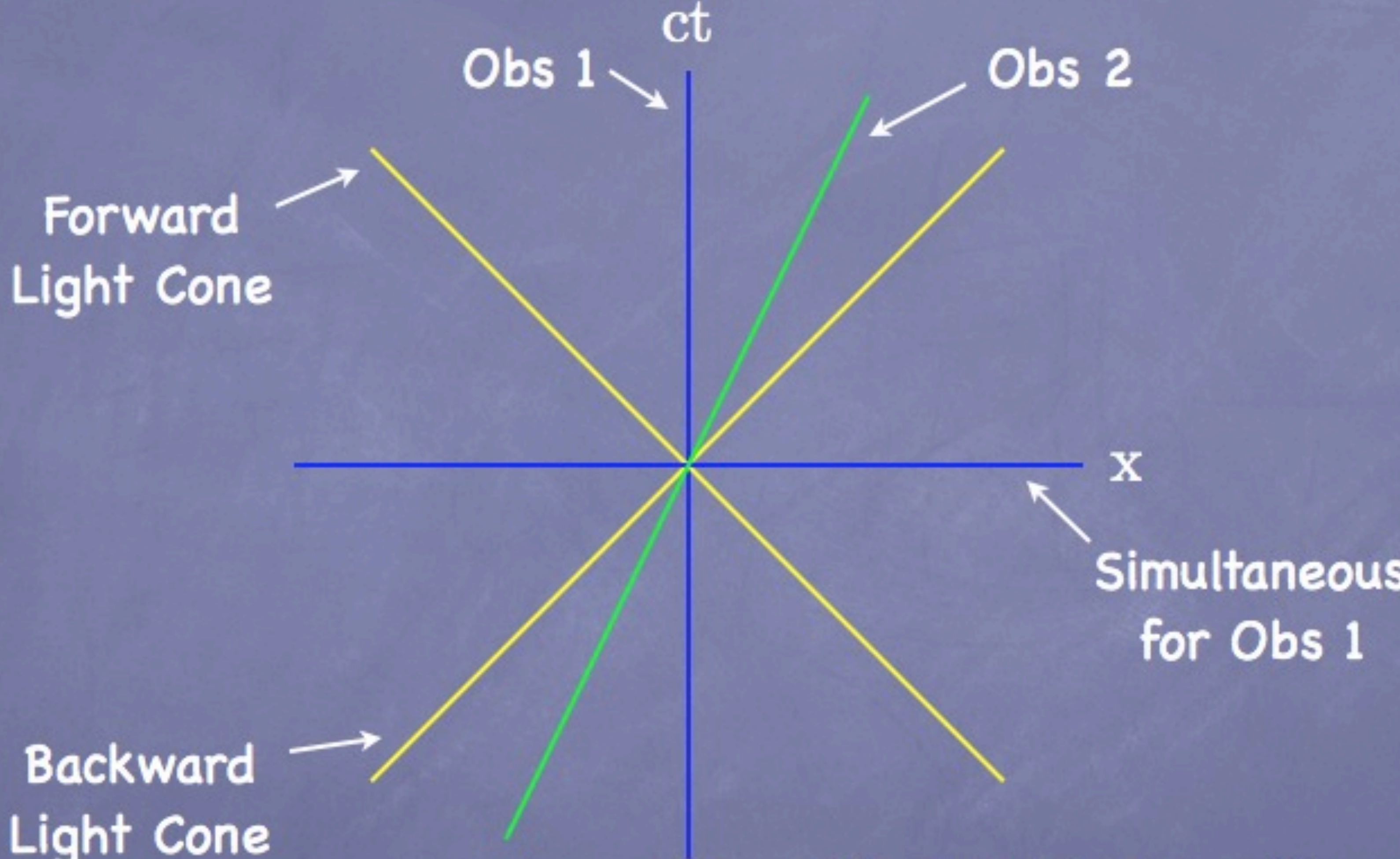
Spacetime Diagram



Observer 1 : $x = 0$

Observer 2 : $x = vt$

Inertial Observers (IOs) :

Do not accelerate. Follow straight lines in spacetime diagrams.

Are equivalent to each other - Relativity Principle.

All see light moving at precisely same speed :

$$c = 299{,}792{,}458 \text{ m/s}$$

Events :

Events are single points in spacetime. They are occurrences that take place over an extremely short time within an extremely small region.

Worldlines:

Worldlines are the curves mapped out by observers. The worldlines of inertial observers are straight lines.

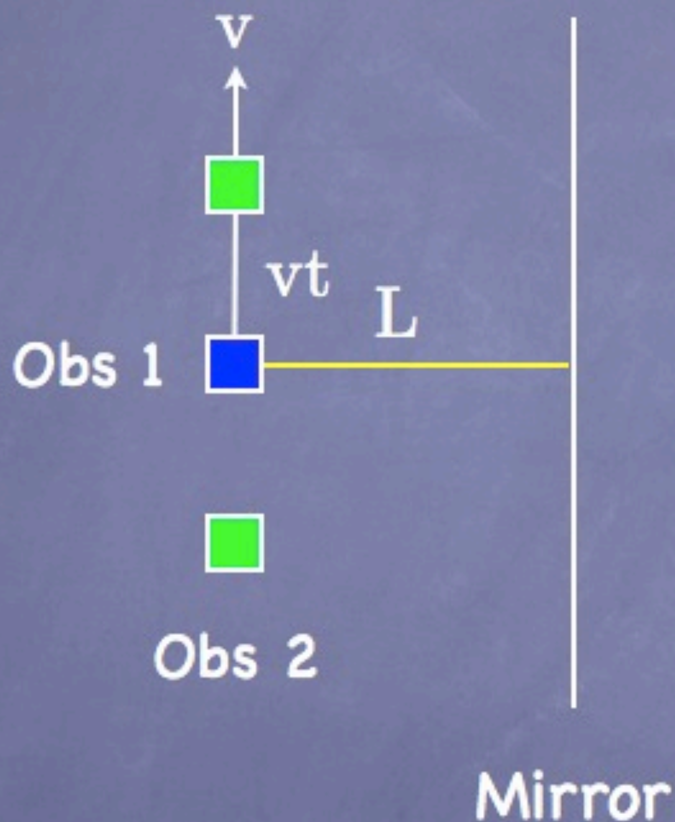
Proper Time :

Proper time between two events is the time measured by an observer which passes through those events.

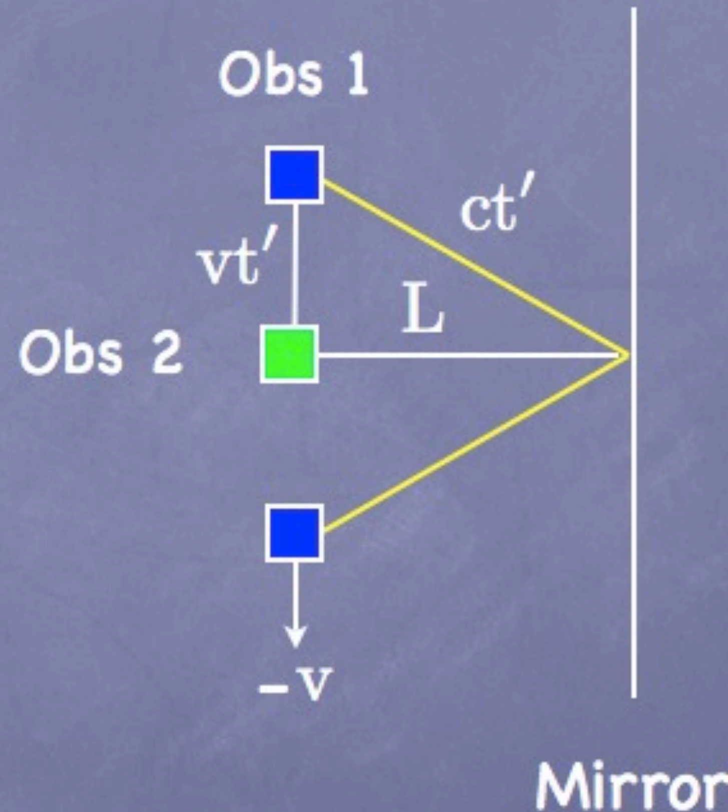
Time Dilation :

The proper time measured between two events on the worldline of an IO is shorter than the time measured between those events by any other IO. Time is dilated (stretched) for IOs which do not pass through both events.

