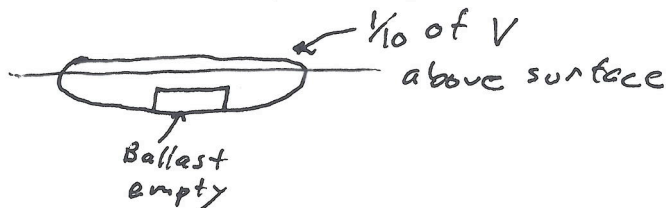


SMU Physics 1307 : Spring 2009

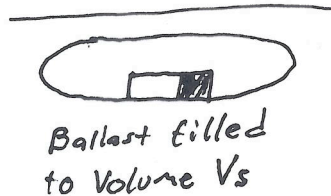
Final

Problem 1 : If a particular submarine has all of its ballast tanks empty of water it has $1/10$ of its total volume (V) above the surface. What proportion, V_S/V , of the total volume of the submarine has to be filled with water in order for the submarine to remain submerged but neither descending nor surfacing. When it completely fills its ballast tanks (which have volume V_B) with water, the submarine descends while submerged with an acceleration $a = -(1/20)g$. Neglecting the mass of the air in an unfilled ballast tank, find the proportion V_B/V of the total volume of the submarine that is taken up by the ballast tanks.

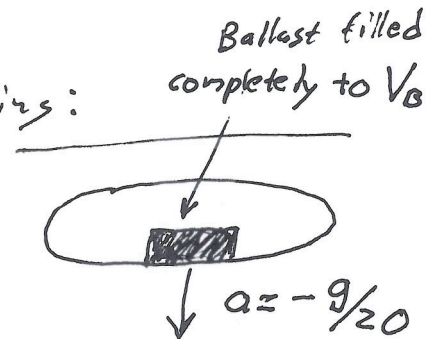
surfaced :
no water
in tanks



submerged but
stationary :



descending :



$$\beta = 9/10$$

$$\alpha = 1/20$$

ballast empty: $Mg = \beta \rho_w V g \Rightarrow M = \beta \rho_w V$

ballast to V_S : $Mg + \rho_w V_S g = \rho_w V g \Rightarrow \beta \rho_w V + \rho_w V_S = \rho_w V$

$$V_S/V = 1 - \beta = \underline{\underline{1/10}}$$

ballast to V_B : $(M + \rho_w V_B) a = -(M + \rho_w V_B) g + \rho_w V g$

$a = -\alpha g$

$$(\beta \rho_w V + \rho_w V_B)(g + a) = \rho_w V g$$

$$(\beta V + V_B)(1 - \alpha) = V$$

$$V_B/V = \frac{1}{1 - \alpha} - \beta = \frac{29}{190} \approx \underline{\underline{0.153}}$$

Problem 2 : The figure below shows a uniform rod ($I_{cm} = \frac{1}{12}ML^2$) of length $L = 2$ m and mass $M = 3$ kg, which rotates around one of its ends and thus forms a pendulum. Given $g = 9.8$ m/s², find the period T of the pendulum. Now a mass m is attached to the bottom of the pendulum. If the period now lengthens to $T' = T\sqrt{11/8}$, find the mass m .



period T

$$I = \frac{1}{12}ML^2 + M(L/2)^2$$

$$= \frac{1}{3}ML^2$$

$$I \frac{d^2\theta}{dt^2} = -MgL/2 \sin\theta$$

so

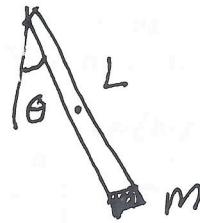
$$\frac{d^2\theta}{dt^2} = -\frac{3g}{2L} \sin\theta$$

$$\approx -\frac{3g}{2L} \theta$$

$$\omega^2 = \frac{3g}{2L}$$

$$T = 2\pi \sqrt{\frac{2L}{3g}}$$

$$= \underline{\underline{2.325}}$$



period T'

$$(I + mL^2) \frac{d^2\theta}{dt^2} = -(MgL/2 + mgL) \sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L} \left(\frac{M/2 + m}{M/3 + m} \right) \sin\theta$$

$$\omega'^2 = \frac{3g}{2L} \left(\frac{M + 2m}{M + 3m} \right)$$

$$= \omega^2 \left(\frac{M + 2m}{M + 3m} \right)$$

$$(T')^2 = \frac{11}{8} T^2 \Rightarrow \omega'^2 = \underline{\underline{\frac{8}{11} \omega^2}}$$

