SMU Physics 1307 : Fall 2008

Exam 2

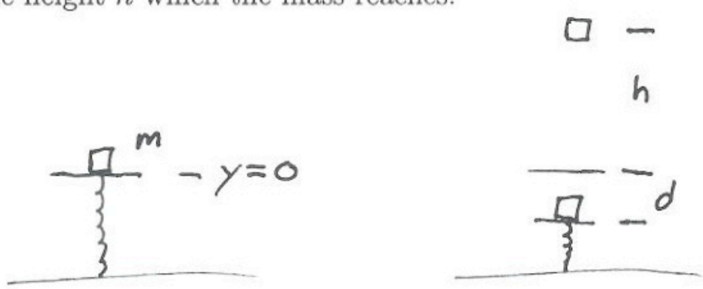
Problem 1 : The figure at left below shows a spring with $k = 20\,\mathrm{N/m}$ held in its uncompressed position with a mass $m = 0.1\,\mathrm{kg}$ resting on top of it. There is a constant gravitational force with $g = 9.8\,\mathrm{m/s^2}$ which points downward in the figure. As shown at right below, the spring and mass are then pulled down by an external force and held at rest at a position of $d = 0.2\,\mathrm{m}$ below the uncompressed position of the spring. The external force is then removed and the mass moves upward until it reaches the uncompressed position of the spring, at which time it comes off the spring and flies upward to a height h.

(a) In compressing the spring, what is the total work done? What is the respective work done by the spring, gravity, and the external force during this motion?

(b) As the mass moves upward, the force of the spring changes. Find the position and velocity of the mass when the force of the spring is equal in magnitude and oppositely directed to that of gravity.

(c) Find the velocity of the mass when it reaches the uncompressed position of the spring.

(d) Find the height h which the mass reaches.



a) $W_{net} = \Delta K = 0$ $\qquad W_{net} = W_g + W_s + W_{ext} = 0$

$W_g = -\Delta U_g = -mg(-d) = mgd = \underline{0.196\,J}$

$W_s = -\Delta U_s = -\frac{1}{2}Kd^2 = \underline{-0.40\,J}$ $\qquad W_{ext} = -W_g - W_s = \underline{0.204\,J}$

b) for $y<0$ $\quad F_{net} = -mg - Ky$ $\qquad y>0$ $\quad E = \frac{1}{2}mv^2 + mgy$

when $y = y_c$ $\quad F_{net} = -mg - Ky_c = 0$ $\qquad y<0$ $\quad E = \frac{1}{2}mv^2 + mgy + \frac{1}{2}Ky^2$