Time Dilation



$$L = ct$$



$$(ct')^2 = (vt')^2 + L^2$$

Time Dilation

Since : $\mathrm{L = ct}$

And : $\mathrm{(ct')^2 = (vt')^2 + L^2}$

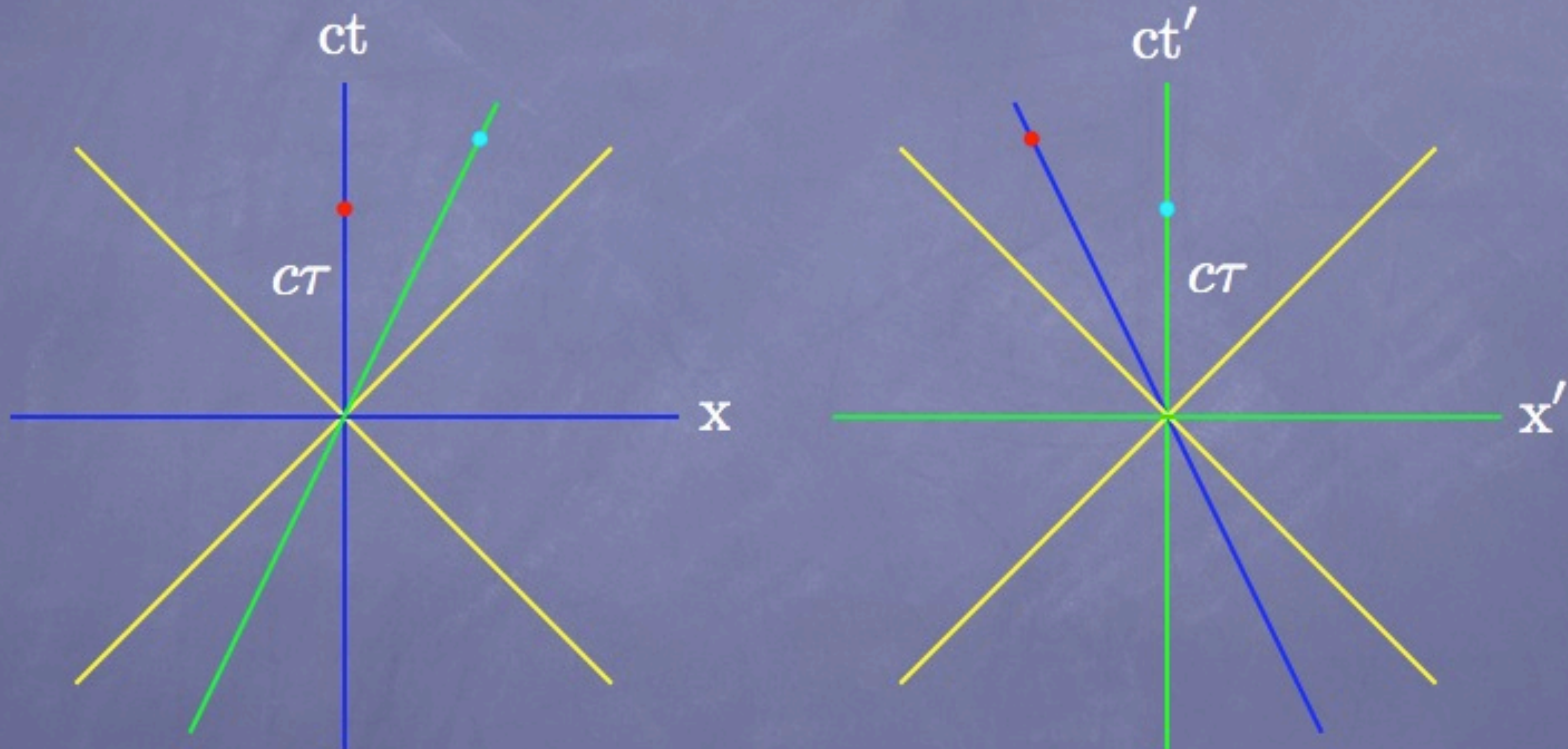
$$\mathrm{t' = \frac{t}{\sqrt{1 - v^2/c^2}} = \gamma t \geq t}$$

Time measured between two events by IOs is shortest for observer which passes through both events. Proper time is shortest.

Events chosen to have same proper time from origin.

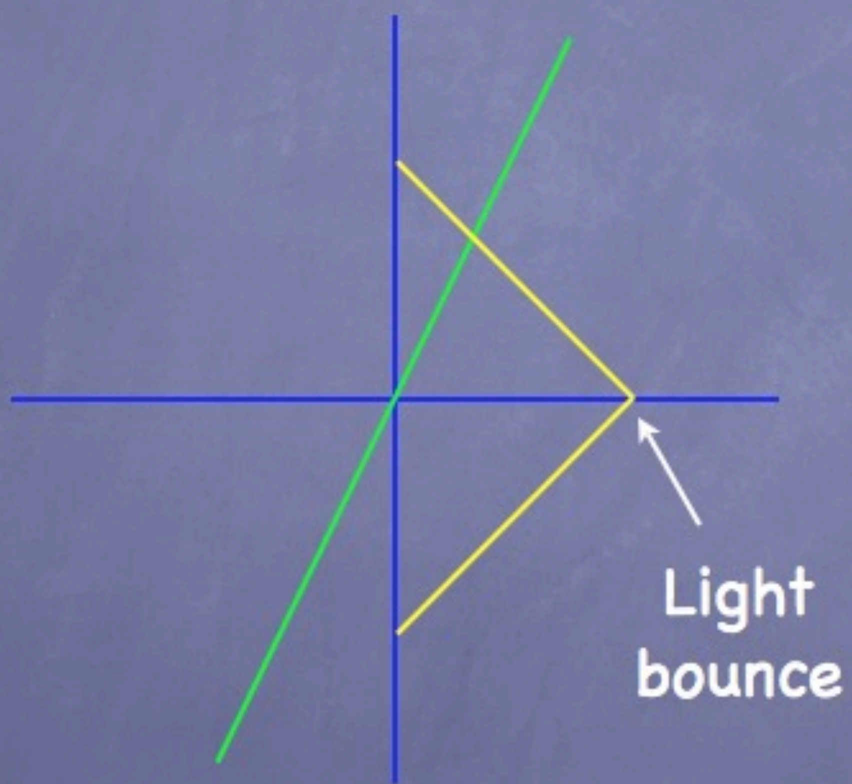
Event ● : $\mathrm{x} = 0 \quad \mathrm{ct} = c\tau < \mathrm{ct}'$

