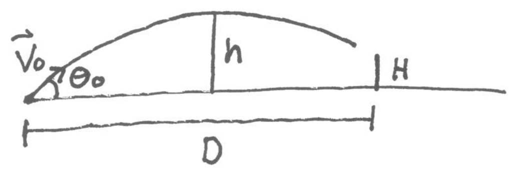
SMU Physics 1307 : Spring 2012

Exam 1

Problem 1 : The figure below shows a ball (projectile) with an initial angle from the horizontal $\theta_0 = 30°$ and initial velocity of unknown magnitude v_0. If the ball reaches a maximum height $h = 20\,\mathrm{m}$, find v_0. There is a fence at a horizontal distance $D = 120\,\mathrm{m}$ which is of height $H = 5\,\mathrm{m}$. Find the time t_f when the ball reaches the fence and height of the ball $y(t_f)$ at this time. Does the ball clear the fence?



$X = V_0 \cos\theta_0 t$

$V_y = V_0 \sin\theta_0 - gt$

$y = V_0 \sin\theta_0 t - \frac{1}{2}gt^2$

$V_y(t_p) = 0 \quad t_p = V_0 \sin\theta_0/g$

$y(t_p) = h = V_0 \sin\theta_0 t_p - \frac{1}{2}g t_p^2 = \frac{1}{2}g t_p^2$ gives $t_p = 2.02\,s$

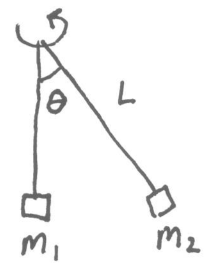
$V_0 = g t_p/\sin\theta_0 = 39.6\,m/s$

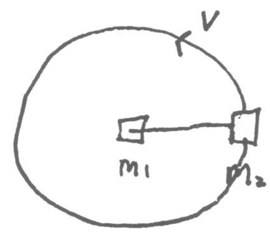
$X(t_f) = D = V_0 \cos\theta_0 t_f$ gives $t_f = 3.50\,s$

$y(t_f) = V_0 \sin\theta_0 t_f - \frac{1}{2}g t_f^2 = 9.28\,m$

does clear

Problem 2 : The figure at left below shows a mass $m_1 = 5\,\text{kg}$ which hangs vertically and is attached by a string to another mass $m_2 = 3\,\text{kg}$ which is executing circular motion as the segment of string of length $L = 1\,\text{m}$ traces out a cone of angle θ. The figure at right shows the same system from above. Find the angle θ, the tension in the string T, the velocity v of mass m_2, and the period t of the circular motion.





$T\hat{y}$

$-m_1 g\hat{y}$

$T\cos\theta\,\hat{y} - T\sin\theta\,\hat{x}$

$-m_2 g\hat{y}$

$\vec{a}_2 = -\frac{v^2}{r}\hat{x}$

$r = L\sin\theta$

$T - m_1 g = 0$

$T = m_1 g = \underline{49\text{N}}$

$T\cos\theta - m_2 g = 0$

$-T\sin\theta = -\frac{m_2 v^2}{L\sin\theta}$

$\cos\theta = \frac{m_2 g}{T} = \frac{m_2}{m_1}$ $\quad \underline{\theta = 53.1^\circ}$

$\underline{\sin\theta = 0.8}$

$m_1 g\sin\theta = \frac{m_2 v^2}{L\sin\theta}$

$v^2 = \frac{m_1}{m_2} g L\sin^2\theta$ $\quad \sin^2\theta = 1 - \cos^2\theta$

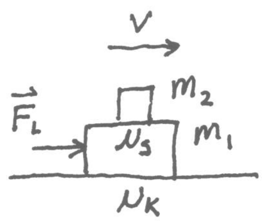
$v^2 = gL\left(\frac{m_1}{m_2} - \frac{m_2}{m_1}\right)$

