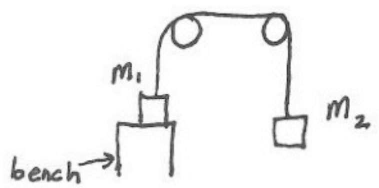
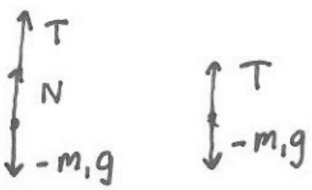
SMU Physics 1307 : Fall 2008

Exam 1

Problem 1 : A person of mass $m_1 = 60\,\text{kg}$ is seated on a weight bench and pulls down on a cable which is connected over a (massless, frictionless) pulley to a weight stack of mass $m_2 = 20\,\text{kg}$. Suppose that the weight stack is lifted upward with acceleration $a_2 = 3\,\text{m/s}^2$. Find the tension T in the cable, and force N that the bench exerts upward on the person in order to maintain $a_1 = 0$. Find the smallest acceleration a_2 required to lift the person off the bench, which is the same as the acceleration for which $N = 0$ and $a_1 = 0$. Find the tension T in this case.



Note: m_1 can draw in cable (length not fixed)



$$T + N - m_1 g = m_1 a_1$$

$$T - m_2 g = m_2 a_2$$

1) $a_1 = 0 \quad a_2 = 3\,m/s^2$

$$T + N = m_1 g = 60 \cdot 9.8 = 588 \text{ Newtons}$$

$$T = m_2(g + a_2) = 20\,(9.8 + 3) = \underline{256 \text{ Newtons}}$$

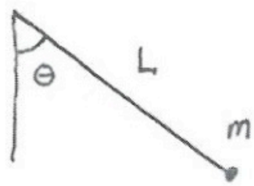
$$N = m_1 g - T = 588 - 256 = \underline{332 \text{ Newtons}}$$

2) $N = 0 \quad a_1 = 0$

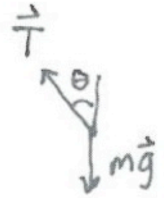
$$T - m_1 g = 0 \qquad T = m_1 g = \underline{588 \text{ Newtons}}$$

$$a_2 = (T - m_2 g)/m_2 = (588 - 20 \cdot 9.8)/20 = \underline{19.6\,m/s^2}$$

Problem 2 : A conical pendulum is formed by a string of length $L = 5\,\mathrm{m}$ which makes an angle $\theta = 25°$ with the vertical and is attached to a weight of mass $m = 3\,\mathrm{kg}$ which swings around in a circle with a velocity of constant magnitude. What are the magnitudes, $|\vec{v}|$ and $|\vec{a}|$, of the velocity and acceleration? Find the period of revolution τ of the pendulum and the tension T in the string.



$\vec{V}$ of m into page.
string sweeps out cone



$\hat{y}$, $\hat{x}$

$|\vec{V}| = V$

$\vec{a} = -\frac{V^2}{r}\hat{x}$

$\hat{y}: \quad T\cos\theta - mg = 0$

$\hat{x}: \quad -T\sin\theta = -\frac{mV^2}{r}$

$r = L\sin\theta$

$T\cos\theta = mg$

$T\sin\theta = mV^2/r$

$\frac{V^2}{gr} = \tan\theta$

$V^2 = gL\sin\theta\tan\theta$

$V^2 = gL\sin^2\theta/\cos\theta = 9.8 \cdot 5 \cdot (\sin 25°)^2/\cos 25° = 9.66\ m^2/s^2$

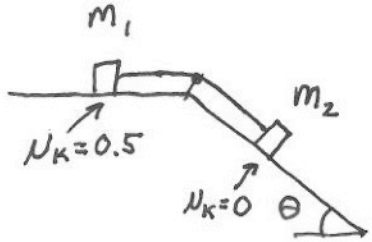
$|\vec{V}| = 3.11\ m/s$

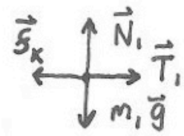
$|\vec{a}| = \frac{V^2}{r} = \frac{V^2}{L\sin\theta} = \frac{9.66}{5 \cdot \sin 25°} = 4.57\ m/s^2$

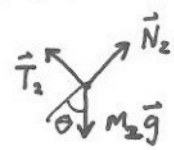
$\tau = \frac{2\pi r}{V} = \frac{2\pi L\sin\theta}{V} = \frac{2\pi \cdot 5 \cdot \sin 25°}{3.11} = 4.27\,s$

$T = \frac{mg}{\cos\theta} = \frac{3 \cdot 9.8}{\cos 25°} = 32.44$ Newtons

Problem 3 : Consider a mass $m_1 = 3\,\mathrm{kg}$ on a level surface with coefficient of kinetic friction $\mu_k = 0.5$. It is connected by a string to a mass $m_2 = 5\,\mathrm{kg}$ which is on a downward sloping frictionless inclined plane with angle from the horizontal of $\theta = 35°$. If the masses are moving together so that m_1 is initially moving toward the incline, find the acceleration a of the system and the tension T in the string. Repeat the calculation of a and T for the masses moving together so that m_1 is initially moving away from the incline. Take a to be positive if the acceleration of m_1 is toward the incline.



