Therefore $E = 0$ when $x = \frac{-4.00 \pm \sqrt{16.0+16.0}}{2} = \boxed{-4.83 \text{ m}}$

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V = \frac{k_e q_1}{x} + \frac{k_e q_2}{2.00 - x} = 0$ or $V = k_e\left(\frac{+q}{x} - \frac{2q}{2.00-x}\right) = 0$

Again solving for x, $2qx = q(2.00 - x)$

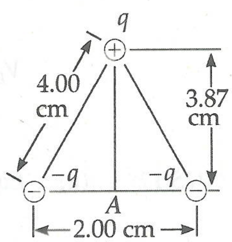
For $0 \le x \le 2.00$ $V = 0$ when $x = \boxed{0.667 \text{ m}}$

and $\frac{q}{|x|} = \frac{-2q}{|2-x|}$ For $x < 0$ $x = \boxed{-2.00 \text{ m}}$

P25.20 The charges at the base vertices are $d/2 = 0.0100$ m from point A, and the charge at the top vertex is

$$\sqrt{(2d)^2 - \left(\frac{d}{2}\right)^2} = \frac{\sqrt{15}}{2}d$$

from point A.



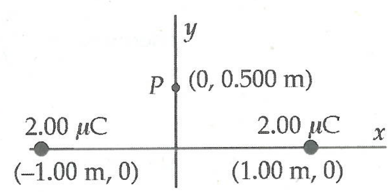
ANS FIG. P25.20

$$V = \sum_i k_e \frac{q_i}{r_i}$$

$$= k_e\left(\frac{-q}{d/2} + \frac{-q}{d/2} + \frac{q}{d\sqrt{15}/2}\right) = k_e \frac{q}{d}\left(-4 + \frac{2}{\sqrt{15}}\right)$$

$$V = \left(8.99 \times 10^9\right)\frac{7.00 \times 10^{-6}}{0.0200}\left(-4 + \frac{2}{\sqrt{15}}\right) = \boxed{-1.10 \times 10^7 \text{ V}}$$

P25.21 (a) $V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = 2\left(\frac{k_e q}{r}\right)$



ANS FIG. P25.21

$$V = 2\left(\frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00 \times 10^{-6} \text{ C}\right)}{\sqrt{(1.00 \text{ m})^2 + (0.500 \text{ m})^2}}\right)$$

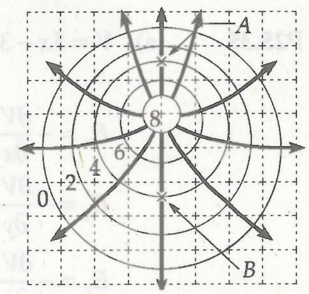
$V = 3.22 \times 10^4 \text{ V} = \boxed{32.2 \text{ kV}}$

(b) $U = qV = \left(-3.00 \times 10^{-6} \text{ C}\right)\left(3.22 \times 10^4 \text{ J/C}\right) = \boxed{-9.65 \times 10^{-2} \text{ J}}$

P25.38 (a) $\boxed{E_A > E_B \text{ since } E = \frac{\Delta V}{\Delta s}}$

(b) $E_B = -\frac{\Delta V}{\Delta s} = -\frac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}$ down

(c) The figure is shown to the right, with sample field lines sketched in.



ANS FIG. P25.38

P25.39 $$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_e Q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$$

$$E_y = \frac{k_e Q}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_e Q}{y\sqrt{\ell^2 + y^2}}}$$

Section 25.5 Electric Potential Due to Continuous Charge Distributions

P25.40 $$V = \int dV = \frac{1}{4\pi \in_0}\int \frac{dq}{r}$$

All bits of charge are at the same distance from O.