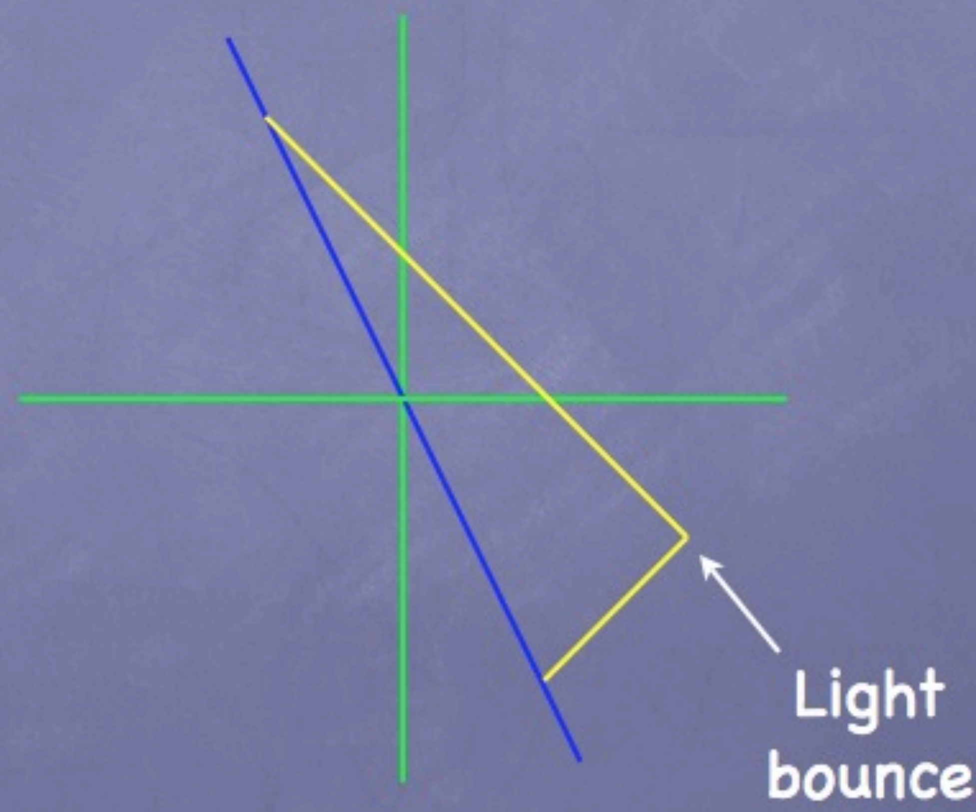
Event ● : $\mathrm{x}' = 0 \quad \mathrm{ct}' = c\tau < \mathrm{ct}$



