} \left(-\frac{1}{2a} \ln \frac{2a+x}{x} \right)_{b-2a}^b$$

$$F = \frac{k_e Q^2}{4a^2} \left(-\ln \frac{2a+b}{b} + \ln \frac{b}{b-2a} \right) = \frac{k_e Q^2}{4a^2} \ln \frac{b^2}{(b-2a)(b+2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2} \right) \ln \left(\frac{b^2}{b^2 - 4a^2} \right)}$$

P23.76 From Figure (a) we have $d \cos 30.0^\circ = 15.0 \text{ cm}$

$$\text{or } d = \frac{15.0 \text{ cm}}{\cos 30.0^\circ}$$

From Figure (b) we have

$$\theta = \sin^{-1} \left(\frac{d}{50.0 \text{ cm}} \right)$$

$$\theta = \sin^{-1} \left(\frac{15.0 \text{ cm}}{50.0 \text{ cm} (\cos 30.0^\circ)} \right) = 20.3^\circ$$

$$\frac{F_q}{mg} = \tan \theta \quad \text{or} \quad F_q = mg \tan 20.3^\circ \quad (1)$$

From Figure (c) we have

$$F_q = 2F \cos 30.0^\circ$$

$$F_q = 2 \left[\frac{k_e q^2}{(0.300 \text{ m})^2} \right] \cos 30.0^\circ \quad (2)$$

Combining equations (1) and (2),

$$2 \left[\frac{k_e q^2}{(0.300 \text{ m})^2} \right] \cos 30.0^\circ = mg \tan 20.3^\circ$$

$$q^2 = \frac{mg(0.300 \text{ m})^2 \tan 20.3^\circ}{2k_e \cos 30.0^\circ}$$

$$q^2 = \frac{(2.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)(0.300 \text{ m})^2 \tan 20.3^\circ}{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \cos 30.0^\circ}$$

$$q = \sqrt{4.20 \times 10^{-14} \text{ C}^2} = 2.05 \times 10^{-7} \text{ C} = \boxed{0.205 \mu\text{C}}$$

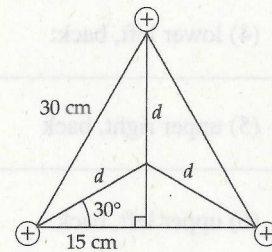


Figure (a)

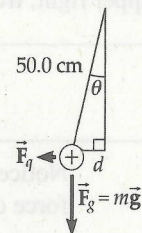


Figure (b)

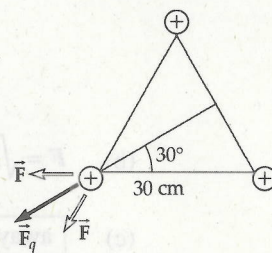


Figure (c)

ANS FIG. P23.76

$$F_e = \left(\frac{mg}{\cos 10^\circ} \right) \sin 10^\circ = mg \tan 10^\circ = (0.001 \text{ kg})(9.8 \text{ m/s}^2) \tan 10^\circ$$

$$F_e \approx 2 \times 10^{-3} \text{ N} \quad \boxed{\sim 10^{-3} \text{ N or 1 mN}}$$

$$(b) \quad F_e = \frac{k_e q^2}{r^2}$$

$$2 \times 10^{-3} \text{ N} \approx \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) q^2}{(0.25 \text{ m})^2}$$

$$q \approx 1.2 \times 10^{-7} \text{ C} \quad \boxed{\sim 10^{-7} \text{ C or 100 nC}}$$

$$(c) \quad E = \frac{k_e q}{r^2} \approx \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.2 \times 10^{-7} \text{ C})}{(0.25 \text{ m})^2} \approx 1.7 \times 10^4 \text{ N/C} \quad \boxed{\sim 10 \text{ kN/C}}$$

$$(d) \quad \Phi_E = \frac{q}{\epsilon_0} \approx \frac{1.2 \times 10^{-7} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2} = 1.4 \times 10^4 \text{ N} \cdot \text{m}^2 / \text{C} \quad \boxed{\sim 10 \text{ kN} \cdot \text{m}^2 / \text{C}}$$