$y_c = -mg/K = \underline{-0.049\,m}$

when $y_1 = -d$ $\quad V_1 = 0$

$E_1 = -mgd + \frac{1}{2}Kd^2$

$E_1 = 0.204\,J$

$= W_{ext}$

when $y = y_c$

$E_c = \frac{1}{2}mV_c^2 + mgy_c + \frac{1}{2}Ky_c^2$

$= \frac{1}{2}mV_c^2 - \frac{1}{2}(mg)^2/K$

$= \frac{1}{2}mV_c^2 - 0.024\,J$

$\Delta E = W_n = 0$

$E_1 = E_c \qquad 0.204\,J = \frac{1}{2} m V_c^2 - 0.024\,J$

$\frac{1}{2} m V_c^2 = 0.228\,J$

$V_c = 2.14\ m/s$

c) $y_2 = 0 \qquad E_2 = \frac{1}{2} m V_2^2 = E_1 = 0.204\,J$

$V_2 = 2.02\ m/s$

d) $y_3 = h \qquad E_3 = mgh = E_1 = 0.204\,J$

$V_3 = 0$

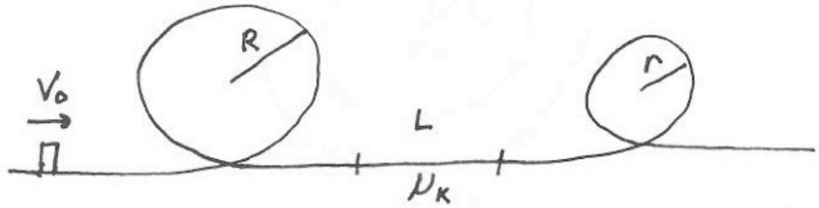
$h = 0.208\,m$

Problem 2 : The figure below shows a pair of frictionless loops around which a mass m will slide. The first loop at left has radius $R = 2\,\mathrm{m}$. The mass is sent in from the left with a velocity v_0, which is chosen so that the mass feels no normal force when it is at the top of the first loop; only gravity provides the required centripetal acceleration at that point. The track is entirely frictionless except for a short section of length $L = 1.5\,\mathrm{m}$ and coefficient of kinetic friction $\mu_k = 0.4$ which lies between the loops. The second loop has a radius r which is chosen so that the mass again feels no normal force at the top of this loop.

(a) Find the velocity v_0.

(b) Find the radius r.

(c) Find the velocity of the mass after it leaves the second loop.



a) top of loop (first): $y = 2R \qquad v^2/R = g$

$$E_0 = \tfrac{1}{2} m V_0^2 = mg\,2R + \tfrac{1}{2} m v^2 = \tfrac{5}{2} mgR$$

$$V_0 = (5gR)^{1/2} = \underline{9.90\ \mathrm{m/s}}$$

b) after μ_K section: $E_1 = E_0 - \mu_K mgL = \tfrac{5}{2} mgR - \mu_K mgL$

top of 2nd loop: $y_2 = 2r \qquad V_2^2/r = g$

$$E_2 = mg\,2r + \tfrac{1}{2} m V_2^2 = \tfrac{5}{2} mgr = E_1$$

$$\tfrac{5}{2} mgr = \tfrac{5}{2} mgR - \mu_K mgL$$

$$\tfrac{5}{2} r = \tfrac{5}{2} R - \mu_K L \qquad r = R - \tfrac{2}{5}\mu_K L = \underline{1.76\ \mathrm{m}}$$

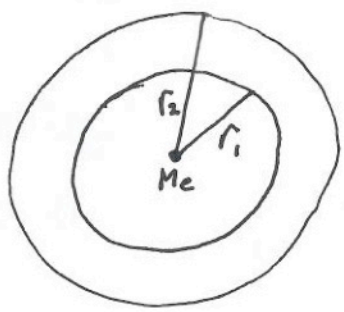
c) after 2nd loop: $y_f = 0 \qquad \tfrac{1}{2} m V_f^2 = E_1 = \tfrac{5}{2} mgr$

$$V_f = (5gr)^{1/2} = \underline{9.29\ \mathrm{m/s}}$$

Problem 3 : The figure below shows two circular orbits of a satellite, one of radius $r_1 = 8 \times 10^6\,\mathrm{m}$ and another of radius $r_2 = 12 \times 10^6\,\mathrm{m}$. The satellite has mass $m_s = 100\,\mathrm{kg}$, the earth has mass $M_e = 6 \times 10^{24}\,\mathrm{kg}$, and Newton's gravitational constant is $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$.

(a) Find the kinetic, potential, and total energy of both orbits.

(b) If an external (non-conservative) force moves the satellite from the smaller to the larger orbit, what is the total work done? What is the work done by gravity? What is the work done by the external force?



a) In general : $U = -\dfrac{GMm}{r}$ $\quad K = \tfrac{1}{2}mv^2$ / $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$

$E = U + K = -\dfrac{GMm}{2r}$ $\quad K = \dfrac{GMm}{2r}$

$U_1 = -\dfrac{GMm}{r_1} = \underline{-5 \times 10^9\,\mathrm{J}}$ $\quad K_1 = \dfrac{GMm}{r_1} = \underline{2.5 \times 10^9\,\mathrm{J}}$ $\quad E_1 = K_1 + U_1 = \underline{-2.5 \times 10^9\,\mathrm{J}}$