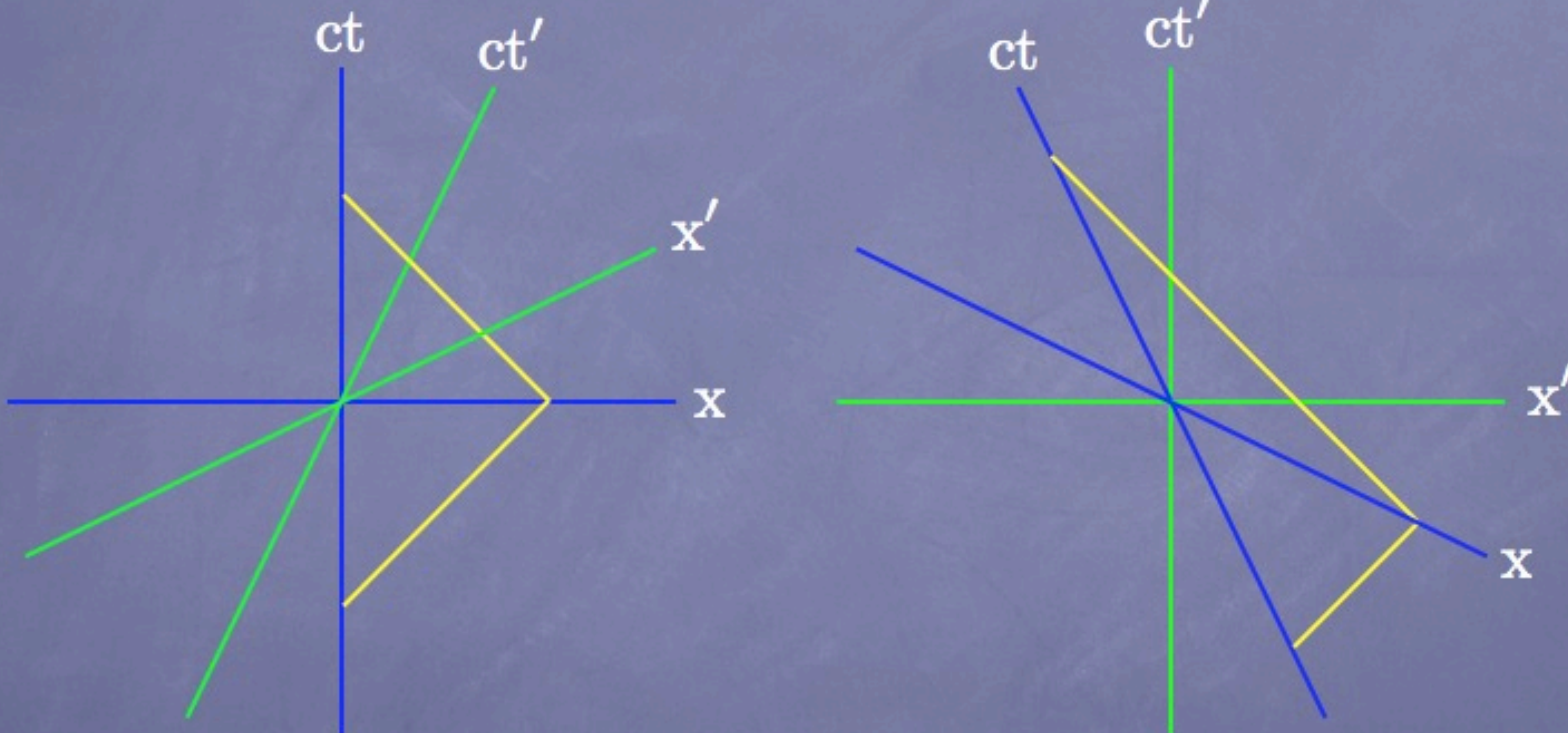
Failure of Simultaneity

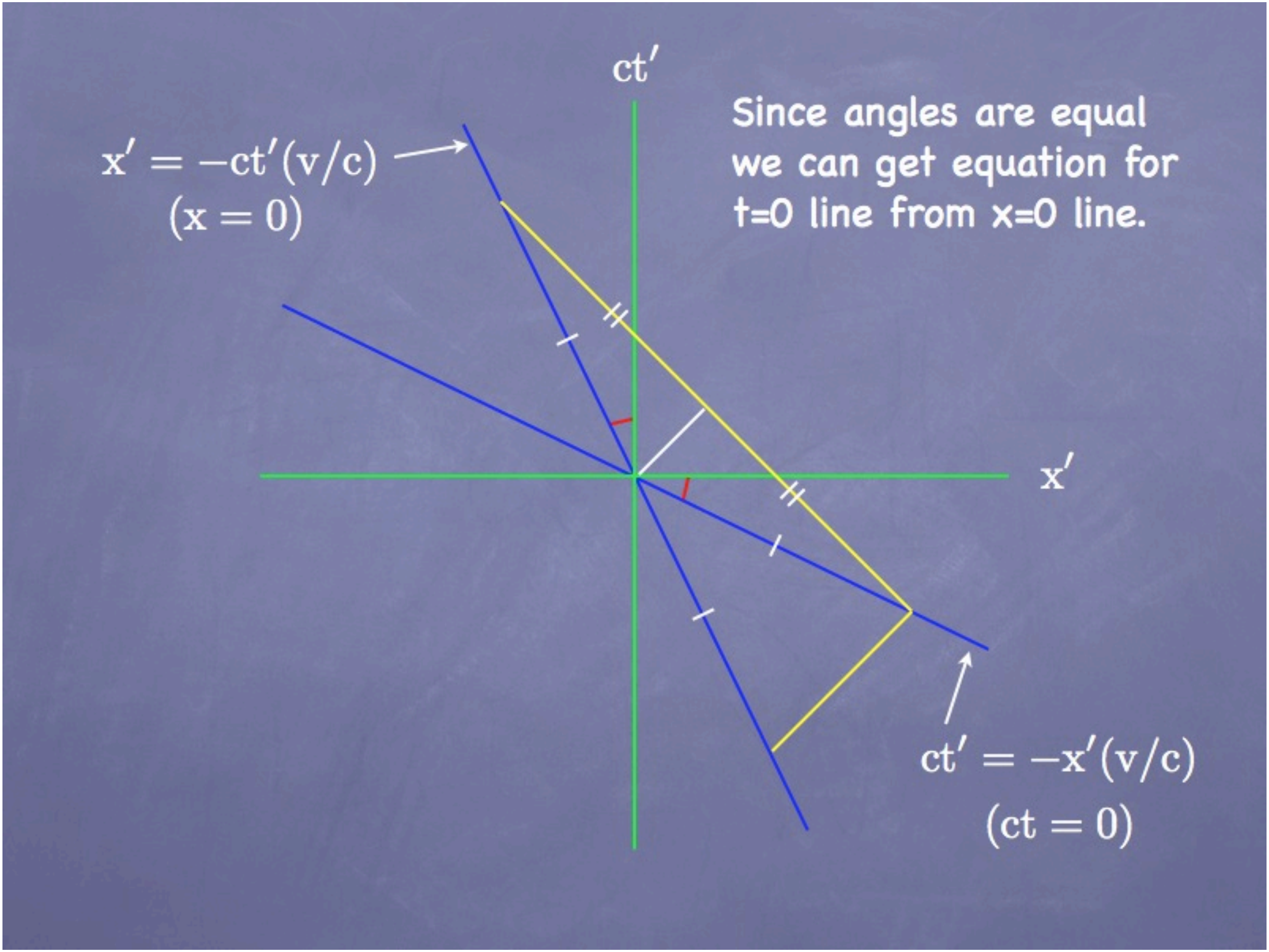
Events that are simultaneous to one inertial observer are not simultaneous to another.





This allows us to draw both axes on both diagrams.

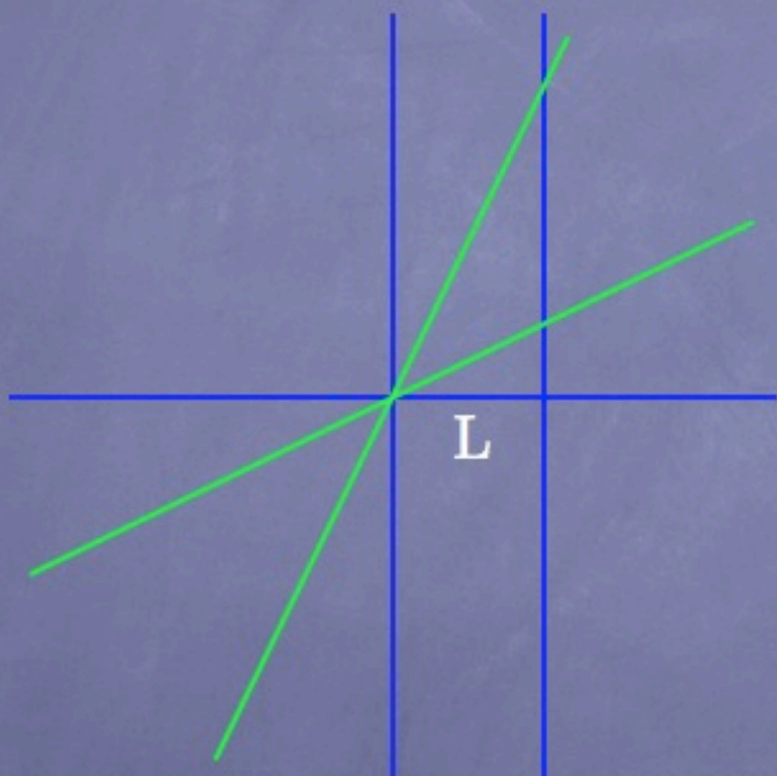




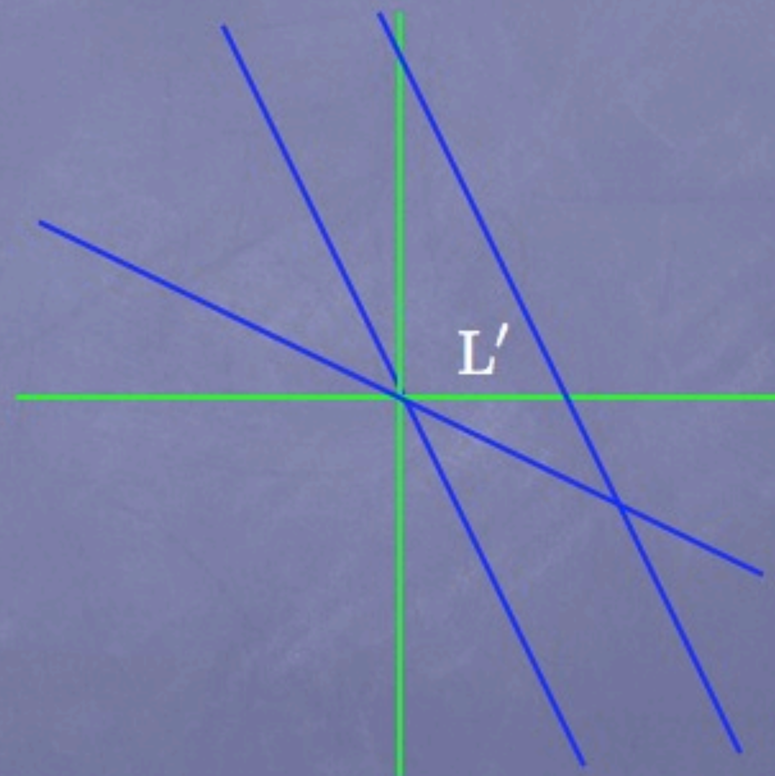
ct'
Since angles are equal we can get equation for t=0 line from x=0 line.
$x' = -ct'(v/c)$
$(x = 0)$
x'
$ct' = -x'(v/c)$
$(ct = 0)$