P24.27 (a) $\vec{E} = \boxed{0}$

$$(b) \quad E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(32.0 \times 10^{-6})}{(0.200)^2} = 7.19 \text{ MN/C}$$

$$\vec{E} = \boxed{7.19 \text{ MN/C radially outward}}$$

P24.28 (a) $\sigma = (8.60 \times 10^{-6} \text{ C/cm}^2) \left(\frac{100 \text{ cm}}{\text{m}} \right)^2 = 8.60 \times 10^{-2} \text{ C/m}^2$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{8.60 \times 10^{-2}}{2(8.85 \times 10^{-12})} = \boxed{4.86 \times 10^9 \text{ N/C away from the wall}}$$

(b) So long as the distance from the wall is small compared to the width and height of the wall, the distance does not affect the field.

P24.29 (a) $E = \frac{2k_e \lambda}{r} = \frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) [(2.00 \times 10^{-6} \text{ C}) / 7.00 \text{ m}]}{0.100 \text{ m}}$

$$E = \boxed{51.4 \text{ kN/C, radially outward}}$$

$$(b) \quad \Phi_E = EA \cos \theta = E(2\pi r \ell) \cos 0^\circ$$

$$\Phi_E = (5.14 \times 10^4 \text{ N/C}) 2\pi (0.100 \text{ m})(0.0200 \text{ m})(1.00) = \boxed{646 \text{ N} \cdot \text{m}^2 / \text{C}}$$

***P24.30** (a) The area of each face is $A = 1.00 \text{ m}^2$.

left face: The angle between the electric field and the normal is 0° :

$$(\Phi_E)_{\text{left face}} = EA \cos \theta = (20.0 \text{ N/C})(1.00 \text{ m}^2) \cos 0^\circ = 20.0 \text{ N} \cdot \text{m}^2 / \text{C}$$

right face: The angle between the electric field and the normal is 180° :

$$(\Phi_E)_{\text{right face}} = EA \cos \theta = (35.0 \text{ N/C})(1.00 \text{ m}^2) \cos 180^\circ = -35.0 \text{ N} \cdot \text{m}^2/\text{C}$$

top face: The angle between the electric field and the normal is 180° :

$$(\Phi_E)_{\text{top face}} = EA \cos \theta = (25.0 \text{ N/C})(1.00 \text{ m}^2) \cos 180^\circ = -25.0 \text{ N} \cdot \text{m}^2/\text{C}$$

bottom face: The angle between the electric field and the normal is 0° :

$$(\Phi_E)_{\text{bottom face}} = EA \cos \theta = (15.0 \text{ N/C})(1.00 \text{ m}^2) \cos 0^\circ = 15.0 \text{ N} \cdot \text{m}^2/\text{C}$$

front face: The angle between the electric field and the normal is 0° :

$$(\Phi_E)_{\text{front face}} = EA \cos \theta = (20.0 \text{ N/C})(1.00 \text{ m}^2) \cos 0^\circ = 20.0 \text{ N} \cdot \text{m}^2/\text{C}$$

back face: The angle between the electric field and the normal is 0° :

$$(\Phi_E)_{\text{back face}} = EA \cos \theta = (20.0 \text{ N/C})(1.00 \text{ m}^2) \cos 0^\circ = 20.0 \text{ N} \cdot \text{m}^2/\text{C}$$

$$\text{Total flux: } \Phi_E = (20.0 - 35.0 - 25.0 + 15.0 + 20.0 + 20.0) \text{ N} \cdot \text{m}^2/\text{C} = \boxed{15.0 \text{ N} \cdot \text{m}^2/\text{C}}$$

$$(b) \quad \Phi_E = \frac{q_{\text{in}}}{\epsilon_0} \rightarrow q_{\text{in}} = \epsilon_0 \Phi_E = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(15.0 \text{ N} \cdot \text{m}^2/\text{C}) = \boxed{1.33 \times 10^{-10} \text{ C}}$$

(c) No; fields on the faces would not be uniform.

