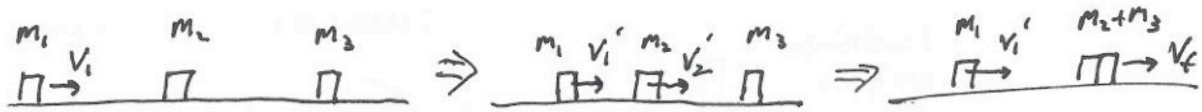
SMU Physics 1307 : Fall 2008

Exam 3

Problem 1 : The mass $m_1 = 3\,\mathrm{kg}$ in the figure below is given an initial velocity $v_1 = 2\,\mathrm{m/s}$ to the right. It then collides elastically with mass $m_2 = 1\,\mathrm{kg}$ which is initially at rest. Find the velocities v_1' and v_2' after this collision. The mass m_2 then collides completely inelastically with a mass m_3. Find the mass m_3 such that the final velocity v_f of the resulting combined mass is equal to v_1'. How much total kinetic energy is lost in this entire process?



$$V_1' = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_1 = \frac{2}{4} V_1 = 1\,\mathrm{m/s}$$

$$V_2' = \frac{2m_1}{m_1 + m_2} V_1 = \frac{6}{4} V_1 = 3\,\mathrm{m/s}$$

$$m_2 V_2' = (m_2 + m_3) V_f = (m_2 + m_3) V_1'$$

$$m_3 = m_2 \left(V_2'/V_1' - 1 \right) = 2\,\mathrm{Kg}$$

$$K_0 = \tfrac{1}{2} m_1 V_1^2$$

$$K_0 = \tfrac{1}{2} (3\,\mathrm{Kg})(4\,\mathrm{m^2/s^2})$$

$$K_0 = 6\,\mathrm{J}$$

$$K_f = \tfrac{1}{2} m_1 V_1'^2 + \tfrac{1}{2}(m_2 + m_3) V_f^2$$

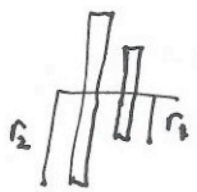
$$K_f = \tfrac{1}{2}(m_1 + m_2 + m_3) V_1'^2$$

$$K_f = \tfrac{1}{2}(6\,\mathrm{Kg})(1\,\mathrm{m^2/s^2}) = 3\,\mathrm{J}$$

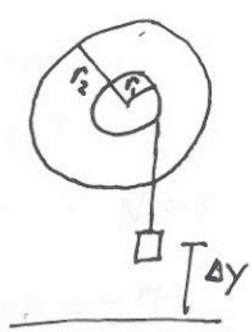
$$K_f - K_0 = -3\,\mathrm{J}$$

Problem 2 : The figure below shows two uniform disks, of masses $m_1 = 1\,\mathrm{kg}$ and $m_2 = 3\,\mathrm{kg}$ with radii $r_1 = 0.15\,\mathrm{m}$ and $r_2 = 0.25\,\mathrm{m}$, which spin freely and initially independently about a common horizontal axis. The moment of inertia about the center of a disk of mass m and radius r is $I = \frac{1}{2}mr^2$. A string attached to a mass $M = 0.5\,\mathrm{kg}$ is initially wrapped around the first disk. This mass drops a distance of $\Delta y = -1\,\mathrm{m}$ before it hits the floor and the string goes slack. Find the tension T in the string and the angular acceleration α_1 of the first disk. Find the angular velocity ω_1 of the first disk after the mass M has hit the floor. Now the disks are brought together and allowed to couple through friction until their angular velocities are equal. Find the final common angular velocity ω_f.

side view:



front view:



$\Rightarrow$

frictional coupling

$I_1 = \frac{1}{2} m_1 r_1^2 = .01125\ \mathrm{Kg\,m^2}$

$I_2 = \frac{1}{2} m_2 r_2^2 = .09375\ \mathrm{Kg\,m^2}$

since $w_2 = 0$:

$E = \frac{1}{2} I_1 w_1^2 + \frac{1}{2} M V^2 + M g y$ (general)

before mass drops : $E = M g y_0$

after mass drops : $E = \frac{1}{2} I_1 w_1^2 + \frac{1}{2} M V^2 + M g (y_0 + \Delta y)$

so, since $V = r_1 w_1$:

$-M g \Delta y = \frac{1}{2}(I_1 + M r_1^2) w_1^2$

$(0.5)(9.8) = (0.5)(0.01125 + 0.01125)\, w_1^2$

$w_1^2 = \frac{9.8}{.0225} = 435.6\ s^{-2}$

since $w_1 < 0$ $\quad \underline{w_1 = -20.87\ s^{-1}}$

$\Delta L = 0$

before coupling: $L = I_1 w_1$

after coupling : $L = (I_1 + I_2) w_f$

$w_f = \frac{I_1}{I_1 + I_2} w_1 = \underline{-2.236\ s^{-1}}$

$\underline{T \text{ and } \alpha_1}$

$T - Mg = Ma$

$-r_1 T = I_1 \alpha_1$

$a = r_1 \alpha_1$

$\Rightarrow T - Mg = M r_1 \alpha_1$

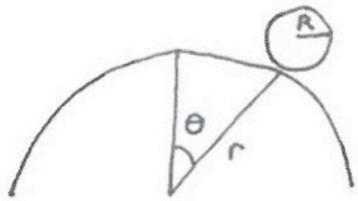
$T = -\frac{I_1 \alpha_1}{r_1}$

$-Mg = (M r_1 + I_1/r_1)\,\alpha_1$

$\alpha_1 = \underline{-32.7\ s^{-2}}$

$T = \underline{2.45\,N}$

Problem 3 : The figure below shows a spherical mass M of radius R ($I = \frac{2}{5}MR^2$) which is initially placed at rest on top of a hemisphere of radius $r = 3R$. The object then rolls without slipping down the hemisphere. Without neglecting the radius of the sphere, find the angle θ from the vertical at which it leaves the hemisphere.



in general:

$$E = \tfrac{1}{2}MV^2 + \tfrac{1}{2}I\omega^2 + Mg\,y_{cm}$$

$$y_{cm} = (r+R)\cos\theta$$

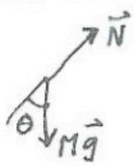
$$\omega = -V/R$$

$$E = \tfrac{1}{2}MV^2\left(1 + I/MR^2\right) + Mg(r+R)\cos\theta$$

Initially: $V = 0 \quad \theta = 0$

$$E = Mg(r+R)$$

when object comes off: $N = 0$



$$N - Mg\cos\theta = \frac{-MV^2}{(r+R)}$$

$$V^2 = g(r+R)\cos\theta$$

$$Mg(r+R) = \tfrac{1}{2}Mg(r+R)\cos\theta\,\left(1 + I/MR^2\right) + Mg(r+R)\cos\theta$$

drop $Mg(r+R)$

$$1 = \tfrac{1}{2}\cos\theta\left(1 + I/MR^2\right) + \cos\theta$$

$$1 = \tfrac{1}{2}\cos\theta\left(3 + I/MR^2\right) = \cos\theta\left(3/2 + 2/10\right)$$

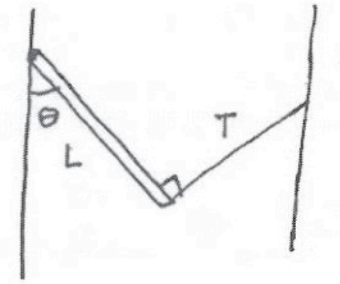
$$\cos\theta = 10/17$$

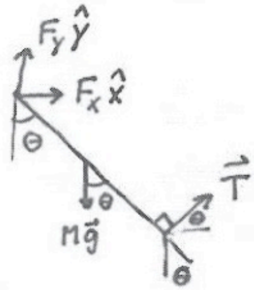
$$\theta = 53.97^\circ$$

depends only on $I/MR^2 = 2/5$

not on r, g, R! or M

Problem 4 : The figure below shows a uniform beam of length L and mass $M = 2\,\mathrm{kg}$ which is attached to a wall at an angle $\theta = 30°$ from the vertical. A wire is attached at a right angle to the lower end of the beam. Find the tension T in the wire and the components F_x and F_y of the force that the wall exerts on the beam.





$$\vec{T} = T\cos\theta\,\hat{x} + T\sin\theta\,\hat{y}$$

$$F_{net\text{-}x} = F_x + T\cos\theta = 0$$

$$F_{net\text{-}y} = F_y - Mg + T\sin\theta = 0$$

$$\tau = LT - \tfrac{1}{2} L Mg\sin\theta = 0$$

$$\underline{T = \tfrac{1}{2} Mg\sin\theta} = \underline{4.9\,N}$$

$$F_x = -T\cos\theta = -\tfrac{1}{2} Mg\sin\theta\cos\theta = \underline{-4.24\,N}$$

$$F_y = Mg \mp T\sin\theta = Mg(1 \mp \tfrac{1}{2}\sin^2\theta) = \underline{17.15\,N}$$