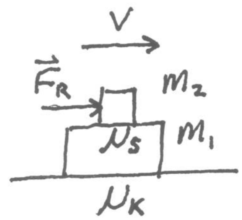
$\underline{v = 3.23\,\text{m/s}}$

$t = \frac{2\pi r}{v} = \frac{2\pi L\sin\theta}{v}$

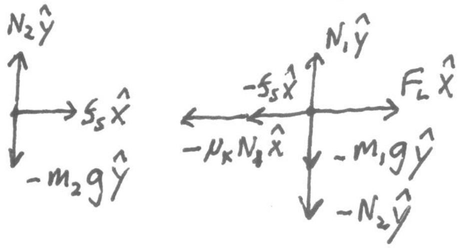
$\underline{t = 1.55\text{s}}$

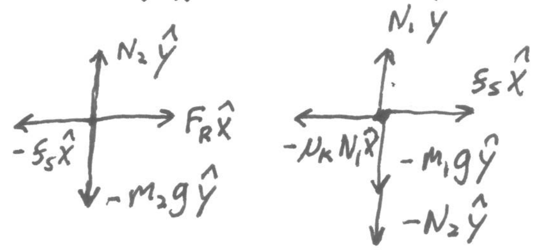
Problem 3 : The figures below show two masses $m_1 = 4\,\mathrm{kg}$ and $m_2 = 3\,\mathrm{kg}$ moving to the right over a horizontal surface. The lower mass m_1 is in contact with the surface, which has coefficient of kinetic friction $\mu_k = 0.3$. The upper mass m_2 is in contact with the lower mass m_1 and the surfaces of the respective masses have a coefficient of static friction $\mu_s = 0.8$. In the figure at left a force F_L is applied to mass m_1 and acts to the right. In the figure at right a force F_R is applied to mass m_2 and also acts to the right. Find the maximum accelerations a_L and a_R that the blocks can experience in both of these cases before the top block slips on the bottom block.





$\vec{a}_1 = \vec{a}_L = a\hat{x}$





$N_2 - m_2 g = 0 \qquad N_1 - N_2 - m_1 g = 0$

$f_s = m_2 a_L \qquad F_L - f_s - \mu_K N_1 = m_1 a_L$

slips : $\quad f_s = \mu_s N_2$

$m_2 a_L = \mu_s m_2 g$

$a_L = \mu_s g = 7.84\,m/s^2$

$N_2 - m_2 g = 0 \qquad N_1 - N_2 - m_1 g = 0$

$F_R - f_s = m_2 a_R \qquad f_s - \mu_K N_1 = m_1 a_R$

$N_1 = (m_1 + m_2) g$

$F_R = \mu_K (m_1 + m_2) g + (m_1 + m_2) a_R$

slips : $\quad f_s = \mu_s N_2$

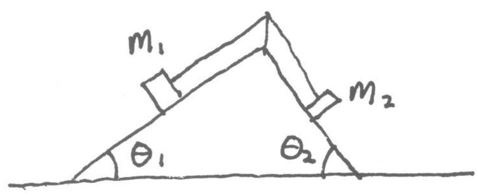
$F_R - m_2 a_R = \mu_s m_2 g$

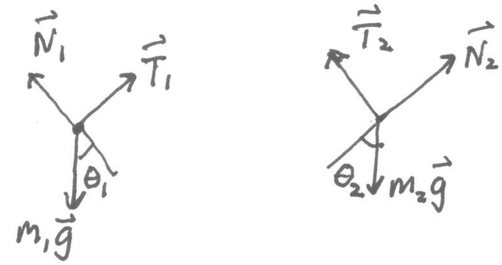
$\mu_K (m_1 + m_2) g + m_1 a_R = \mu_s m_2 g$

$a_R = \frac{m_2 \mu_s}{m_1} g - \mu_K \left(1 + \frac{m_2}{m_1}\right) g$

$a_R = 0.74\,m/s^2$

Problem 4 : The figure below shows two masses $m_1 = 3\,\text{kg}$ and $m_2 = 2\,\text{kg}$ connected by a string which rest on opposing frictionless inclined planes of angles $\theta_1 = 30°$ and $\theta_2 = 40°$. Find the acceleration a of m_1 with positive a taken up the inclined plane. Also find the tension T in the string.





$$|\vec{T}_1| = |\vec{T}_2| = T$$

$$T - m_1 g \sin\theta_1 = m_1 a \qquad m_2 g \sin\theta_2 - T = m_2 a$$

$$m_2 g \sin\theta_2 - m_1 g \sin\theta_1 = (m_1 + m_2) a$$

$$a = g(m_2 \sin\theta_2 - m_1 \sin\theta_1)/(m_1+m_2) = \underline{-0.42\ m/s^2}$$

$$m_2 T + m_1 T - m_1 m_2 g \sin\theta_1 - m_1 m_2 g \sin\theta_2 = 0$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2}(\sin\theta_1 + \sin\theta_2) = \underline{13.4N}$$