Length Contraction

Obs 1 at Rest



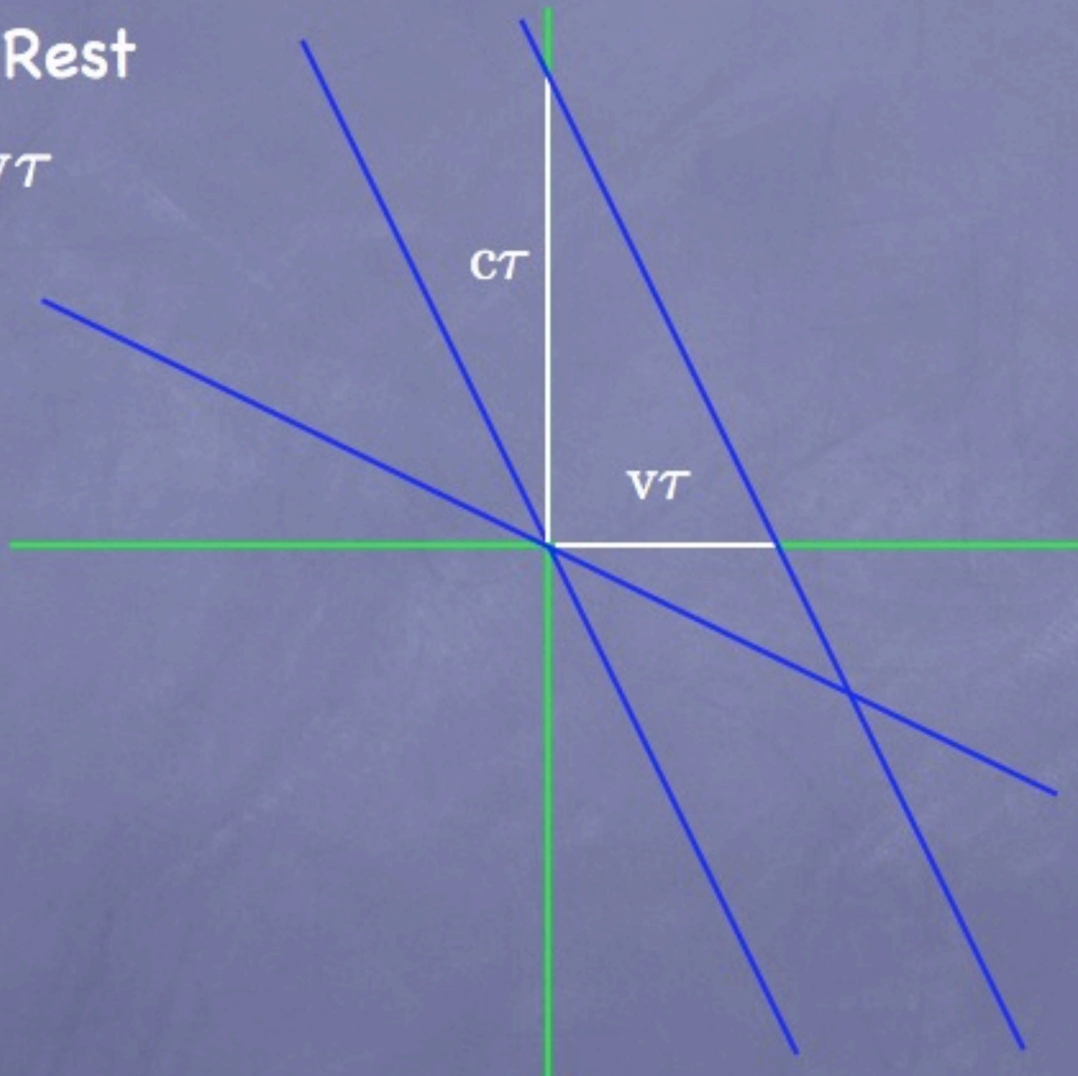
Obs 2 at Rest



Length Contraction

Obs 2 at Rest

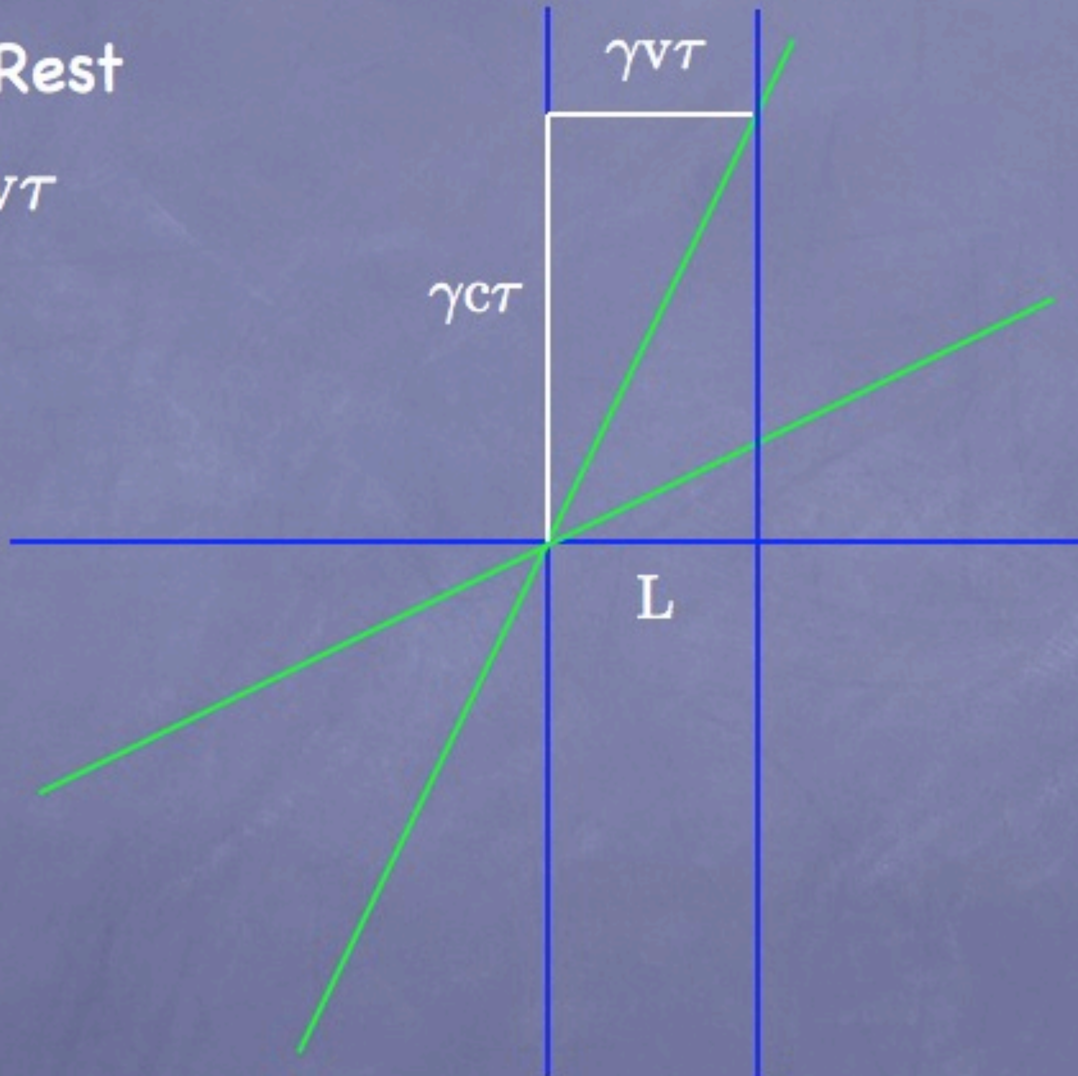
$$L' = v\tau$$



Length Contraction

Obs 1 at Rest

$$L = \gamma v \tau$$



Length Contraction

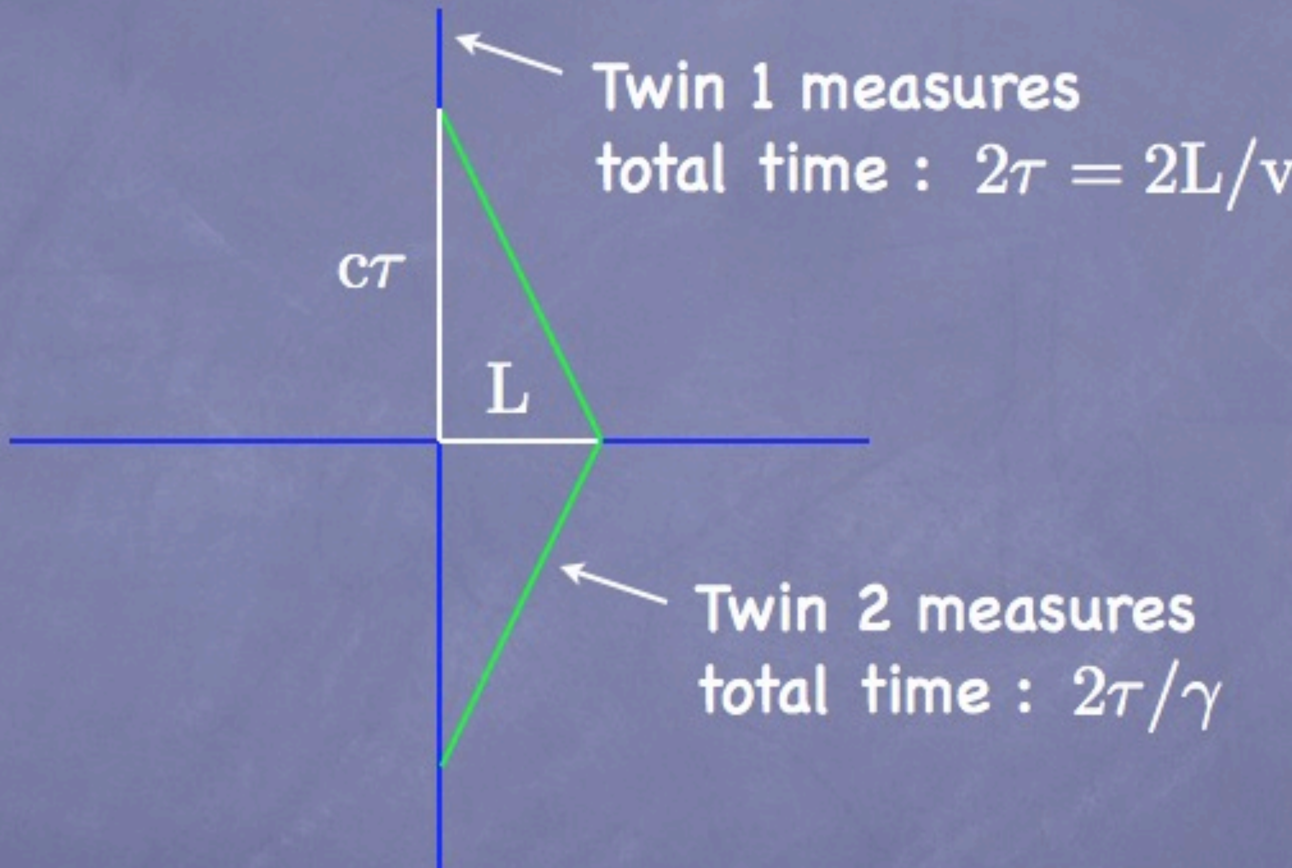
$$L' = v\tau$$

$$L = \gamma v\tau$$

$$L' = L\sqrt{1 - v^2/c^2} < L$$

The length of an object is longest for an inertial observer which is at rest with respect to the object. Proper length is longest.

Twin "Paradox"



There is no spacetime diagram like this for twin 2 at rest since he is not inertial. He does not follow a straight line in spacetime.

