SMU Physics 1307 : Spring 2012

Exam 2

Problem 1 : The figure below shows two masses of equal mass M and radius R which are isolated from any other mass (including earth). If the respective centers are initially separated by a distance r with the masses at rest, find the (equal) magnitude of the velocities v_c of the two objects when they collide. Find v_c as $r \to \infty$ and note that it approaches a finite value v_∞. Find v_∞ for two bowling balls with $M = 5\,\text{kg}$ and $R = 0.1\,\text{m}$. Also find v_∞ for two earths with $M = 6 \times 10^{24}\,\text{kg}$ and $R = 6.4 \times 10^6\,\text{m}$.

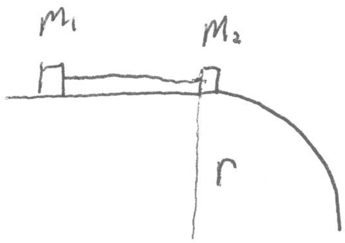
$$U_0 = -\frac{GM^2}{r} \qquad U_f = -\frac{GM^2}{2R}$$

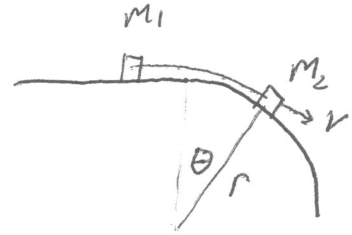
$$K_0 = 0 \qquad K_f = 2\left(\tfrac{1}{2} M V_c^2\right)$$

$$-\frac{GM^2}{r} = -\frac{GM^2}{2R} + MV_c^2$$

$r \to \infty$ $\qquad V_c^2 = \dfrac{GM}{2R}$

Problem 2 : The figure below shows two masses $m_1 = 3\,\mathrm{kg}$ and $m_2 = 2\,\mathrm{kg}$ connected by a string on a frictionless surface. The mass m_2 begins from rest and slides down a hill of circular profile with radius $r = 1\,\mathrm{m}$. Find magnitude of the velocity of the masses v_θ as a function of θ as m_2 slides down the hill. Find the angle θ_c at which m_2 leaves the surface.





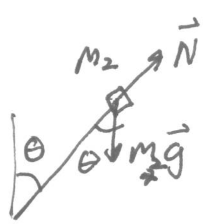
$U_0 = 0 \quad K_0 = 0$

$U_\theta = -m_2 g (r - r\cos\theta)$

$K_\theta = \frac{1}{2}(m_1 + m_2) V_\theta^2$

$U_0 + K_0 = U_\theta + K_\theta$

$$\frac{1}{2}(m_1 + m_2) V_\theta^2 = m_2 g r (1 - \cos\theta)$$



$N - m_2 g \cos\theta = -m_2 V_\theta^2 / r$

$N = 0 \qquad V_\theta^2 = g r \cos\theta$

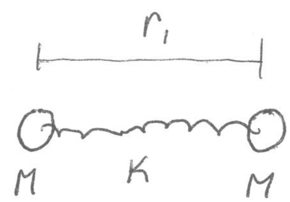
$$\frac{1}{2}(m_1 + m_2) g r \cos\theta_c = m_2 g r (1 - \cos\theta)$$

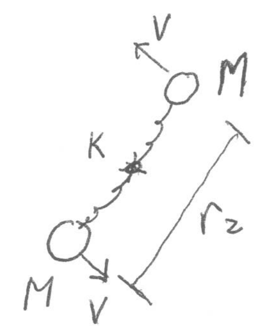
$$(m_1/m_2 + 1)\cos\theta_c = 2 - 2\cos\theta$$

$$(m_1/m_2 + 3)\cos\theta_c = 2$$

$$\cos\theta_c = \frac{2}{3 + m_1/m_2}$$

Problem 3 : The figure below shows a spring of equilibrium length $r_{eq} = 1\,\mathrm{m}$ and unknown spring constant k. Thus if the spring is extended to a distance $r > r_{eq}$ there will be a tension in the spring given by $k(r - r_{eq})$. In the first of two experiments two equal masses of unknown mass M are attached to opposite end of the spring which is now of length $r_1 = 0.95\,\mathrm{m}$ due to the gravitational attraction of the masses. In the second experiment the spring and mass system is rotating around the center of the spring with period $t = 0.1\,\mathrm{s}$ and the spring is now of length $r_2 = 1.08\,\mathrm{m}$. Find k and M.





