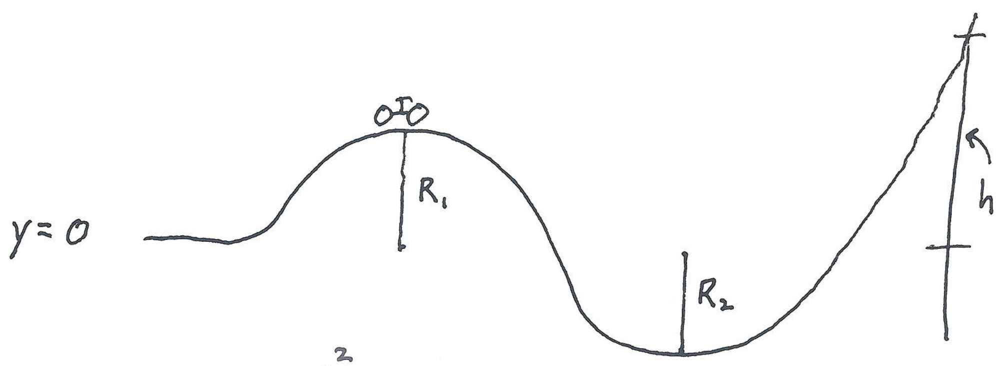
SMU Physics 1307 : Spring 2011

Exam 2

Problem 1 : The figure below shows a bicycle of mass $m = 100\,\text{kg}$ cresting a hill of circular profile with radius $r_1 = 10\,\text{m}$ above ground level. This hill is followed by a trough of circular profile with radius $r_2 = 6\,\text{m}$ below ground level, followed by a long hill of constant slope. If the bicycle feels a normal force $N_1 = mg/3$ while cresting the first hill, find the velocity v_1 at this point. Assuming the rider of the bicycle does no work during the process, also find the normal force N_2 and velocity v_2 at the bottom of the trough. Finally, find the height h that the bicycle ascends the final hill before coming to a stop. You will need $g = 9.8\,\text{m/s}^2$.



$$N_1 - mg = -m\frac{V_1^2}{R_1}$$

$$N_1 = mg/3$$

$$\frac{mV_1^2}{R_1} = \frac{2}{3}mg$$

$$V_1^2 = \frac{2}{3} g R_1 \,]$$

$$E = mgR_1 + \frac{1}{2}mV_1^2$$

$$E = \frac{4}{3} mgR_1$$

$$E = -mgR_2 + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}mV_2^2 = mgR_2 + \frac{4}{3}mgR_1 \,]$$

$$N_2 - mg = mV_2^2/R_2$$

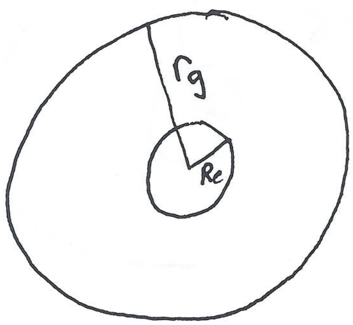
$$N_2 = mg + 2(mgR_2 + \frac{4}{3}mgR_1)/R_2$$

$$N_2 = 3mg + \frac{8}{3}mg\,R_1/R_2 \,]$$

$$E = mgh = \frac{4}{3}mgR_1$$

$$h = \frac{4}{3}R_1$$

Problem 2 : A geosynchronous orbit around the earth is one that has an orbital period which is equal to the rotational period $T = 24 \cdot 3600\,\mathrm{s}$ of the earth. Find the radius r_g of the geosynchronous orbit. Find the potential U, kinetic K, and total mechanical energy E of an object of mass $m = 1000\,\mathrm{kg}$ in such an orbit. Assuming the object takes off from the north pole, where it has no initial velocity, find the work W_{np} required to place it in a geosynchronous orbit. Now assume that the object takes off from the equator, where it has some intital velocity due to the rotation of the earth, and find the work W_{eq} required to place it in a geosynchronous orbit. You will need $G = 6.67 \times 10^{-11}\,\mathrm{Nm^2/kg^2}$ and the radius of the earth $R_e = 6.37 \times 10^6\,\mathrm{m}$.