Lorentz Transformation

Given: $\beta = \mathrm{v}/\mathrm{c}$ $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

Lorentz : $(\mathrm{x}, \mathrm{ct}) \rightarrow (\mathrm{x}', \mathrm{ct}')$

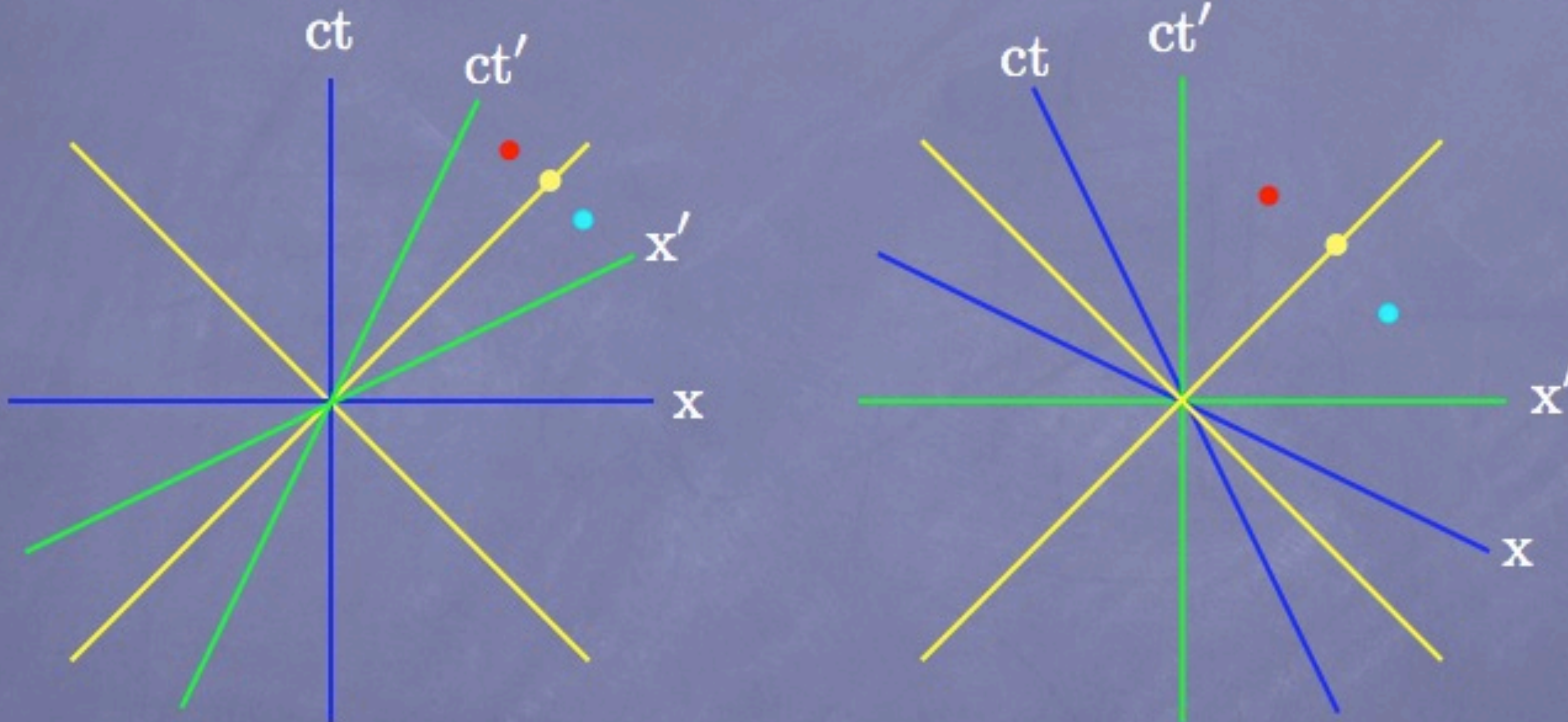
$$\mathrm{x}' = \gamma\,(\mathrm{x} - \beta \mathrm{ct}) \qquad \mathrm{ct}' = \gamma\,(\mathrm{ct} - \beta \mathrm{x})$$

Inverse LT : $(\mathrm{x}', \mathrm{ct}') \rightarrow (\mathrm{x}, \mathrm{ct})$

$$\mathrm{x} = \gamma\,(\mathrm{x}' + \beta \mathrm{ct}') \qquad \mathrm{ct} = \gamma\,(\mathrm{ct}' + \beta \mathrm{x}')$$

Lorentz Transformation preserves interval :