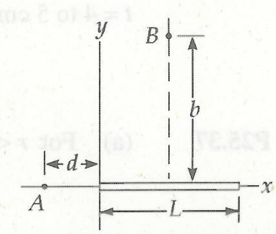
So

$$V = \frac{1}{4\pi \in_0}\left(\frac{Q}{R}\right) = \left(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-7.50 \times 10^{-6}\text{ C}}{0.140\text{ m}/\pi}\right) = \boxed{-1.51\text{ MV}}$$

P25.41 $$\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R}\left(\frac{1}{\sqrt{5}} - 1\right) = \boxed{-0.553\frac{k_e Q}{R}}$$

P25.42 (a) $$[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\text{C}}{\text{m}}\cdot\left(\frac{1}{\text{m}}\right) = \boxed{\frac{\text{C}}{\text{m}^2}}$$

(b) $$V = k_e\int \frac{dq}{r} = k_e\int \frac{\lambda dx}{r} = k_e\alpha\int_0^L \frac{x dx}{d + x} = \boxed{k_e\alpha\left[L - d\ln\left(1 + \frac{L}{d}\right)\right]}$$



ANS FIG. P25.42

P25.43 $$V = \int \frac{k_e dq}{r} = k_e\int \frac{\alpha x dx}{\sqrt{b^2 + (L/2 - x)^2}}$$

Let $z = \frac{L}{2} - x$.

Then $x = \frac{L}{2} - z$, and $dx = -dz$

$$V = k_e\alpha\int\frac{(L/2-z)(-dz)}{\sqrt{b^2+z^2}} = -\frac{k_e\alpha L}{2}\int\frac{dz}{\sqrt{b^2+z^2}} + k_e\alpha\int\frac{z\,dz}{\sqrt{b^2+z^2}}$$

$$= -\frac{k_e\alpha L}{2}\ln\left(z+\sqrt{z^2+b^2}\right) + k_e\alpha\sqrt{z^2+b^2}$$

$$V = -\frac{k_e\alpha L}{2}\ln\left[\left(\frac{L}{2}-x\right)+\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\right]_0^L + k_e\alpha\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\,\Bigg|_0^L$$

$$V = -\frac{k_e\alpha L}{2}\ln\left[\frac{L/2-L+\sqrt{(L/2)^2+b^2}}{L/2+\sqrt{(L/2)^2+b^2}}\right] + k_e\alpha\left[\sqrt{\left(\frac{L}{2}-L\right)^2+b^2}-\sqrt{\left(\frac{L}{2}\right)^2+b^2}\right]$$

$$V = \boxed{-\frac{k_e\alpha L}{2}\ln\left[\frac{\sqrt{b^2+(L^2/4)}-L/2}{\sqrt{b^2+(L^2/4)}+L/2}\right]}$$

P25.44 $V = k_e\int_{\text{all charge}}\frac{dq}{r} = k_e\int_{-3R}^{-R}\frac{\lambda dx}{-x} + k_e\int_{\text{semicircle}}\frac{\lambda ds}{R} + k_e\int_R^{3R}\frac{\lambda dx}{x}$

$$V = -k_e\lambda\ln(-x)\Big|_{-3R}^{-R} + \frac{k_e\lambda}{R}\pi R + k_e\lambda\ln x\Big|_R^{3R}$$

$$V = k_e\ln\frac{3R}{R} + k_e\lambda\pi + k_e\ln 3 = \boxed{k_e\lambda(\pi + 2\ln 3)}$$

Section 25.6 Electric Potential Due to a Charged Conductor

P25.45 Substituting given values into $V = \frac{k_e q}{r}$

$$7.50\times10^3\text{ V} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q}{0.300\text{ m}}$$

Substituting $q = 2.50\times10^{-7}$ C,

$$N = \frac{2.50\times10^{-7}\text{ C}}{1.60\times10^{-19}\text{ C}/e^-} = \boxed{1.56\times10^{12}\text{ electrons}}$$

***P25.46** [No.] A conductor of any shape forms an equipotential surface. If the conductor is a sphere of radius R, and if it holds charge Q, the electric field at its surface is $E = k_eQ/R^2$ and the potential of the surface is $V = k_eQ/R$; thus, if we know E and R, we can fund V. However, if the surface varies in shape, there is no clear way to relate electric field at a point on the surface to the potential of the surface.

