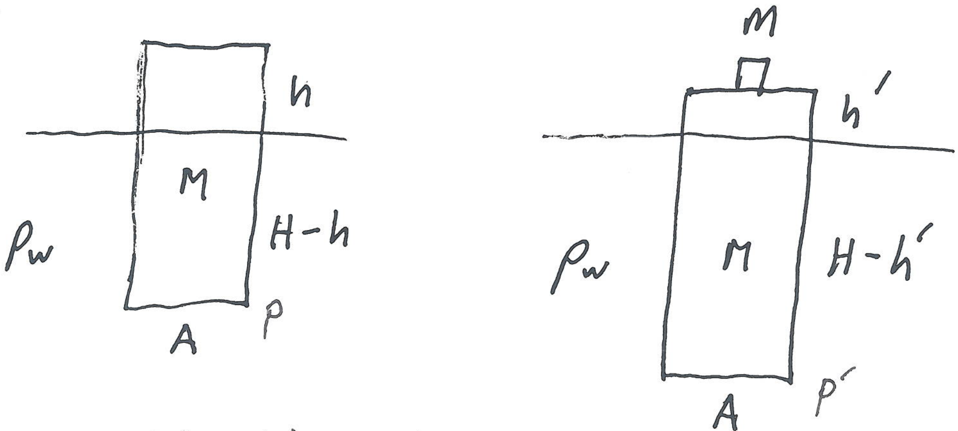
SMU Physics 1307 : Spring 2011

Final Exam

Problem 1 : The figure below shows a block of unknown mass M, height $H = 1\,\mathrm{m}$ and cross-sectional area $A = 0.3\,\mathrm{m}^2$ floating in water. If the block extends $h = 0.2\,\mathrm{m}$ above the surface while floating, find the mass M. Now a mass $m = 25\,\mathrm{kg}$ is added to the top of the block. Find the new position h' of the block. If this mass is suddenly lifted off of the block, find the acceleration a of the block at this moment. Also show that the buoyancy force for both h and h' is consistent with the pressure at the bottom of the block in both cases. Assume the density and pressure of the atmosphere both vanish. The density of water is $\rho_w = 10^3\,\mathrm{kg/m^3}$.



(1) $-Mg + \rho_w A(H-h)g = 0$

$M = \rho_w A(H-h)$

$0 = p - \rho_w g(H-h)$

$pA = \rho_w g(H-h)A = F_B$

(2) $-mg - Mg + \rho_w A(H-h')g = 0$

$m + \rho_w A(H-h) = \rho_w A(H-h')$

$m = \rho_w A(h-h')$

$h' = h - \dfrac{m}{\rho_w A}$

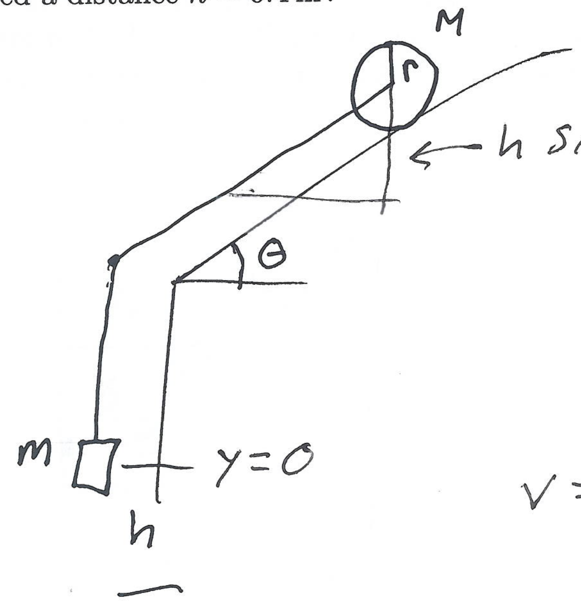
$0 = p' - \rho_w g(H-h')$

$p'A = \rho_w g(H-h')A = F_B'$

(3) $-Mg + \rho_w A(H-h')g = Ma$

$a = g\left(\dfrac{\rho_w A}{M}(H-h') - 1\right)$

Problem 2 : The figure below shows a disk ($I = \frac{1}{2}Mr^2$) of radius $r = 0.05\,\mathrm{m}$ and mass $M = 5\,\mathrm{kg}$ rolling down an incline of angle $\theta = 30°$. It is attached via a string to a mass $m = 7\,\mathrm{kg}$ which is free to move vertically. If the system starts from rest, find the velocity of the mass m when it has dropped a distance $h = 0.4\,\mathrm{m}$.