$$V = \frac{2\pi(r_2/2)}{t}$$

$$-K(r_1 - r_{eq}) - \frac{GM^2}{r_1^2} = 0$$

$$① \quad \frac{GM^2}{K} = r_1^2(r_{eq} - r_1)$$

$$-K(r_2 - r_{eq}) - \frac{GM^2}{r_2^2} = \frac{MV^2}{(r_2/2)}$$

$$-K(r_2 - r_{eq}) - \frac{GM^2}{r_2^2} = M\left(\frac{2\pi}{t}\right)^2\left(\frac{r_2}{2}\right)$$

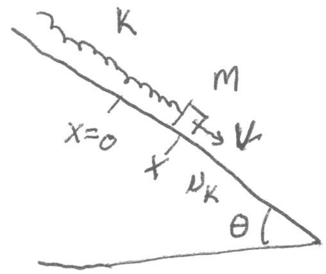
$$② \quad -Kr_2^2(r_2 - r_{eq}) - GM^2 = M\left(\frac{2\pi}{t}\right)^2 \frac{r_2^3}{2}$$

① and ②

$$\hookrightarrow \quad -GM^2\frac{r_2^2(r_2 - r_{eq})}{r_1^2(r_{eq} - r_1)} - GM^2 = M\left(\frac{2\pi}{t}\right)^2 \frac{r_2^3}{2}$$

this fixes M then get K from ①

Problem 4 : The figure below shows a mass $m = 1\,\mathrm{kg}$ attached to a spring with $k = 10^2\,\mathrm{N/m}$ which is initially in its uncompressed position on an inclined plane of angle $\theta = 30°$. If the surface on which the mass rests has coefficient of kinetic friction $\mu_k = 0.4$, find the velocity v as a function of position x measured down the ramp. Find the quadratic equation for the maximum distance $x_{\rm max}$ that the mass slides down the ramp for the first time. Assuming $x_{\rm max}$ is known, find the quadratic equation for the position of the mass $x_{\rm min}$ when it next reaches its highest point.



$$y(x) = -x\sin\theta$$

$$U_0 = 0$$

$$U(x) = mg\,y(x) + \tfrac{1}{2}Kx^2$$
$$= -mgx\sin\theta + \tfrac{1}{2}Kx^2$$

$$K_0 = 0 \quad K(x) = \tfrac{1}{2}mV^2$$

$$W_{nc} + U_0 + K_0 = K(x) + U(x)$$

$$\tfrac{1}{2}mV^2 = mgx\sin\theta - \tfrac{1}{2}Kx^2 + W_{nc}$$

$$W_{nc} = -\mu_K N x = -\mu_K mg\cos\theta\, x$$

$$\tfrac{1}{2}mV^2 = mg(\sin\theta - \mu_K\cos\theta)x - \tfrac{1}{2}Kx^2$$

$\underline{V=0}$

$$\tfrac{1}{2}Kx_{max} = mg(\sin\theta - \mu_K\cos\theta)$$

at x_{max} $\quad E(x_{max}) = \underset{\substack{\parallel \\ 0}}{K(x_{max})} + U(x_{max}) = -mgx_{max}\sin\theta + \tfrac{1}{2}Kx_{max}^2$

at x_{min} $\quad E(x_{min}) = \underset{\substack{\parallel \\ 0}}{K(x_{min})} + U(x_{min}) = -mgx_{min}\sin\theta + \tfrac{1}{2}Kx_{min}^2$

$$E(x_{min}) - E(x_{max}) = -\mu_K mg\cos\theta\,(x_{max} - x_{min})$$

quadratic for x_{min}