P24.31

$$(a) \quad E = \frac{k_e Q r}{a^3} = \boxed{0}$$

$$(b) \quad E = \frac{k_e Q r}{a^3} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = \boxed{365 \text{ kN/C}}$$

$$(c) \quad E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = \boxed{1.46 \text{ MN/C}}$$

$$(d) \quad E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = \boxed{649 \text{ kN/C}}$$

The direction for each electric field is radially outward.

P24.32

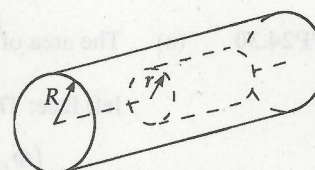
$$(a) \quad E = \frac{2k_e \lambda}{r} \rightarrow 3.60 \times 10^4 = \frac{2(8.99 \times 10^9)(Q/2.40)}{0.190}$$

$$Q = +9.13 \times 10^{-7} \text{ C} = \boxed{+913 \text{ nC}}$$

$$(b) \quad \vec{E} = \boxed{0}$$

P24.33

If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r , contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho \pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps;



ANS FIG. P24.33

The flux at $x = 0.3 \text{ m} + dx$ is out of the box:

$$\Phi_E = +EA = +(6\,000 \text{ N/C} \cdot \text{m}^2)(0.3 \text{ m} + dx)^2 A = +(540 \text{ N/C}) A + (3\,600 \text{ N/C} \cdot \text{m}) dx A$$

(The term in $(dx)^2$ is negligible.)

The charge in the box is $\rho A dx$ where ρ is the unknown. Applying Gauss's law:

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0}$$

$$-(540 \text{ N/C}) A + (540 \text{ N/C}) A + (3\,600 \text{ N/C} \cdot \text{m}) dx A = \rho A dx / \epsilon_0$$

$$\text{Then } \rho = (3\,600 \text{ N/C} \cdot \text{m}) \epsilon_0 = (3\,600 \text{ N/C} \cdot \text{m})(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = \boxed{31.9 \text{ nC/m}^3}$$

- (b) No; then the field would have to be zero.

P24.43

- (a) Inside surface: consider a cylindrical gaussian surface of arbitrary length ℓ within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0} \rightarrow 0 = \frac{(\lambda + \lambda_{\text{inner}})\ell}{\epsilon_0}, \text{ so the inside } \lambda_{\text{inner}} = \boxed{-\lambda}.$$

- (b) Outside surface: consider a cylindrical gaussian surface of arbitrary length ℓ outside the metal. The total charge within the gaussian surface is

$$q_{\text{wire}} + q_{\text{cylinder}} = q_{\text{wire}} + (q_{\text{inner surface}} + q_{\text{outer surface}})$$

$$\lambda\ell + 2\lambda\ell = \lambda\ell + (-\lambda\ell + \lambda_{\text{outer}}\ell) \rightarrow \lambda_{\text{outer}} = \boxed{3\lambda}$$

- (c) Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E2\pi r\ell = \frac{3\lambda\ell}{\epsilon_0} \rightarrow E = 2 \frac{3\lambda}{4\pi\epsilon_0 r} = \boxed{6k_e \frac{\lambda}{r}}, \text{ radially outward}$$

P24.44

- (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2} \left(\frac{q}{A} \right) = \frac{1}{2} \frac{(4.00 \times 10^{-8} \text{ C})}{(0.500 \text{ m})^2} = 8.00 \times 10^{-8} \text{ C/m}^2 = \boxed{80.0 \text{ nC/m}^2}$$

$$(b) \quad \vec{E} = \left(\frac{\sigma}{\epsilon_0} \right) \hat{k} = \left(\frac{8.00 \times 10^{-8} \text{ C/m}^2}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} \right) \hat{k} = \boxed{(9.04 \text{ kN/C}) \hat{k}}$$

$$(c) \quad \vec{E} = \boxed{(-9.04 \text{ kN/C}) \hat{k}}$$

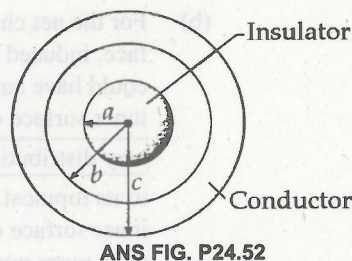
ADDITIONAL PROBLEMS

- *P24.45 The electric field makes an angle of 60.0° with to the normal. The square has side $d = 5.00 \text{ cm}$.