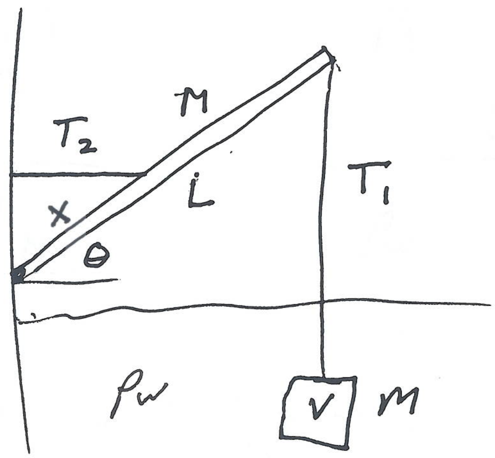


$v = \omega r$

$$0 = \tfrac{1}{2} m v^2 + \tfrac{1}{2} I \omega^2 + \tfrac{1}{2} M v^2 - mgh - Mgh\sin\theta$$

$$v^2 = \frac{2(m + M\sin\theta)gh}{(m + M + I/r^2)}$$

Problem 3 : The figure below shows the uniform beam of a crane of mass $M = 10^4\,\mathrm{kg}$ and length $L = 30\,\mathrm{m}$ which makes an angle $\theta = 40°$ with the horizontal. It is attached to a vertical cable with which it is holding a mass $m = 3 \times 10^3\,\mathrm{kg}$ which has volume $V = 1\,\mathrm{m}^3$ and which is completely submerged in water. In addition a horizontal cable is attached to the beam a distance $x = 12\,\mathrm{m}$ from the left end. Find the tension T_1 and T_2 for the vertical and horizontal cables respectively. Also find the horizontal F_H and vertical F_V components of the force on the left end of the beam.

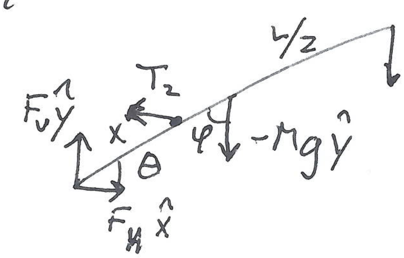


$$T_1 - mg + \rho_w V g = 0$$ gives T_1

$$F_V - Mg - T_1 = 0$$ gives F_V

$$F_H - T_2 = 0$$ $F_H = T_2$

Torques : about left end :

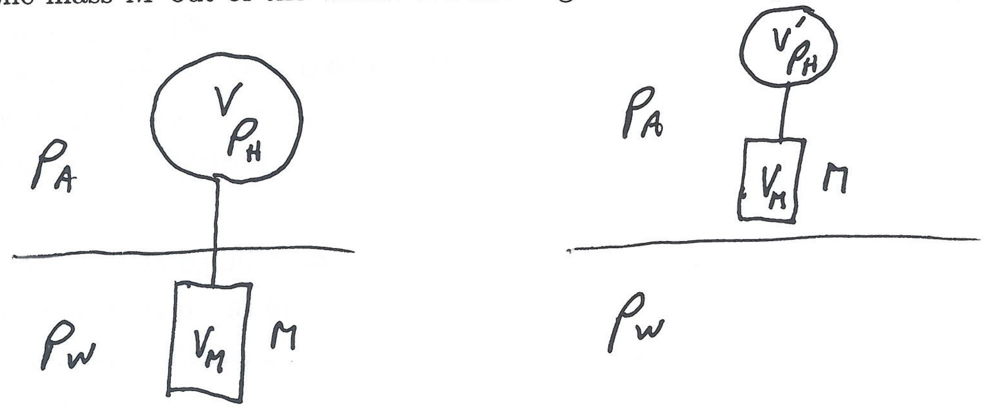


$$T_2 \sin\theta\, x - Mg\cos\theta\, L/2 - T_1 L\cos\theta = 0$$

gives T_2

$\sin\varphi = \cos\theta$

Problem 4 : The figure below shows a balloon of volume $V = 10^3\,\mathrm{m}^3$ which is filled with helium $\rho_{He} = \frac{2}{7}\rho_A$, where the density of air is $\rho_A = 1.23\,\mathrm{kg/m}^3$. The balloon is attached by a cable to an object of mass $M = 2000\,\mathrm{kg}$ and unknown volume V_M which is completely submerged in water. If the system has zero acceleration, find the volume V_M. Again, assuming that the system has zero acceleration, find the volume of helium V' necessary to completely lift the mass M out of the water. Do not neglect the density of air in any part of this problem.



$$-\rho_H V g + \rho_A V g - Mg + \rho_W V_M g = 0$$

$$\rho_W V_M = M\cancel{g} - (\rho_A - \rho_H) V$$ gives V_M

$$-\rho_H V' g + \rho_A V' g - Mg + \rho_A V_M g = 0$$

$$(\rho_A - \rho_H) V' = M + \rho_A V_M$$ gives V'