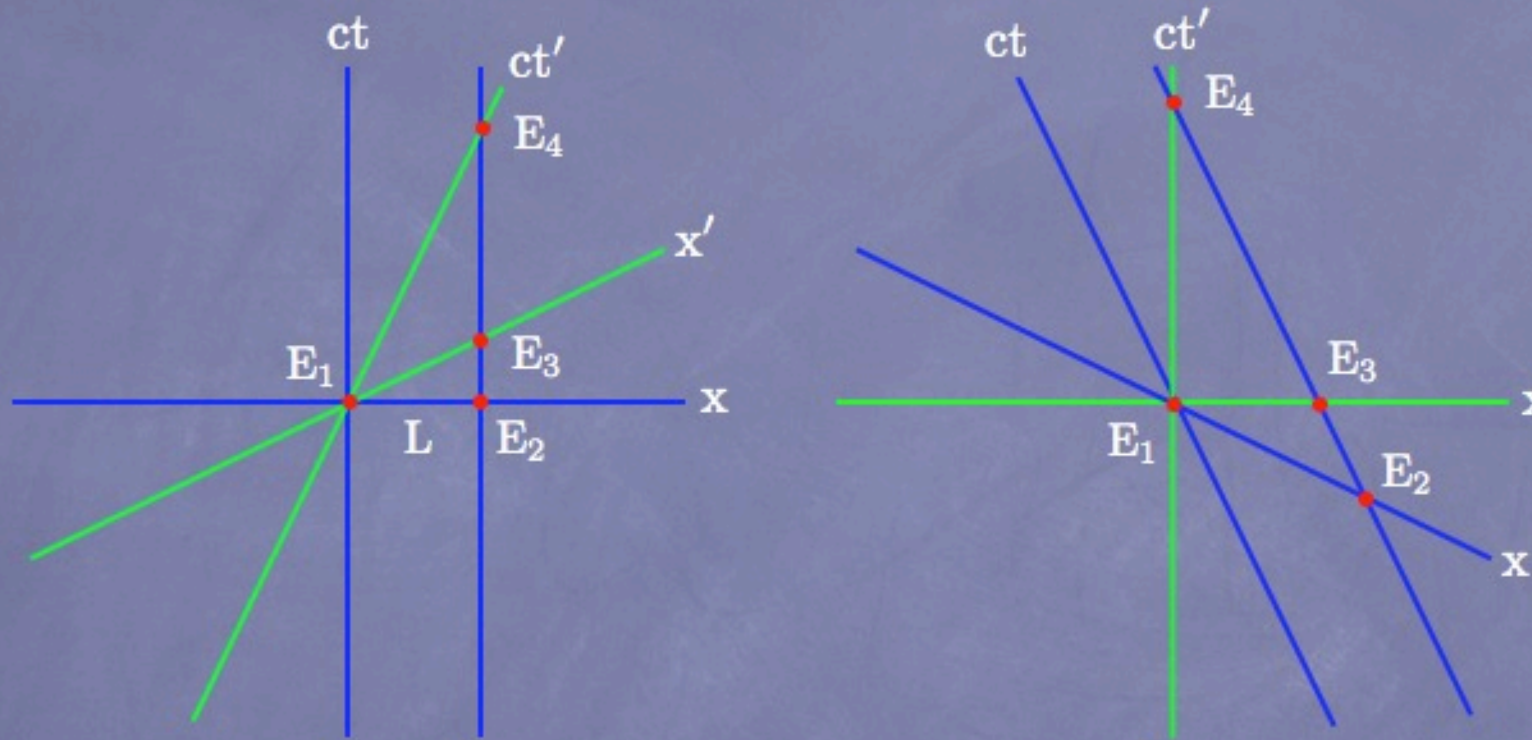
(from origin) $$I = -c^2t^2 + x^2 = c^2t'^2 - x'^2$$



- Timelike Interval : $I < 0$
- Lightlike Interval : $I = 0$
- Spacelike Interval : $I > 0$

$$I = -c^2t^2 + x^2 = c^2t'^2 - x'^2$$

$$x' = \gamma\,(x - \beta ct) \qquad ct' = \gamma\,(ct - \beta x)$$



E_1	E_2	E_3	E_4
$ct = 0 \quad x = 0$	$ct = 0 \quad x = L$	$x = L \quad ct' = 0$	$x = L \quad x' = 0$
$ct' = 0$	$ct' = -\gamma\beta L$	$ct = \beta L$	$ct = L/\beta$
$x' = 0$	$x' = \gamma L$	$x' = L/\gamma$	$ct' = L/\beta\gamma$
$I = 0$	$I = L^2$	$I = L^2/\gamma^2$	$I = -L^2/\beta^2\gamma^2$

Addition of Velocities

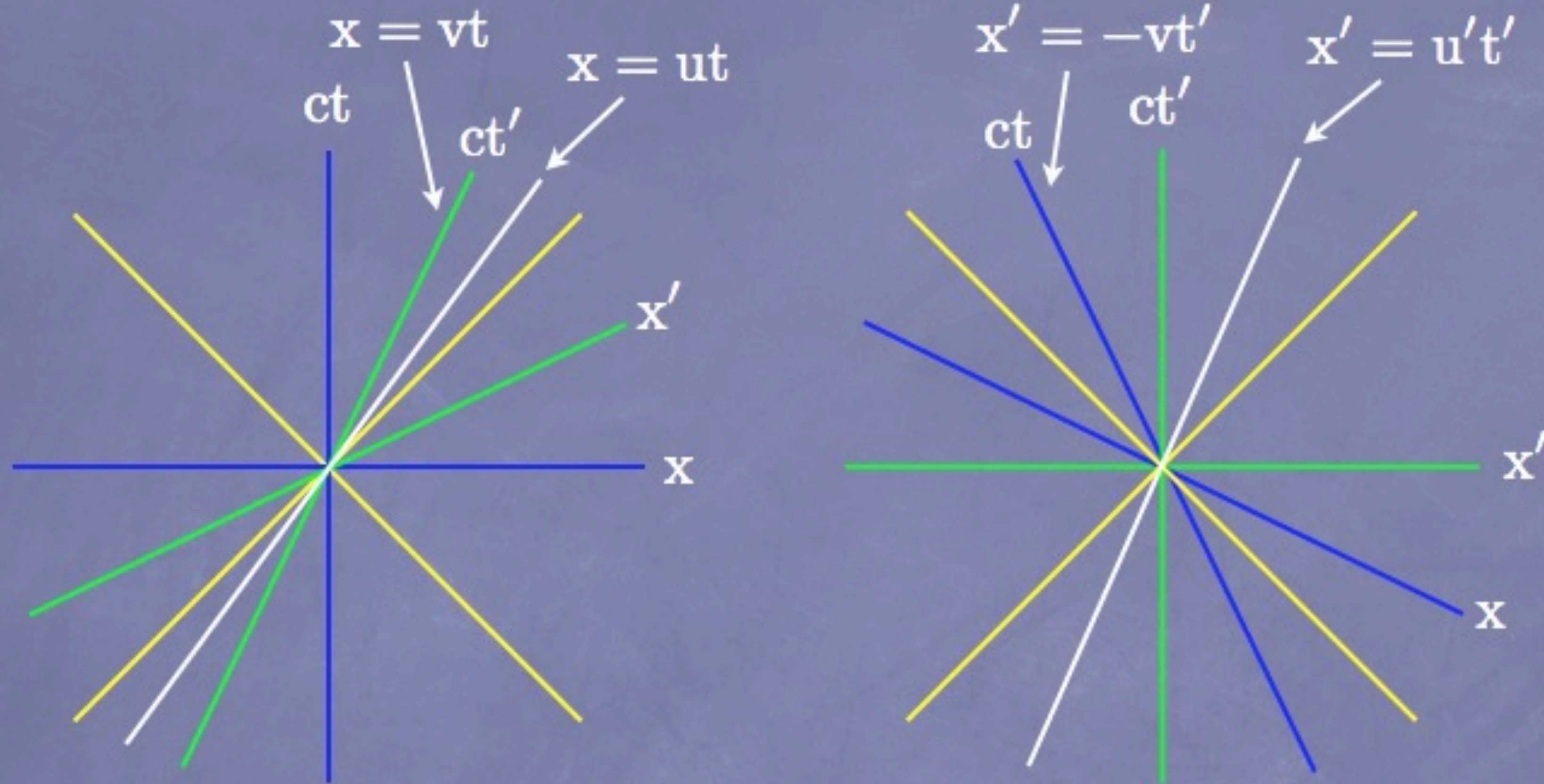
Given: $\beta = \mathrm{v/c}$ $\qquad \gamma = \dfrac{1}{\sqrt{1-\beta^2}}$

$$\mathrm{dx'} = \gamma\,(\mathrm{dx} - \beta \mathrm{cdt}) \qquad \mathrm{cdt'} = \gamma\,(\mathrm{cdt} - \beta \mathrm{dx})$$