$$\Phi_E = EA \cos \theta = (3.50 \times 10^2 \text{ N/C})(5.00 \times 10^{-2} \text{ m})^2 \cos 60.0^\circ = \boxed{0.438 \text{ N} \cdot \text{m}^2/\text{C}}$$

P24.52 (a) For $r < a$,

$$q_{\text{in}} = \rho \left(\frac{4}{3} \pi r^3 \right) = \left(\frac{Q}{\frac{4}{3} \pi R^3} \right) \left(\frac{4}{3} \pi r^3 \right) = \boxed{Q \left(\frac{r}{R} \right)^3}$$



(b) Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \left(\frac{r}{a} \right)^3 \rightarrow E = \frac{1}{4\pi \epsilon_0} \frac{Qr}{a^3} = \boxed{k_e \frac{Qr}{a^3}}$$

(c) For $a < r < b$, the total charge is contained in the solid, insulating sphere: $q_{\text{in}} = \boxed{Q}$.

(d) Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} = \boxed{k_e \frac{Q}{r^2}}$$

(e) For $b \leq r \leq c$, $\boxed{E = 0}$ because there is no electric field inside a conductor.

(f) For $b \leq r \leq c$, we know $E = 0$. Assume the inner surface of the hollow sphere holds charge Q_{inner} . By Gauss's law,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$0 = \frac{Q + Q_{\text{inner}}}{\epsilon_0} \rightarrow Q_{\text{inner}} = \boxed{-Q}$$

(g) The total charge on the hollow sphere is zero; therefore, charge on the outer surface is opposite to that on the inner surface: $Q_{\text{outer}} = -Q_{\text{inner}} = \boxed{+Q}$.

(h) A surface of area A holding charge Q has surface charge $\sigma = q/A$. The solid, insulating sphere has small surface charge because its total charge Q is uniformly distributed throughout its volume. The inner surface of radius b has smaller surface area, and therefore larger surface charge, than the outer surface of radius c .

P24.53

Consider the charge distribution to be an unbroken charged spherical shell with uniform charge density σ and a circular disk with charge per area $-\sigma$. The total field is that due to the whole sphere,

$$E_{\text{sphere}} = \frac{Q}{4\pi \epsilon_0 R^2} = \frac{4\pi R^2 \sigma}{4\pi \epsilon_0 R^2} = \frac{\sigma}{\epsilon_0} \text{ outward}$$

plus the field of the disk

$$E_{\text{disk}} = -\frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0}, \text{ radially inward}$$

CHALLENGE PROBLEMS

P24.58 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge ($Q = +e$) with radius R . From Gauss's law,

$$\begin{aligned}\int \vec{E} \cdot d\vec{A} &= \frac{q_{\text{in}}}{\epsilon_0} \\ (4\pi r^2)E &= \frac{q_{\text{in}}}{\epsilon_0} = \frac{1}{\epsilon_0} \rho V = \frac{1}{\epsilon_0} \left(\frac{+e}{\frac{4}{3}\pi R^3} \right) \frac{4}{3}\pi r^3 \\ \rightarrow (4\pi r^2)E &= \left(\frac{e}{\epsilon_0 R^3} \right) r^3 \\ \rightarrow E &= \left(\frac{e}{4\pi \epsilon_0 R^3} \right) r, \text{ directed outward}\end{aligned}$$

(a) The force exerted on a point charge $q = -e$ located at distance r from the center is then

$$F = qE = -e \left(\frac{e}{4\pi \epsilon_0 R^3} \right) r = - \left(\frac{e^2}{4\pi \epsilon_0 R^3} \right) r = \boxed{-Kr}$$

(b) From (a), $K = \frac{e^2}{4\pi \epsilon_0 R^3} = \boxed{\frac{k_e e^2}{R^3}}$