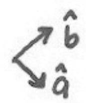




$a_1 = a_2 = a$

$\vec{a}_1 = a_1\hat{x} = a\hat{x}$

$\vec{T}_1 = T\hat{x}$



$\vec{a}_2 = a_2\hat{a} = a\hat{a}$

$\vec{T}_2 = -T\hat{a}$

$\hat{x}: \quad T - f_K = m_1 a$

$\hat{y}: \quad N_1 - m_1 g = 0$

$\hat{a}: \quad -T + m_2 g\sin\theta = m_2 a$

$\hat{b}: \quad N_2 - m_2 g\cos\theta = 0$

① $\vec{V}_1 = V_1\hat{x} \quad \underline{V_1 > 0}$

$\vec{V}_2 = V_2\hat{a}$

$\underline{f_K = \mu_K N_1} \Rightarrow \quad T - m_1 a = \mu_K m_1 g$

$-T + m_2 g\sin\theta = m_2 a$

$m_2 g\sin\theta - \mu_K m_1 g = (m_1 + m_2)a$

$T = m_2 g\sin\theta - m_2 a = \underline{19.73\ \text{Newtons}}$ $\quad \boxed{a = \underline{1.68\ m/s^2}}$

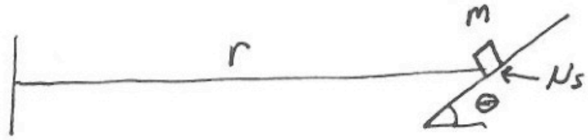
② $V_1 < 0 \quad f_K = -\mu_K N_1$

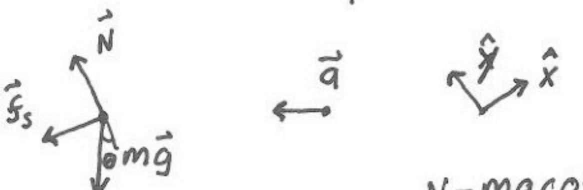
$m_2 g\sin\theta + \mu_K m_1 g = (m_1 + m_2)a$

$a = \underline{5.35\ m/s^2}$

$T = m_2 g\sin\theta - m_2 a = \underline{1.35\ \text{Newton}}$

Problem 4 : A banked roadway is being built which has a surface with lateral coefficient of static friction $\mu_s = 0.65$. The intention is for vehicles to experience no lateral frictional forces when moving at $v_1 = 30\,\mathrm{m/s}$, and for the maximum speed at which a car may go around the turn without its wheels slipping to be $v_2 = 60\,\mathrm{m/s}$. Find the radius of curvature r and banking angle θ of the road. There should be two solutions for r and θ; express both of them. The mass of a typical car may be taken to be $m = 1000\,\mathrm{kg}$, but this will not be required to solve for r and θ.





$$\vec{a} = -\frac{v^2}{r}\cos\theta\,\hat{x} + \frac{v^2}{r}\sin\theta\,\hat{y}$$

$$N - mg\cos\theta = m\frac{v^2}{r}\sin\theta$$

$$-f_s - mg\sin\theta = -m\frac{v^2}{r}\cos\theta$$

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① $f_s = 0 \quad V = V_1$ $\qquad mg\sin\theta = m\frac{V_1^2}{r}\cos\theta \qquad \underline{r\tan\theta = V_1^2/g}$

② $f_s = \mu_s N \quad V = V_2$

$$m\frac{V_2^2}{r}\cos\theta - mg\sin\theta = \mu_s\left(m\frac{V_2^2}{r}\sin\theta + mg\cos\theta\right)$$

divide by $mg\cos\theta$ $\rightarrow$ $\frac{V_2^2}{gr} - \tan\theta = \mu_s\left(\frac{V_2^2}{gr}\tan\theta + 1\right)$

multiply by r^2 and use $r\tan\theta = V_1^2/g$ $\rightarrow$ $\frac{V_2^2}{g}r - \frac{V_1^2}{g}r = \mu_s\frac{V_2^2}{g}\frac{V_1^2}{g} + r^2\mu_s$

quadratic equation:

$Ar^2 + Br + C = 0$

$A = \mu_s = 0.65$

$B = -(V_2^2 - V_1^2)/g = -275.5$ meters

$C = \mu_s\frac{V_1^2}{g}\frac{V_2^2}{g} = 2.19\times10^4\,m^2$

two roots: $\underline{\underline{r_+ = 317.7\,m}} \qquad \underline{\underline{r_- = 106.2\,m}}$

$$r_\pm\tan\theta_\pm = V_1^2/g$$

$\underline{\underline{\theta_+ = 16.1^\circ}} \qquad \underline{\underline{\theta_- = 40.9^\circ}}$