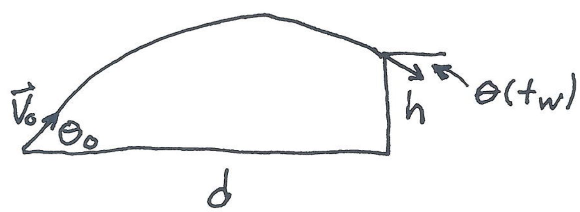
SMU Physics 1307 : Fall 2011

Exam 1

Problem 1: The figure below shows a ball hit from ground level with initial angle from the horizontal $\theta_0 = 50°$. The ball then flies a horizontal distance $d = 60\,\text{m}$ and passes through a window at a vertical height $h = 20\,\text{m}$. Find the magnitude $v_0 = |\vec{v}_0|$ of the initial velocity. Find the time t_w that it takes to reach the window. Also find the magnitude $|\vec{v}(t_w)|$ of the velocity and the angle $\theta(t_w)$ that $\vec{v}(t_w)$ makes with the horizontal when the ball passes through the window.



$$y = x\tan\theta_0 - \tfrac{1}{2}g x^2/(V_0^2\cos^2\theta_0)$$

set $y=h,\ x=d$

$$V_0^2 = \frac{\tfrac{1}{2}g x^2}{\cos^2\theta_0(x\tan\theta_0 - y)} \qquad \underline{V_0 = 28.8\,\text{m/s}}$$

$$X = V_0\cos\theta_0\, t \qquad t_w = \frac{d}{V_0\cos\theta_0} = \underline{3.24\,\text{s}}$$

$V_x = V_0\cos\theta_0 = 18.5\,\text{m/s}$

$V_y = V_0\sin\theta_0 - gt$

$V_y(t_w) = -9.72\,\text{m/s}$

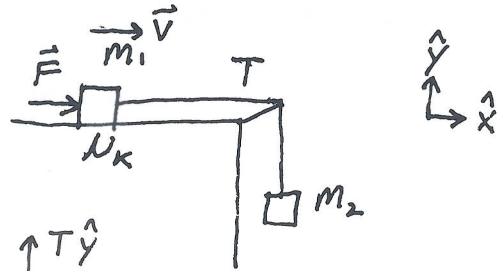
$$\tan\theta(t_w) = \frac{V_y(t_w)}{V_x}$$

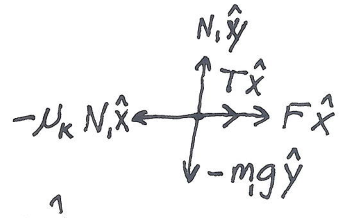
$\underline{\theta(t_w) = -27.7°}$

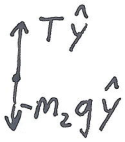
$(V_y^2(t_w) + V_x^2)^{1/2} = |\vec{V}(t_w)| = \underline{20.92\,\text{m/s}}$

Problem 2:

The figure below shows a block of mass $m_1 = 5\,\mathrm{kg}$ sliding to the right on a surface of coefficient of kinetic friction $\mu_k = 0.6$. This block is attached by a string to a block of mass $m_2 = 2\,\mathrm{kg}$ which hangs vertically. If a force $\vec{F} = F\hat{x}$ with $F = 20\,\mathrm{N}$ acts on the first block as shown, find the tension T in the string and the vectors $\vec{a}_1$ and $\vec{a}_2$ in the $\hat{x}$-$\hat{y}$ frame shown in the figure. What value of F would make the string go slack? What value of F would make $\vec{a}_1 = \vec{a}_2 = 0$?







$\vec{a}_1 = a\hat{x}$ $\qquad$ $N_1 - m_1 g = 0$ $\qquad$ $\vec{a}_2 = a\hat{y}$

$$F + T - \mu_K m_1 g = m_1 a \qquad T - m_2 g = -m_2 a$$

$$F + m_2 g - \mu_K m_1 g = (m_1 + m_2) a$$

① $F = 20N$ $\qquad$ $\underline{a = 1.46\,m/s^2}$

$$T = m_2(g - a) = \underline{16.7N}$$

② $T = 0 \quad a = g$

$$F = m_1 g(1 + \mu_K) = \underline{78.4N}$$

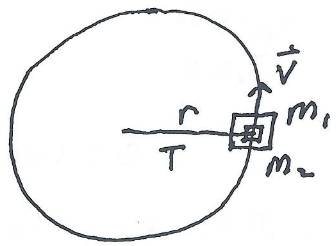
③ $T = m_2 g$

$$F = \mu_K m_1 g - m_2 g = \underline{9.8N}$$

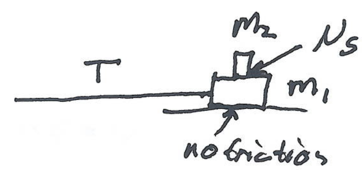
Problem 3:

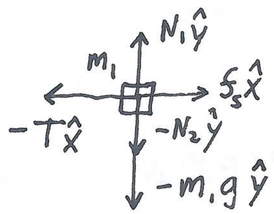
The figure below shows two blocks traveling in a horizontal circle of radius $r = 1\,\text{m}$. The bottom block of mass $m_1 = 3\,\text{kg}$ is attached to the center of rotation by a string, and experiences zero friction with the horizontal surface on which it rides. The top block of mass $m_2 = 2\,\text{kg}$ has coefficient of static friction $\mu_s = 0.7$ with the bottom block. Suppose the string is capable of sustaining a maximum tension $T_{\text{max}} = 40\,\text{N}$. If the velocity of the circular motion is steadily increased from zero, does the string break before the top block slides on the bottom block? Whichever happens first, find the velocity v of circular motion, the magnitude of the static frictional force $|\vec{f_s}|$ between the blocks, and the tension T in the string at that moment.

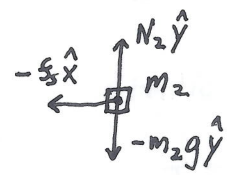
top view



side view







$$N_1 - N_2 - m_1 g = 0$$

$$-T + f_s = -m_1 v^2/r$$

$$T = (m_1 + m_2)\,v^2/r$$

$$v^2 = \frac{Tr}{(m_1 + m_2)}$$

string breaks at: $\underline{v = 2.83\ \text{m/s}}$

$$N_2 - m_2 g = 0$$

$$-f_s = -m_2 v^2/r$$

$$f_s \le \mu_s N_2$$

$$m_2 v^2/r \le \mu_s m_2 g$$

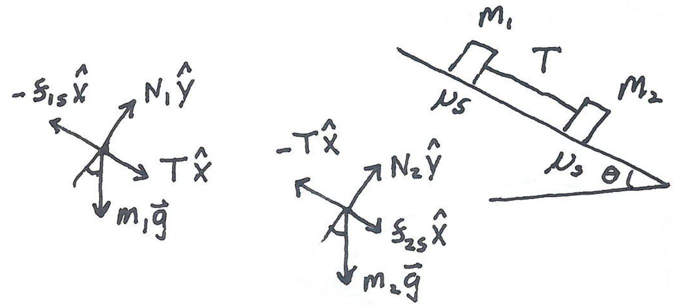
$$v^2 \le \mu_s g r$$

block slides at: $\underline{v = 2.62\ \text{m/s}}$

$\underline{T = 34.3\,\text{N}}$

$f_s = \mu_s m_2 g = \underline{13.7\,\text{N}}$

Problem 4: The figure below shows two people having a tug-of-war contest on an inclined plane of angle $\theta = 20°$. The person on the upper part of the slope has a mass $m_1 = 40\,\mathrm{kg}$ and the person on lower part of the slope has a mass $m_2 = 20\,\mathrm{kg}$. They both have identical coefficients of static friction $\mu_s = 0.75$. This value of μ_s is sufficient for both people to maintain their footing without slipping while the rope is slack. As the tension T is increased which of the people will slip first? What is the value of T when this happens? What value of θ would be necessary for an exact tie? What would be the value of T when they slip in the event of a tie?



$N_1 - m_1 g\cos\theta = 0$

$T - f_{1s} + m_1 g\sin\theta = \cancel{m_1 a_1} = 0$

static when:

$f_{1s} \le \mu_s N_1$

$T + m_1 g\sin\theta \le \mu_s m_1 g\cos\theta$

$T \le m_1 g(\mu_s\cos\theta - \sin\theta)$

slips when: $T = 142.2\,N$

$N_2 - m_2 g\cos\theta = 0$

$-T + f_{2s} + m_2 g\sin\theta = \cancel{m_2 a_2} = 0$

static when: $f_{2s} \le \mu_s N_2$

$T - m_2 g\sin\theta \le \mu_s m_2 g\cos\theta$

$T \le m_2 g(\mu_s\cos\theta + \sin\theta)$

slips when: $T = 205.2\,N$

m_2 wins

exact tie:

$$m_1 g(\mu_s\cos\theta - \sin\theta) = m_2 g(\mu_s\cos\theta + \sin\theta)$$

$$\frac{m_1}{m_2} = \frac{\mu_s + \tan\theta}{\mu_s - \tan\theta}$$

$$\tan\theta = \mu_s\frac{(m_1 - m_2)}{(m_1 + m_2)}$$

$\theta = 14.0°$ $\quad T = 190.1\,N$