Define: $\mathrm{u} = \dfrac{\mathrm{dx}}{\mathrm{dt}}$ $\qquad \mathrm{u'} = \dfrac{\mathrm{dx'}}{\mathrm{dt'}}$

Derive:

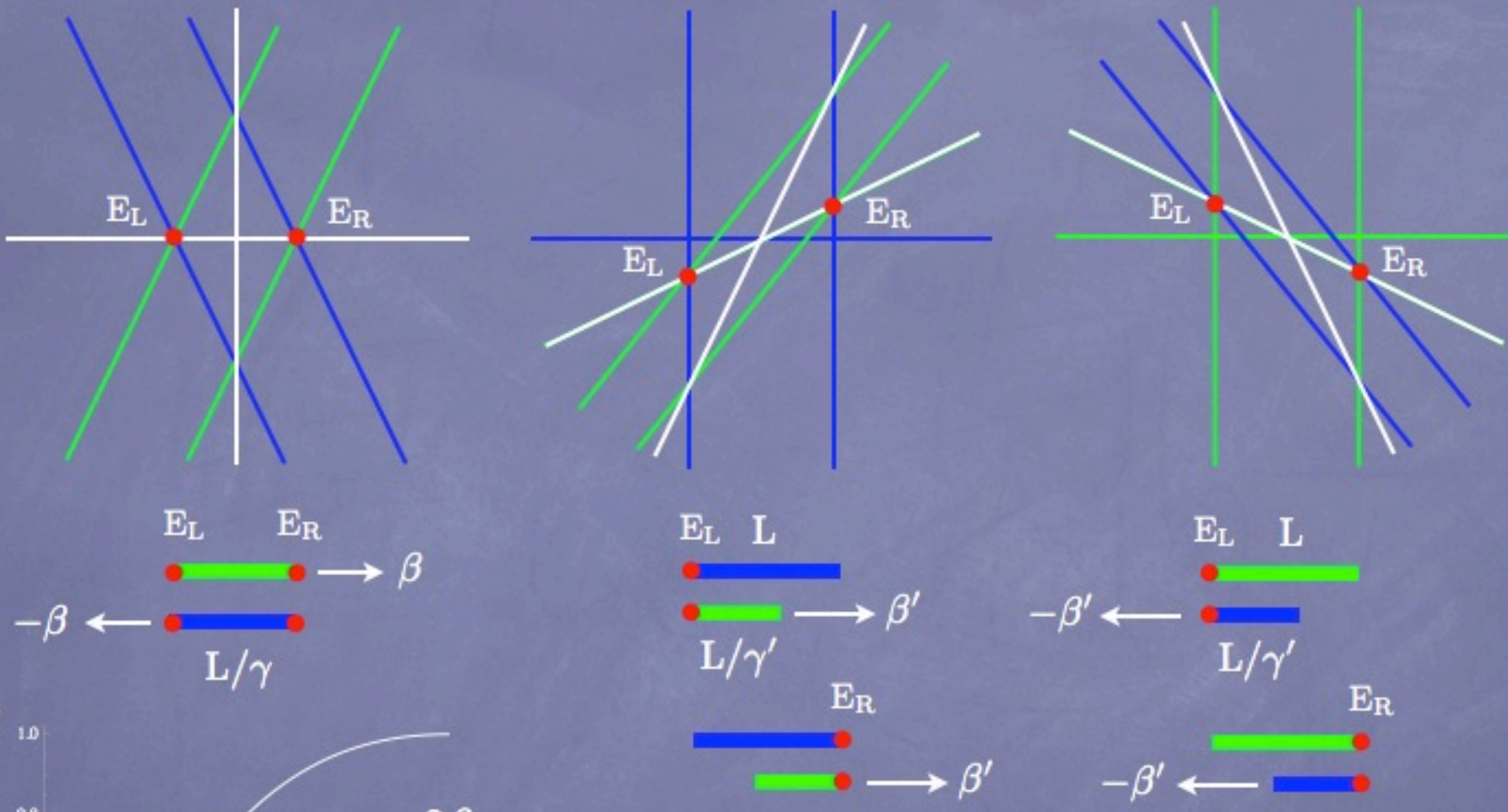
$$\mathrm{u'} = \frac{\mathrm{u - v}}{1 - \mathrm{uv/c^2}} \qquad \mathrm{u} = \frac{\mathrm{u' + v}}{1 + \mathrm{u'v/c^2}}$$

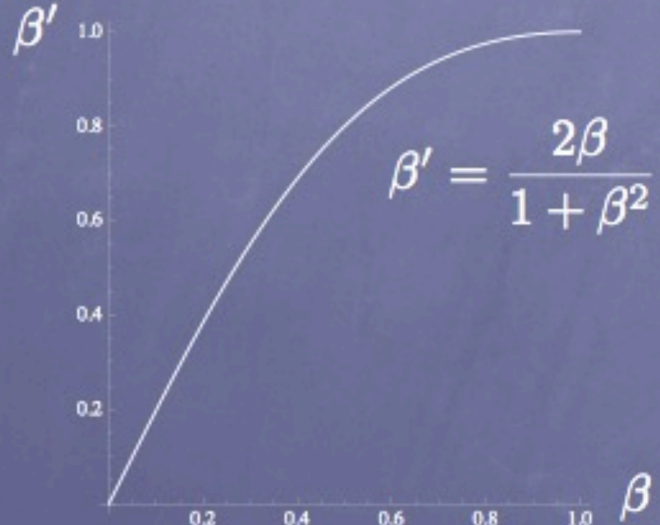


$$u' = \frac{u - v}{1 - uv/c^2} \qquad u = \frac{u' + v}{1 + u'v/c^2}$$

If $u \rightarrow c$ then $u' \rightarrow c$ for all v

If $v \rightarrow c$ then $u' \rightarrow c$ and $u \rightarrow c$





$$\beta' = \frac{2\beta}{1+\beta^2}$$

$$v = \beta' c$$

$$u = \beta c$$

$$u' = -\beta c$$

$$u' = \frac{u - v}{1 - uv/c^2}$$

Relativistic Momentum : $p = \gamma m v$

Relativistic Energy : $E = \gamma m c^2$

$$E^2 = p^2 c^2 + m^2 c^4$$

Velocity : $v = pc^2/E$ Rest Mass : m

Rest energy ($E = mc^2$) includes all binding energy (chemical, nuclear, etc.)

Photon ($m = 0$) has $v = c$ and $E = pc$

Relativistic Kinetic Energy : $\mathrm{K = E - mc^2}$

$$\mathrm{K = mc^2\,(\gamma - 1) \simeq \tfrac{1}{2}mv^2 + \tfrac{3}{8}mv^4/c^2 + \ldots}$$