P25.47 (a) Both spheres must be at the same potential according to $\dfrac{k_e q_1}{r_1} = \dfrac{k_e q_2}{r_2}$

where also $q_1 + q_2 = 1.20 \times 10^{-6}$ C

Then $q_1 = \dfrac{q_2 r_1}{r_2}$

$$\frac{q_2 r_1}{r_2} + q_2 = 1.20 \times 10^{-6} \text{ C}$$

$$q_2 = \frac{1.20 \times 10^{-6} \text{ C}}{1 + 6 \text{ cm}/2 \text{ cm}} = 0.300 \times 10^{-6} \text{ C on the smaller sphere}$$

$$q_1 = 1.20 \times 10^{-6} \text{ C} - 0.300 \times 10^{-6} \text{ C} = 0.900 \times 10^{-6} \text{ C}$$

$$V = \frac{k_e q_1}{r_1} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(0.900 \times 10^{-6} \text{ C})}{6 \times 10^{-2} \text{ m}} = \boxed{1.35 \times 10^5 \text{ V}}$$

(b) Outside the larger sphere,

$$\vec{\mathbf{E}}_1 = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}} = \frac{V_1}{r_1}\hat{\mathbf{r}} = \frac{1.35 \times 10^5 \text{ V}}{0.06 \text{ m}}\hat{\mathbf{r}} = \boxed{2.25 \times 10^6 \text{ V/m away}}$$

Outside the smaller sphere,

$$\vec{\mathbf{E}}_2 = \frac{1.35 \times 10^5 \text{ V}}{0.02 \text{ m}}\hat{\mathbf{r}} = \boxed{6.74 \times 10^6 \text{ V/m away}}$$

The smaller sphere carries less charge but creates a much stronger electric field than the larger sphere.

P25.48 (a) $E = \boxed{0}$

$$V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{0.140} = \boxed{1.67 \text{ MV}}$$

(b) $$E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)^2} = \boxed{5.84 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{0.200} = \boxed{1.17 \text{ MV}}$$

(c) $$E = \frac{k_e q}{R^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.140)^2} = \boxed{11.9 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$$

(d) From the kinetic energy of part (c),

$$K = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(7.81\times10^{-17}\ \text{J})}{1.67\times10^{-27}\ \text{kg}}} = 3.06\times10^5\ \text{m/s} = \boxed{306\ \text{km/s}}$$

(e) Using the constant-acceleration equation: $v_f^2 = v_i^2 + 2a(x_f - x_i)$

$$(3.06\times10^5\ \text{m/s})^2 = 0 + 2a(0.120\ \text{m})$$

$$a = \boxed{3.90\times10^{11}\ \text{m/s}^2\ \text{toward the negative plate}}$$

(f) $\sum F = ma = (1.67\times10^{-27}\ \text{kg})(3.90\times10^{11}\ \text{m/s}^2) = \boxed{6.51\times10^{-16}\ \text{N}}$ toward the negative plate

(g) $E = \dfrac{F}{q} = \dfrac{6.51\times10^{-16}\ \text{N}}{1.60\times10^{-19}\ \text{C}} = \boxed{4.07\ \text{kN/C}}$

(h) They are the same.

P25.66 (a) $V_B - V_A = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is

$$E = \frac{\lambda}{2\pi\in_0 r} = \frac{2k_e\lambda}{r}$$

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

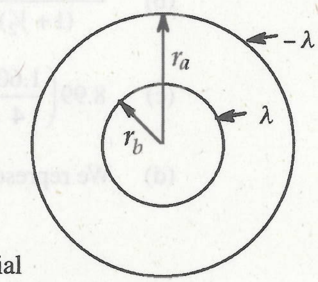


FIG. P25.66

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r}\,dr = 2k_e\lambda\ln\left(\frac{r_a}{r_b}\right)$$

or

$$\boxed{\Delta V = 2k_e\lambda\ln\left(\frac{r_a}{r_b}\right)}$$

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda\ln\left(\frac{r_a}{r}\right)$$

The field at r is given by

$$E = -\frac{\partial V}{\partial r} = -2k_e\lambda\left(\frac{r}{r_a}\right)\left(-\frac{r_a}{r^2}\right) = \frac{2k_e\lambda}{r}$$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$.

Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$.

according to $T\cos\theta = mg$. Dividing the expression for the horizontal component by that for the vertical component, we find that

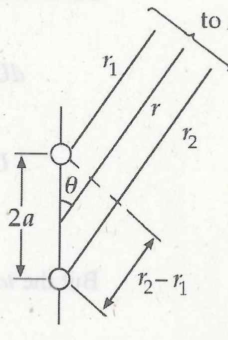
$$\tan\theta = \frac{4V_0^2 xR}{k_e d^2 mg} = \frac{x}{L} \quad \rightarrow \quad V_0 = \left(\frac{k_e d^2 mg}{4RL}\right)^{1/2} \text{ for small } x$$

If V_0 is less than this value, the only equilibrium position of the ball is hanging straight down. If V_0 exceeds this value, the ball will swing over to one plate or the other.

P25.71 (a) $V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2}(r_2 - r_1)$

From the figure, for $r >> a$, $r_2 - r_1 \approx 2a\cos\theta$. Note that r_1 is approximately equal to r_2.

Then $V \approx \frac{k_e q}{r_1 r_2} 2a\cos\theta \approx \frac{k_e p\cos\theta}{r^2}$



ANS FIG. P25.71

(b) $E_r = -\frac{\partial V}{\partial r} = \boxed{\frac{2k_e p\cos\theta}{r^3}}$

In spherical coordinates, the θ component of the gradient is $-\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$.

Therefore, $E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$

(c) For $r >> a$, $\theta = 90°$: $E_r(90°) = 0$, $E_\theta(90°) = \frac{k_e p}{r^3}$

For $r >> a$, $\theta = 0°$: $E_r(0°) = \frac{2k_e p}{r^3}$, $E_\theta(0°) = 0$

[Yes,] these results are reasonable.

(d) [No,] because as $r \to 0$, $E \to \infty$. The magnitude of the electric field between the charges of the dipole is not infinite.

(e) Substituting $r_1 \approx r_2 \approx r = (x^2 + y^2)^{1/2}$ and $\cos\theta = \frac{y}{(x^2+y^2)^{1/2}}$ into $V = \frac{k_e p\cos\theta}{r^2}$ gives

$$\boxed{V = \frac{k_e p y}{(x^2+y^2)^{3/2}}}$$

and

$$\boxed{E_x = -\frac{\partial V}{\partial x} = \frac{3k_e pxy}{(x^2+y^2)^{5/2}}}$$

$$\boxed{E_y = -\frac{\partial V}{\partial y} = \frac{k_e p(2y^2 - x^2)}{(x^2+y^2)^{5/2}}}$$

P25.72 $dU = Vdq$ where the potential $V = \frac{k_e q}{r}$.

The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho\left(4\pi r^2 dr\right)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho\int_0^r r^2\,dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e\rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho\left(4\pi r^2 dr\right) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4\,dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4\,dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

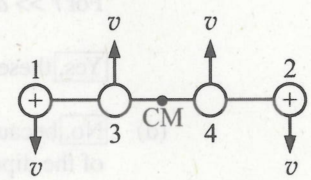
But the *total* charge, $Q = \rho\frac{4}{3}\pi R^3$. Therefore, $\boxed{U = \frac{3}{5}\frac{k_e Q^2}{R}}$.

P25.73 For an element of area which is a ring of radius r and width dr, $dV = \frac{k_e dq}{\sqrt{r^2 + x^2}}$.

$$dq = \sigma dA = Cr\left(2\pi r dr\right) \text{ and}$$

$$V = C\left(2\pi k_e\right)\int_0^R \frac{r^2\,dr}{\sqrt{r^2 + x^2}} = \boxed{\pi k_e C\left[R\sqrt{R^2 + x^2} + x^2\ln\left(\frac{x}{R + \sqrt{R^2 + x^2}}\right)\right]}$$

P25.74 Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal y-components of velocity. The maximum-kinetic-energy point is illustrated. System energy is conserved because it is isolated:



ANS FIG. P25.74

$$K_i + U_i = K_f + U_f$$
$$0 + U_i = K_f + U_f$$
$$\to U_i = K_f + U_f$$

$$\frac{k_e q^2}{a} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{k_e q^2}{3a}$$

$$\frac{2k_e q^2}{3a} = 2mv^2 \quad \to \quad \boxed{v = \sqrt{\frac{k_e q^2}{3am}}}$$