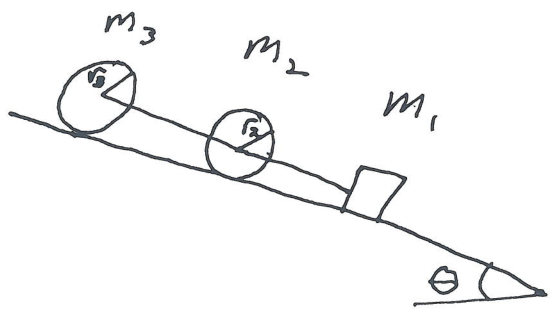
SMU Physics 1307 : Summer 2010

Final Exam

Problem 1 : The figure below shows an inclined plane of angle $\theta = 30°$ with three objects moving down it while attached with strings. The lower object is a sliding block of mass $m_1 = 3\,\text{kg}$. The middle object is a ball of mass $m_2 = 2\,\text{kg}$ and radius $r_2 = 0.1\,\text{m}$ ($I = \frac{2}{5}m_2r_2^2$). The upper object is a disk of mass $m_3 = 1\,\text{kg}$ and radius $r_3 = 0.05\,\text{m}$ ($I = \frac{1}{2}m_3r_3^2$). If the upper and middle objects roll without slipping, find the velocity v of the block after it has moved a vertical distance $h = 3\,\text{m}$ downward.

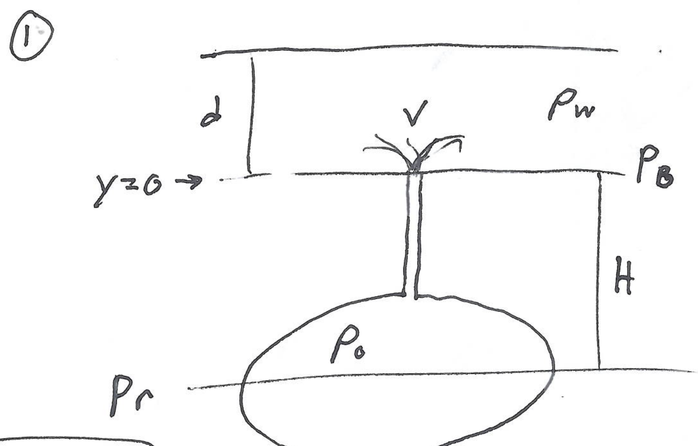


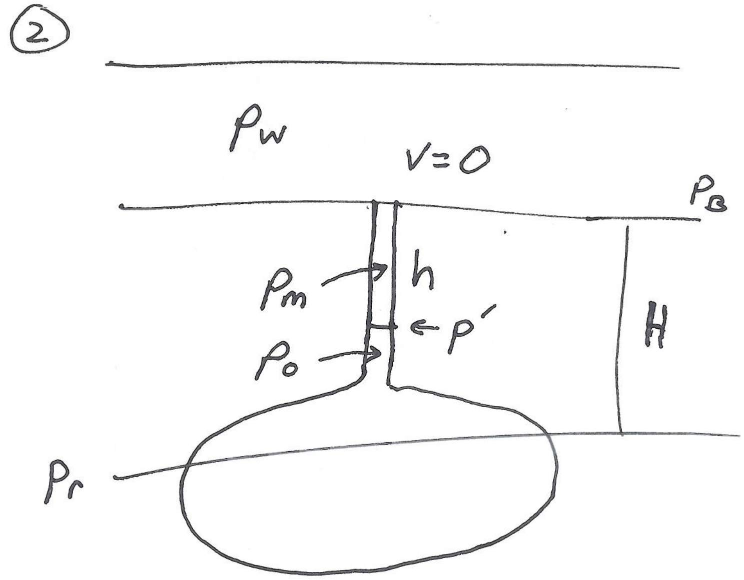
$\Delta E = 0$ $\quad V_3 = \omega_3 r_3 \quad V_2 = \omega_2 r_2 \quad V = V_2 = V_3$

$$(m_3 + m_2 + m_1) g h = \tfrac{1}{2}\left(m_3 + I_3/r_3^2 + m_2 + I_2/r_2^2 + m_1\right) V^2$$

solve for V: $\quad V = 6.52\ \text{m/s}$

Problem 2 : The figure at left below shows a blown-out oil well on the floor of the ocean at a depth $d = 2000\,\mathrm{m}$. Disregarding the existence of the well, find the pressure p_b on the ocean floor. As shown in the figure, the oil ($\rho_o = 0.7\rho_w$) originates in a reservoir which has its largest cross-sectional area at a depth $H = 1000\,\mathrm{m}$ below the ocean floor. The well is spewing oil into the water at a velocity $v = 40\,\mathrm{m/s}$. Ignoring the velocity of the oil at the widest part of the reservoir, find the pressure p_r at this point. In the figure at right the well has been plugged with a column of mud ($\rho_m = 2.0\rho_w$) of depth h. Using $\rho_w = 10^3\,\mathrm{kg/m^3}$ find the depth h.





