SMU Physics 1307 : Fall 2008

Final

Problem 1 : You are shown an irregularly shaped mass of unknown uniform density ρ. You are also shown a vertically hanging spring with unknown spring constant k, which we will not determine since this would require the unknown volume V. When the mass is attached to the spring, the new equilibrium position of the spring (with respect to the unstretched position at $y = 0$) is $y_e = -0.04\,\mathrm{m}$. The mass, still attached to the spring, is then entirely submerged in water. Its new equilibrium position is $y_e' = -0.025\,\mathrm{m}$. If the density of water is $\rho_w = 10^3\,\mathrm{kg/m^3}$, find the density ρ. What are the periods of oscillation T_a and T_w in air and water, respectively? Take $g = 9.8\,\mathrm{m/s^2}$.

in air

$$m\frac{d^2y}{dt^2} = -ky - mg \qquad \omega_a^2 = k/m \qquad y_e = -mg/k$$

in water

$$m\frac{d^2y}{dt^2} = -ky - (m - m_w)g \qquad \omega_w^2 = k/m \text{ (same)} \qquad y_e' = -(m - m_w)g/k$$

$$y_e'/y_e = (m - m_w)/m = 1 - \rho_w/\rho$$

$$\rho = \frac{\rho_w}{(1 - y_e'/y_e)} = \underline{2.67 \times 10^3\ \mathrm{kg/m^3}}$$

$$\omega_a^2 = \omega_w^2 = k/m = -g/y_e$$

$$T_a = T_w = \frac{2\pi}{\omega_a} = \underline{0.40\,\mathrm{s}}$$

Problem 2 : A pipe has been buried at an unknown depth H below ground. It is known to be $R = 0.025\,\mathrm{m}$ in radius, and to have a pressure at depth H of $4p_a$, where $p_a = 10^5\,\mathrm{N/m^2}$ is the atmospheric pressure. The pipe emerges from the ground and rises to a height $h = 1.0\,\mathrm{m}$ above ground where it narrows to $r = 0.01\,\mathrm{m}$ before water flows out onto the ground. Looking at the stream of water as it exits the pipe you find that it is moving at $v = 10\,\mathrm{m/s}$. Find the depth of the pipe H and the velocity v' of the water at this depth.

$$4P_a + \tfrac{1}{2}\rho_w v'^2 - \rho_w g H = P_a + \tfrac{1}{2}\rho_w v^2 + \rho_w g h$$

$$\frac{3P_a}{\rho_w} + \tfrac{1}{2}(v'^2 - v^2) - gh = gH$$

$$AV = A'V' \Rightarrow \pi r^2 v = \pi R^2 v' \qquad v' = v\left(\frac{r^2}{R^2}\right) = \underline{1.6\ m/s}$$

$$v'^2 - v^2 = -v^2\left(1 - r^4/R^4\right) = -97.44\ m^2/s^2$$

$$3P_a/\rho_w = 300\ m^2/s^2 \qquad gh = 9.8\ m^2/s^2$$

$$\underline{H = 24.64\ m}$$

Problem 3 : A basketball player can exert a force F_e through a distance $d_p = 0.33\,\mathrm{m}$ and can can leap to a height of $h_p = 1\,\mathrm{m}$. Since energy conservation leads to $(F_e - m_p g)d_p = m_p g h_p$, using $m_p = 100\,\mathrm{kg}$, we find $F_e = m_p g(1 + h_p/d_p) = 3920\,\mathrm{N}$. Assume a dolphin can exert the same force F_e through a distance of $d_d = 5\,\mathrm{m}$ under water. If the dolphin has mass $m_p = 400\,\mathrm{kg}$, and a density $\rho = 0.9\rho_w$, how high h_d can the dolphin leap out of the water?

$$(F_e - M_d g + M_w g)\,d_d = M_d g h_d \qquad \frac{m_w}{m_d} = \frac{1}{0.9}$$

$$h_d = d_d\left(F_e/M_d g - 1 + 1/0.9\right) \qquad \frac{F_e}{M_d g} = 1$$

$$h_d = d_d/0.9 = \underline{5.56\,\mathrm{m}}$$

dolphin can exert a force equal only to his weight on land and jump this high because of his buoyancy.

Problem 4 : A mass of $m = 1\,\mathrm{kg}$ drops from a height of $h = 0.1\,\mathrm{m}$ onto a spring with $k = 0.25\,\mathrm{N/m}$. Find the angular frequncy ω and the equilibrium position y_e of the mass on the spring, taking $y = 0$ to be the uncompressed position. Taking $t = 0$ when the mass strikes the spring, so $y_0 = 0$, use energy conservation to find the velocity v_0 at that time. From the general equations $y = y_e + A\cos(\omega t + \phi)$ and $v = -\omega A \sin(\omega t + \phi)$, use the initial conditions to find the constants A and ϕ. Be careful about the signs of $\sin\phi$ and $\cos\phi$ in determining ϕ. How much total time t_s does the mass remain on the spring? Hint : Be careful to determine which quadrant both ϕ and $\omega t_s + \phi$ lie in (draw a picture). Remember that ωt_s is positive. It is perhaps easier to find v at t_s using energy conservation, and then determine the angle $\omega t_s + \phi$ through this knowledge and that of the position of the mass when it leaves the spring.

$$m\frac{d^2y}{dt^2} = -mg - kx$$

$$\omega^2 = k/m$$

$$\omega = \sqrt{k/m} = \sqrt{.25} = .5\,s^{-1}$$

$$y = y_e + A\cos(\omega t + \phi)$$

$$v = -\omega A \sin(\omega t + \phi)$$

$$y_e = -\frac{mg}{k} = -\frac{9.8}{.25} = -39.2\,m$$

$t=0$

$$V_0 = -\sqrt{2gh} = -\omega A \sin\phi$$

$$y_0 = 0 = y_e + A\cos\phi$$

$$A\cos\phi = -y_e = mg/k > 0$$

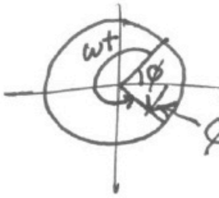
$$A\sin\phi = \sqrt{2gh}/\omega > 0$$

ϕ in first quadrant

$$A^2(\cos^2\phi + \sin^2\phi) = A^2 = (mg/k)^2 + 2gh/\omega^2$$

$$A = 39.299\,m$$

$$\phi = \cos^{-1}(-y_e/A) = 0.071 \quad (4.09^\circ)$$



$t = t_s$

$$V = -V_0 = +\sqrt{2gh} = -\omega A \sin(\phi + \omega t_s)$$

$$y = 0 = y_e + A\cos(\phi + \omega t_s)$$

$$\breve{\phi} = \phi + \omega t_s \qquad A\cos\breve{\phi} = -y_e = mg/k > 0$$

$$A\sin\breve{\phi} = -\sqrt{2gh}/\omega < 0$$

$\breve{\phi}$ in fourth quadrant

$$\breve{\phi} = -\phi$$

make angles positive:

$$\phi + \omega t_s + \phi = 2\pi$$

$$\omega t_s = 6.14$$

$$t_s = 12.28\,s$$