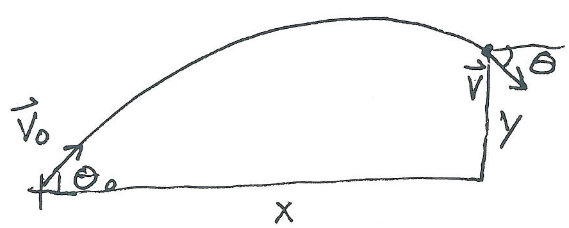
SMU Physics 1307 : Spring 2011

Exam 1

Problem 1 : The figure below shows a ball that has been hit from ground level at angle $\theta_0 = 40°$ and observed a time $t = 3.6\,\mathrm{s}$ later to have a velocity vector which makes an angle $\theta = -10°$ with the horizontal. Express the angle θ in terms of the velocity vector components v_x and v_y at time t. Then express these components in terms of θ_0 and the unknown magnitude $|\vec{v}_0|$ of the initial velocity vector. Thus compute $|\vec{v}_0|$, and use it to find x,y, and v_y at time t.



$V_x = V_{0x} = V_0 \cos\theta_0$

$V_y = V_{0y} - gt$

$= V_0 \sin\theta_0 - gt$

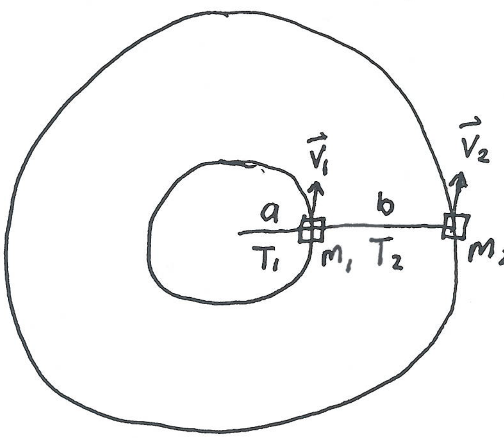
$\tan\theta = \frac{V_y}{V_x} = \frac{V_0 \sin\theta_0 - gt}{V_0 \cos\theta_0}$

solve for V_0. Then: $V_y = V_0 \sin\theta_0 - gt$

$X = V_0 \cos\theta_0 t$

$y = V_0 \sin\theta_0 - \frac{1}{2} g t^2$

Problem 2 : The figure below shows a view from above of two masses which move in concentric circles in a horizontal plane. As shown, the mass $m_1 = 2\,\mathrm{kg}$ is attached by a string of length $a = 1\,\mathrm{m}$ to the origin, and by another string of length $b = 2\,\mathrm{m}$ to a mass $m_2 = 3\,\mathrm{kg}$. If the masses each take a time $t = 0.1\,\mathrm{s}$ for each full revolution, find the velocities v_1 and v_2 of the masses, and find the tensions T_1 and T_2 of the respective strings.



$r_1 = a \qquad r_2 = a + b$

$$t = \frac{2\pi r_1}{V_1} = \frac{2\pi r_2}{V_2}$$

$$V_1 = \frac{2\pi r_1}{t} \qquad V_2 = \frac{2\pi r_2}{t}$$

$$T_2 - T_1 = -m_1 V_1^2 / r_1 = -m_1 \left(\frac{2\pi}{t}\right)^2 r_1$$

$$-T_2 = -m_2 V_2^2 / r_2 = -m_2 \left(\frac{2\pi}{t}\right)^2 r_2$$

$$T_1 = \left(\frac{2\pi}{t}\right)^2 (m_1 r_1 + m_2 r_2)$$

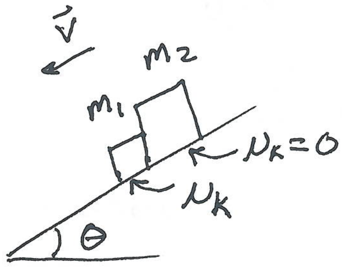
$$T_2 = \left(\frac{2\pi}{t}\right)^2 m_2 r_2$$

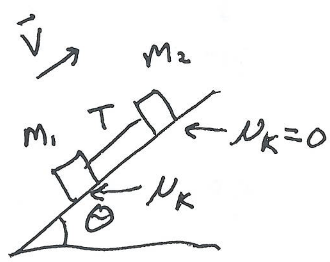
Problem 3 : The figure at left below shows two masses $m_1 = 2\,\mathrm{kg}$ and $m_2 = 5\,\mathrm{kg}$ which are directly touching and sliding together down an inclined plane of angle $\theta = 30°$. If m_1 has a coefficient of kinetic friction $\mu_k = 0.6$ and m_2 experiences no friction, find the acceleration a of the masses, with a positive when the acceleration is down the ramp. Also find the normal force component F which m_2 exerts on m_1.