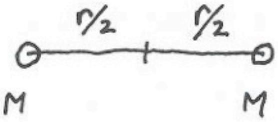
$U_2 = -\dfrac{GMm}{r_2} = \underline{-3.34 \times 10^9\,\mathrm{J}}$ $\quad K_2 = \dfrac{GMm}{r_2} = \underline{1.67 \times 10^9\,\mathrm{J}}$ $\quad E_2 = K_2 + U_2 = \underline{-1.67 \times 10^9}$.

b) $W_{net} = \Delta K = K_2 - K_1 = \underline{-8.34 \times 10^8\,\mathrm{J}}$

$W_g = -\Delta U = -(U_2 - U_1) = \underline{-1.67 \times 10^9\,\mathrm{J}}$

$W_{nc} = \Delta E = E_2 - E_1 = \underline{8.34 \times 10^8\,\mathrm{J}} = \Delta K + \Delta U$

Problem 4 : Two stars of equal mass M orbit around each other at a distance $r = 2.5\times10^{10}$ m , with each executing a circular orbit around a point equidistant $(r/2)$ between the stars. The period of the orbit is $\tau = 150$ days. Find the mass M. Now suppose that there are three stars of mass M arranged in an equilateral triangle which orbit around their common center, which is again at a distance $r/2$ from each star. Find the the period τ' of the orbit of this three star system.

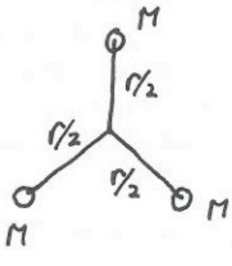


$$V = \frac{2\pi(r/2)}{\tau}$$

$$\frac{GM^2}{r^2} = \frac{MV^2}{(r/2)} = \left(\frac{2\pi}{\tau}\right)^2 r/2\, M$$

$$\left(\frac{\tau}{2\pi}\right)^2 = \frac{r^3}{2GM}$$

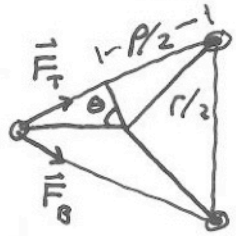
$$M = \left(\frac{2\pi}{\tau}\right)^2 \frac{r^3}{2G} = 2.75\times10^{28}\ \text{Kg}$$



force on M:

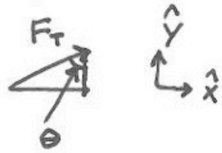
$$V' = \frac{2\pi\, r/2}{\tau'}$$

$$|\vec{F}| = \frac{MV^2}{(r/2)} = M\left(\frac{2\pi}{\tau'}\right)^2 r/2$$



$\theta = 60°$

$\rho/2 = r/2 \sin\theta$



$$\vec{F}_T = \frac{GM^2}{\rho^2}(\sin\theta\,\hat{x} + \cos\theta\,\hat{y})$$

$$\vec{F}_B = \frac{GM^2}{\rho^2}(\sin\theta\,\hat{x} - \cos\theta\,\hat{y})$$

$$\vec{F} = \vec{F}_T + \vec{F}_B = \frac{2GM^2}{\rho^2}\sin\theta\,\hat{x}$$

$$\sin 60° = \sqrt{3}/2$$

$$|\vec{F}| = \frac{2GM^2}{r^2\sin\theta} = M\left(\frac{2\pi}{\tau'}\right)^2 r/2$$

$$\left(\frac{\tau'}{2\pi}\right)^2 = \frac{r^3}{2GM}\left(\frac{\sin\theta}{2}\right)$$

$$= \left(\frac{\tau}{2\pi}\right)^2 \sqrt{3}/4$$

$$\tau' = \tau\,(3)^{1/4}/2$$

$\tau' = 99$ days