(c) $F_r = m_e a_r = - \left(\frac{k_e e^2}{R^3} \right) r$, so $a_r = - \left(\frac{k_e e^2}{m_e R^3} \right) r = -\omega^2 r$

Thus, the motion is simple harmonic with frequency $f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi} \sqrt{\frac{k_e e^2}{m_e R^3}}}$

(d) $f = 2.47 \times 10^{15} \text{ Hz} = \frac{1}{2\pi} \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})R^3}}$

which yields $R^3 = 1.05 \times 10^{-30} \text{ m}^3$, or $R = \boxed{1.02 \times 10^{-10} \text{ m}}$

P24.59 Consider the gaussian surface described in the solution to problem 59.

(a) For $x > \frac{d}{2}$, $dq = \rho dV = \rho A dx = CAx^2 dx$

$$\begin{aligned}\int \vec{E} \cdot d\vec{A} &= \frac{1}{\epsilon_0} \int dq \\ EA &= \frac{CA}{\epsilon_0} \int_0^{d/2} x^2 dx = \frac{1}{3} \left(\frac{CA}{\epsilon_0} \right) \left(\frac{d^3}{8} \right)\end{aligned}$$

$$E = \frac{Cd^3}{24 \epsilon_0} \quad \text{or} \quad \boxed{\vec{E} = \frac{Cd^3}{24 \epsilon_0} \hat{i} \text{ for } x > \frac{d}{2}; \quad \vec{E} = -\frac{Cd^3}{24 \epsilon_0} \hat{i} \text{ for } x < -\frac{d}{2}}$$

$$(b) \quad \text{For } -\frac{d}{2} < x < \frac{d}{2} \quad \int \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \int dq = \frac{CA}{\epsilon_0} \int_0^x x^2 dx = \frac{CAx^3}{3\epsilon_0}$$

$$\vec{E} = \frac{Cx^3}{3\epsilon_0} \hat{i} \text{ for } x > 0; \quad \vec{E} = -\frac{Cx^3}{3\epsilon_0} \hat{i} \text{ for } x < 0$$

P24.60

The resultant field within the cavity is the superposition of two fields, one \vec{E}_+ due to a uniform sphere of positive charge of radius $2a$, and the other \vec{E}_- due to a sphere of negative charge of radius a centered within the cavity.

$$\frac{4}{3} \left(\frac{\pi r^3 \rho}{\epsilon_0} \right) = 4\pi r^2 E_+ \quad \text{so} \quad \vec{E}_+ = \frac{\rho r}{3\epsilon_0} \hat{r} = \frac{\rho \vec{r}}{3\epsilon_0}$$

$$-\frac{4}{3} \left(\frac{\pi r_1^3 \rho}{\epsilon_0} \right) = 4\pi r_1^2 E_- \quad \text{so} \quad \vec{E}_- = \frac{\rho r_1}{3\epsilon_0} (-\hat{r}_1) = \frac{-\rho}{3\epsilon_0} \vec{r}_1$$

$$\text{Since } \vec{r} = \vec{a} + \vec{r}_1, \quad \vec{E}_- = \frac{-\rho(\vec{r} - \vec{a})}{3\epsilon_0}$$

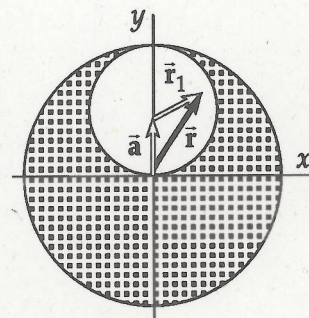
$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\rho \vec{r}}{3\epsilon_0} - \frac{\rho \vec{r}}{3\epsilon_0} + \frac{\rho \vec{a}}{3\epsilon_0} = \frac{\rho \vec{a}}{3\epsilon_0} = 0\hat{i} + \frac{\rho a}{3\epsilon_0} \hat{j}$$

Thus,

$$E_x = 0$$

and

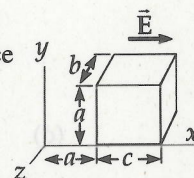
$$E_y = \frac{\rho a}{3\epsilon_0} \text{ at all points within the cavity}$$