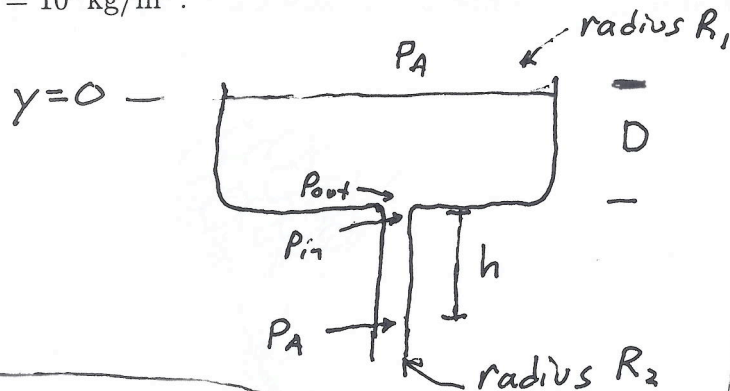
$$\frac{8}{11} = \frac{M + 2m}{M + 3m}$$

$$8M + 24m = 11M + 22m$$

$$\underline{\underline{2m = 3M}}$$

$$2 \quad m = \frac{3}{2}M = \underline{\underline{4.5 \text{ Kg}}}$$

Problem 3 : The figure below shows a circular reservoir of radius $R_1 = 40$ m and depth $D = 20$ m. It is being drained through a vertical pipe of radius $R_2 = 0.85$ m. If the downward velocity of the water in the reservoir is $v_1 = 0.01$ m/s, find the velocity v_2 of the water in the pipe. If the pressure at the surface of the reservoir is that of the atmosphere $p_A = 1 \times 10^5$ N/m², find the pressure p_{out} just outside the pipe. Also find the pressure p_{in} just inside the pipe; note that both of these pressures are to be computed at the depth D . The above values have been chosen so that $p_{in} < p_A$. Find the depth d below the entrance to the pipe at which the pressure returns to p_A . You will need $g = 9.8$ m/s², and the density of water $\rho_w = 10^3$ kg/m³.



$$V_1 A_1 = V_2 A_2$$

$$V_1 R_1^2 = V_2 R_2^2$$

$$V_2 = \frac{R_1^2}{R_2^2} V_1 = \underline{\underline{22.1 \text{ m/s}}}$$

$$p_A + \frac{1}{2} \rho_w V_1^2 = p_{out} + \frac{1}{2} \rho_w V_2^2 - \rho_w g D$$

$$p_{out} = p_A + \rho_w g D$$

$$p_{out} = 2.96 \times 10^5 \text{ N/m}^2$$

$$p_{in} + \frac{1}{2} \rho_w V_2^2 - \rho_w g D = p_{out} + \frac{1}{2} \rho_w V_1^2 - \rho_w g D$$

$$p_{in} = p_{out} - \frac{1}{2} \rho_w (V_2^2 - V_1^2)$$

$$p_{in} = 5.08 \times 10^4 \text{ N/m}^2$$

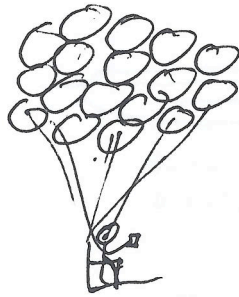
$$p_{in} + \frac{1}{2} \rho_w V_2^2 - \rho_w g D = p_A + \frac{1}{2} \rho_w V_2^2 - \rho_w g (D+h)$$

$$\rho_w g h = p_A - p_{in}$$

$$h = \frac{p_A - p_{in}}{\rho_w g} = \underline{\underline{5.02 \text{ m}}}$$

Problem 4 : In 1982 Los Angeles resident Larry Walters purchased 45 small weather balloons from an Army surplus store, tied them to his favorite lawn chair, inflated them with helium, and then soared 16000 feet into the sky in the direction of Long Beach California. Assume that Larry, his lawn chair, the balloons before being filled, the pellet gun he used for altitude control, and the beer and sandwiches he carried along for the ride had a combined mass of $M = 150 \text{ kg}$. If helium has a mass density which is $\rho_H = (2/7)\rho_A$ that of air, where you may take $\rho_A = 1.2 \text{ kg/m}^3$, find the minimum volume per balloon V_m which would have allowed Larry to leave the ground. In the actual circumstance, Larry filled the balloons to $V = 8 \text{ m}^3$ each. Given $g = 9.8 \text{ m/s}^2$, what was his acceleration a_{Larry} as he left the ground?

$$N = \# \text{ of balloons} = 45$$



for just enough to lift off:

$$(M + \rho_H N V_m) g = \rho_A N V_m g$$

$$M = (\rho_A - \rho_H) N V_m = \frac{5}{7} \rho_A N V_m$$

$$V_m = \frac{7}{5} M / (\rho_A N) = \underline{3.89 \text{ m}^3}$$

for $V = 8 \text{ m}^3$

$$(M + \rho_H N V) a = -(M + \rho_H N V) g + \rho_A N V g$$

$$a = -g + \frac{\rho_A N V g}{(M + \rho_H N V)} = \underline{5.68 \text{ m/s}^2}$$