set $V_r = 0$

① $P_A + \rho_w g d = P_B = 1.97 \times 10^7\ \mathrm{N/m^2}$

$$P_B + \tfrac{1}{2}\rho_o V^2 = P_r - \rho_o g H$$

$$P_r = P_B + \tfrac{1}{2}\rho_o V^2 + \rho_o g H = 2.71 \times 10^7\ \mathrm{N/m^2}$$

②

$P_B = p' - \rho_m g h$ (in mud)

$P_r - \rho_o g H = p' - \rho_o g h$ (in oil)

$\Rightarrow P_r - \rho_o g H$

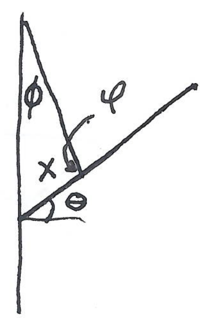
$= P_B + (\rho_m - \rho_o) g h$

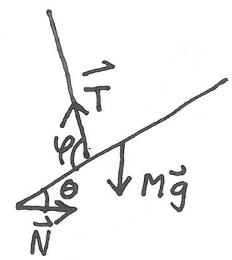
so, from above:

$\tfrac{1}{2}\rho_o V^2 = (\rho_m - \rho_o) g h$

$$h = \frac{\tfrac{1}{2}\rho_o V^2}{(\rho_m - \rho_o) g} = 81.6\,\mathrm{m}$$

Problem 3 : In the figure below a beam of mass $m = 100\,\text{kg}$ and length $L = 3\,\text{m}$ rests against a frictionless wall (so it can provide only a normal force) with angle $\theta = 30°$ as shown. A cable is to be attached to the beam at a distance x from the left end and inserted into the wall at an angle $\phi = 20°$ as shown. Find the distance x such that the beam is in equilibrium, and find the tension T in the cable.





$$\varphi = 90 + \theta - \phi = 100°$$

$$T\cos\phi - mg = 0$$

$$N - T\sin\phi = 0$$

$$Tx\sin\varphi - mg\,L/2\,\cos\theta = 0$$

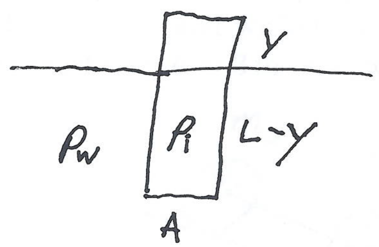
$$x\,\frac{mg\sin\varphi}{\cos\phi} = mg\,L/2\,\cos\theta \qquad \underline{T = \frac{mg}{\cos\phi}}$$

$$\underline{x = L/2\,\frac{\cos\theta\cos\phi}{\sin\varphi}}$$

$$\underline{T = 1043\,N} \qquad \underline{x = 1.24\,m}$$

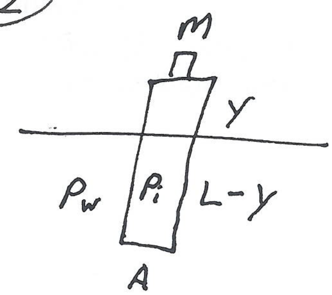
Problem 4 : The figure below shows a block of ice ($\rho_i = 0.86\rho_w$) with cross-sectional area $A = 4\,\mathrm{m}^2$ and length $L = 5\,\mathrm{m}$. Find the equilibrium position y_{eq} of the block of ice, taking y to be the distance from the water surface to the top of the block, and find the period of oscillation T of the block as it bobs in the water. Now suppose a polar bear of mass $m = 1000\,\mathrm{kg}$ rests on top of the block of ice. What is the new equilibrium position y'_{eq} and period of oscillation T'?

1



$\Rightarrow$

2



1

$$-\rho_i L A g + \rho_w (L-y) A g = \rho_i L A a \qquad \omega^2 = \frac{\rho_w}{\rho_i}\, g/L$$

$$a = -\frac{\rho_w}{\rho_i}\frac{g}{L}\, y + (\rho_w/\rho_i - 1)\, g \qquad c = (\rho_w/\rho_i - 1)\, g$$

$$y_{eq} = c/\omega^2 = L(1 - \rho_i/\rho_w) = \underline{0.7\,m}$$

$$T = 2\pi/\omega = \underline{4.16\,s}$$

2

$$-mg - \rho_i L A g + \rho_w (L-y) A g = (m\cancel{g} + \rho_i L A)\, a$$

$$a = -\frac{\rho_w A g}{(m + \rho_i L A)}\, y + \frac{((\rho_w - \rho_i) L A - m) g}{(m + \rho_i L A)} = -\omega'^2 y + c'$$

$$\omega'^2 = \frac{\rho_w A g}{m + \rho_i L A} \qquad y'_{eq} = c'/\omega'^2 = \frac{(\rho_w - \rho_i) L A - m}{\rho_w A}$$

$$T' = \frac{2\pi}{\omega'} = \underline{4.28\,s} \qquad y'_{eq} = \underline{0.45\,m}$$