ANS FIG. P24.60

P24.61

- (a) The electric field throughout the region is directed along x ; therefore, \vec{E} will be perpendicular to normal dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to normal dA over the two faces which are parallel to the yz plane. Therefore,



ANS FIG. P24.61

$$\Phi_E = -(E_x|_{x=a})A + (E_x|_{x=a+c})A$$

$$\Phi_E = -(3 + 2a^2)ab + [3 + 2(a+c)^2]ab$$

$$\Phi_E = 2abc(2a+c)$$

Substituting the given values for a , b , and c , we find $\Phi_E = 0.269 \text{ N} \cdot \text{m}^2 / \text{C}$.

$$(b) \quad \Phi_E = \frac{q_{in}}{\epsilon_0} \rightarrow q_{in} = \epsilon_0 \Phi_E = 2.38 \times 10^{-12} \text{ C}$$

P24.62

Spherical symmetry allows the gaussian surface integral to be simplified to $\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2)$.

By Gauss's law,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \rightarrow E(4\pi r^2) = \frac{q_{in}}{\epsilon_0} \rightarrow E = \frac{q_{in}}{\epsilon_0 (4\pi r^2)}$$

P24.65 In this case the charge density is *not uniform*, and Gauss's law is written as $\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \int \rho dV$.

We use a gaussian surface which is a cylinder of radius r , length ℓ , and is coaxial with the charge distribution.

- (a) When $r < R$, this becomes $E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0} \int_0^r \left(a - \frac{r}{b}\right) dV$. The element of volume is a cylindrical shell of radius r , length ℓ , and thickness dr so that $dV = 2\pi r\ell dr$.

$$E(2\pi r\ell) = \left(\frac{2\pi r^2 \ell \rho_0}{\epsilon_0}\right) \left(\frac{a}{2} - \frac{r}{3b}\right) \text{ so inside the cylinder, } E = \boxed{\frac{\rho_0 r}{2\epsilon_0} \left(a - \frac{2r}{3b}\right)}$$

- (b) When $r > R$, Gauss's law becomes

$$E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0} \int_0^R \left(a - \frac{r}{b}\right) (2\pi r\ell dr) \text{ or outside the cylinder, } E = \boxed{\frac{\rho_0 R^2}{2\epsilon_0 r} \left(a - \frac{2R}{3b}\right)}$$

- P24.66** (a) Consider a cylindrical shaped gaussian surface perpendicular to the yz plane with its left end in the yz plane and its right end at distance x :

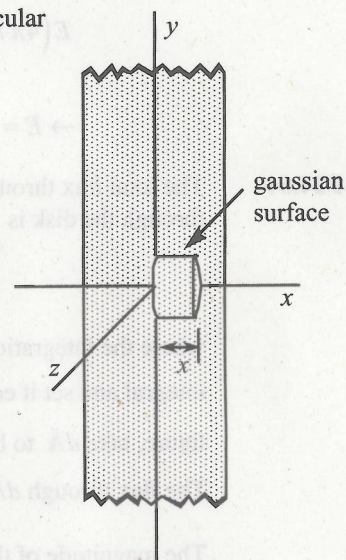
Use Gauss's law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

By symmetry, the electric field is zero in the yz plane and is perpendicular to $d\vec{A}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point x :

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \text{ or } EA = \frac{\rho(Ax)}{\epsilon_0}$$

so that at distance x from the mid-line of the slab,

$$\boxed{E = \frac{\rho x}{\epsilon_0}}$$



ANS FIG. P24.66

- (b) $a = \frac{F}{m_e} = \frac{(-e)E}{m_e} = -\left(\frac{\rho e}{m_e \epsilon_0}\right)x$

The acceleration of the electron is of the form

$$a = -\omega^2 x \text{ with } \omega = \sqrt{\frac{\rho e}{m_e \epsilon_0}}$$

Thus, the motion is simple harmonic with frequency

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi} \sqrt{\frac{\rho e}{m_e \epsilon_0}}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P24.2 355 kN · m²/C

P24.4 (a) -2.34 kN · m²/C; (b) +2.34 kN · m²/C; (c) 0