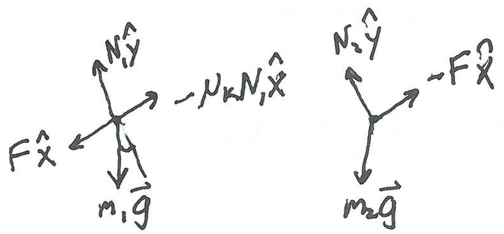
The figure at right below shows two masses $m_1 = 2\,\mathrm{kg}$ and $m_2 = 5\,\mathrm{kg}$ attached by a string and sliding together up an inclined plane of angle $\theta = 30°$. If, as at left, m_1 has a coefficient of kinetic friction $\mu_k = 0.6$ and m_2 experiences no friction, find the acceleration a of the masses, with a positive when the acceleration is down the ramp. Also find the tension T in the string.

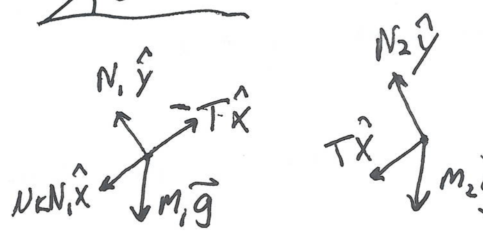
$\hat{x}$

$\vec{a} = a\hat{x}$









$$N_1 - m_1 g \cos\theta = 0$$

$$F - \mu_k N_1 + m_1 g \sin\theta = m_1 a$$

$$N_2 - m_2 g \cos\theta = 0$$

$$-F + m_2 g \sin\theta = m_2 a$$

$$(m_1 + m_2) g \sin\theta - \mu_k m_1 g \cos\theta = (m_1 + m_2) a$$

$$a = g \sin\theta - \frac{\mu_k m_1 g}{(m_1 + m_2)} \cos\theta$$

$$F = m_2 g \sin\theta - m_2 a$$

$$= \frac{\mu_k m_1 m_2 g}{(m_1 + m_2)} \cos\theta$$

$$N_1 - m_1 g \cos\theta = 0$$

$$-T + \mu_k N_1 + m_1 g \sin\theta = m_1 a$$

$$N_2 - m_2 g \cos\theta = 0$$

$$T + m_2 g \sin\theta = m_2 a$$

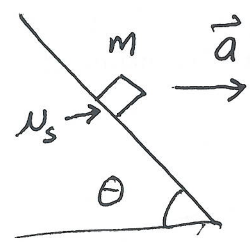
$$(m_1 + m_2) g \sin\theta + \mu_k m_1 g \cos\theta = (m_1 + m_2) g$$

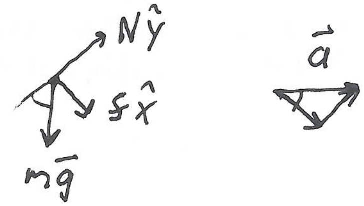
$$a = g \sin\theta + \frac{\mu_k m_1 g}{m_1 + m_2} \cos\theta$$

$$T = m_2 a - m_2 g \sin\theta$$

$$= \frac{\mu_k m_1 m_2 g}{(m_1 + m_2)} \cos\theta$$

Problem 4 : The figure below shows an inclined plane of angle $\theta = 30°$ which is moving to the right with acceleration $\vec{a}$. A block of mass $m = 1\,\mathrm{kg}$ has coefficient of static friction $\mu_s = 0.7$, and is assumed to have the same $\vec{a}$ while it remains static with respect to the surface of the plane. Find the minimum magnitude $|\vec{a}|$ such that the block begins to slide up the plane.





slips when:

$$f = \mu_s N$$

$$N - mg\cos\theta = ma\sin\theta$$

$$f + mg\sin\theta = ma\cos\theta$$

$$ma\cos\theta - mg\sin\theta = \mu_s(mg\cos\theta + ma\sin\theta)$$

$$a(\cos\theta - \mu_s\sin\theta) = g(\sin\theta + \mu_s\cos\theta)$$

$$a = g\,\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$$