$$\left(\frac{T}{2\pi}\right)^2 = \frac{r_g^3}{GM} \qquad \underline{r_g^3 = (T/2\pi)^2 GM}$$

$$U = -\frac{GM_e m}{r_g}$$

$$K = \frac{GM_e m}{2r_g}$$

$$E = -\frac{GM_e m}{2r_g}$$

① from ~~equator~~ north pole

$$E_0 = -\frac{GMm}{R_e}$$

$$\underline{W_{np}} = \frac{GMm}{R_e} - \frac{GMm}{r_g}$$

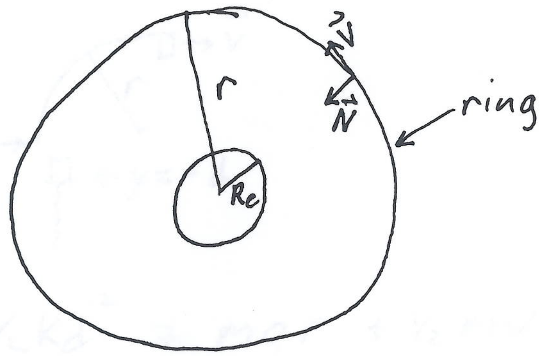
② from equator $\qquad v = 2\pi R_e/T$

$$E_0 = -\frac{GMm}{R_e} + \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 R_e^2$$

$$\underline{W_{eq}} = \frac{GMm}{R_e} - \frac{GMm}{r_g} - \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 R_e^2$$

$$W_{np} - W_{eq} = \frac{1}{2} m \left(\frac{2\pi}{T}\right)^2 R_e^2 \;\; 2 = \frac{GMm}{2} \frac{R_e^2}{r_g^3}$$

Problem 3 : Imagine a future civilization building a spinning ring of radius $r = 9 \times 10^6\,\mathrm{m}$ which encircles the earth and serves as a habitat for humans. As depicted below, the people will live on the inner surface of this ring, which spins at a rate such that the normal force felt by a person of mass $m = 120\,\mathrm{kg}$ is given by mg, exactly the same as that felt on earth. Find the velocity v of objects on the ring, and find the mechanical energy E of the person. Compare this energy to the energy E_{orb} of the person in an orbit of the same radius.



$$-N - \frac{GmM}{r^2} = -\frac{mv^2}{r} \qquad \underline{N = \frac{GmM}{R^2}}$$

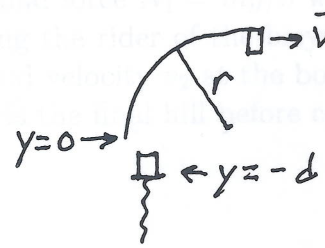
$$v^2/r = \frac{GM}{R^2} + \frac{GM}{r^2}$$

$$v^2 = GM\left(1/r + r/R^2\right)$$

$$E = -\frac{GmM}{r} + \tfrac{1}{2}mv^2 = -\frac{GmM}{2r} + \frac{GMm}{2}\,r/R^2$$

$$E_{orb} = -\frac{GmM}{2r} \qquad \underline{E > E_{orb}}$$

Problem 4 : The figure below shows a vertically mounted spring with $k = 10^4$ N/m which is intended to shoot an object of mass $m = 10$ kg around a semi-circular loop of radius $r = 1$ m. If, when the spring is pulled down to $y = -d$, the object leaves the semi-circle with velocity $v = 20$ m/s, find the distance d. Also find the normal force N just before the object leaves the semi-circle.



$$-mgd + \tfrac{1}{2}kd^2 = mgr + \tfrac{1}{2}mv^2$$

quadratic for $d>0$

$$-N - mg = -mv^2/r$$

$$N = mv^2/r - mg$$