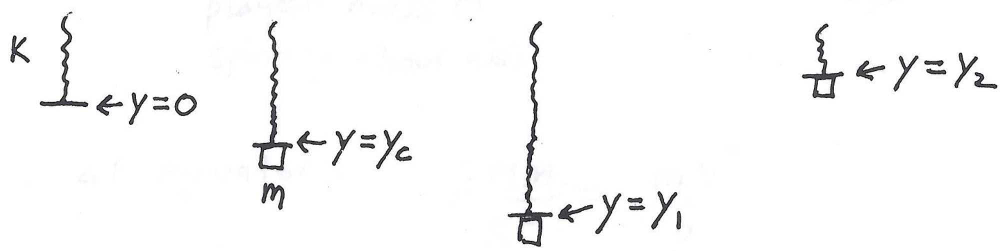
SMU Physics 1307 : Fall 2009

Exam 2

Problem 1 : The figure at left below shows a vertically hanging spring in its unstretched position, which may be taken to be $y = 0$. The second figure from the left shows a mass $m = 2\,\mathrm{kg}$ hanging from rest on the spring, which is now stretched to $y_c = -0.1\,\mathrm{m}$. Find k for the spring. As shown in the third figure from the left, the spring is now pulled down to a point $y_1 = -0.4\,\mathrm{m}$ and released. As illustrated in the figure at right, find the maximum height y_2 that the mass attains. You will have to use $g = 9.8\,\mathrm{m/s^2}$, and will need to make use of the quadratic equation.



$$-mg - Ky_c = 0$$

$$y_c = -mg/K$$

$$K = -mg/y_c = \underline{196\ \mathrm{N/m}}$$

$$E_1 = mgy_1 + \tfrac{1}{2}Ky_1^2$$

$$= E_2 = mgy_2 + \tfrac{1}{2}Ky_2^2$$

divide by K: $\quad \frac{mg}{K}y_1 + \tfrac{1}{2}y_1^2 = \frac{mg}{K}y_2 + \tfrac{1}{2}y_2^2$

since $y_c = -mg/K$: $\quad \tfrac{1}{2}y_2^2 - y_cy_2 + y_cy_1 - \tfrac{1}{2}y_1^2 = 0$

Quadratic:

$A = \tfrac{1}{2}$

$B = -y_c$

$C = y_cy_1 - \tfrac{1}{2}y_1^2$

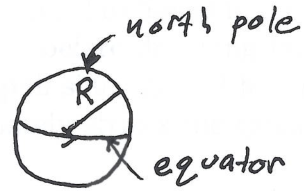
positive root:

$$y_2 = y_c + \left(y_c^2 + y_1^2 - 2y_1y_c\right)^{1/2}$$

$$= y_c + \left((y_c - y_1)^2\right)^{1/2} = 2y_c - y_1$$

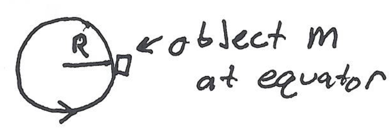
$$= \underline{0.2\,\mathrm{m}}$$

Problem 2 : A spherical planet has a mass $M = 1.0 \times 10^{23}\,\mathrm{kg}$. It spins around its axis at a rate such that objects at the equator feel no normal force. Thus objects which follow the surface of the planet at the equator are essentially in orbit. If the length of the planet's day, and thus the period of the orbit, is $T = 3000\,\mathrm{s}$, find the radius of R. What is the acceleration due to gravity g on the planet's surface? You will need to use $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$.



planet mass M
spinning about axis

top view:



orbit at equator: $\dfrac{GMm}{R^2} = \dfrac{mv^2}{R}$

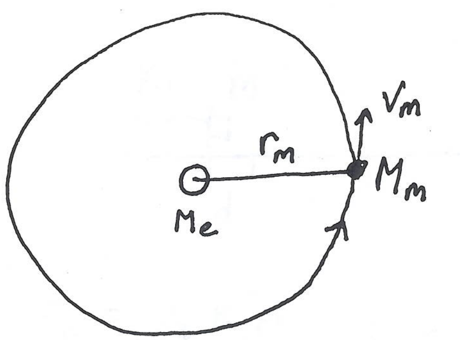
use $v = \dfrac{2\pi R}{T}$

and drop m $\qquad \dfrac{GM}{R^2} = \left(\dfrac{2\pi}{T}\right)^2 R$

$$R^3 = GM\left(\frac{T}{2\pi}\right)^2 \qquad R = \underline{1.15 \times 10^6\ \mathrm{m}}$$

$$g = \frac{GM}{R^2} = \underline{5.04\ \mathrm{m/s^2}}$$

Problem 3 : The moon ($M_m = 7.35 \times 10^{22}$ kg) executes a roughly circular orbit around the earth ($M_e = 5.97 \times 10^{24}$ kg) at a radius of $r_m = 3.85 \times 10^8$ m. Find the the total energy E, the kinetic energy K, the potential energy U, and the velocity v_m of this orbit, all under the assumption that M_e is much greater than M_m. Assume that as in the figure at right, the moon suddenly changes direction (for example through a collision) and begins moving directly away from the earth while maintaining the same total energy E. Thus the kinetic energy just before and just after the change in direction are equal. Find the maximum radius r_{max} that the moon achieves before dropping back toward the earth. Note that the moon will not escape the earths pull since $E < 0$ for the circular orbit. Also find the velocity v_c with which the moon ultimately strikes the earth ($R_e = 6.37 \times 10^6$ m).



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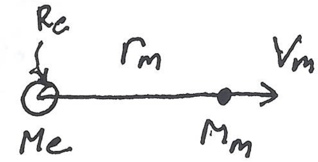


(Just after change in direction)

$$E = -\frac{GM_eM_m}{2r_m} = -3.8 \times 10^{28}\ \mathrm{J}$$

$$U = -\frac{GM_eM_m}{r_m} = -7.6 \times 10^{28}\ \mathrm{J}$$

$$\tfrac{1}{2}M_mV_m^2 = K = +\frac{GM_eM_m}{2r_m} = +3.8 \times 10^{28}\ \mathrm{J}$$

$$V_m = (2K/M_m)^{1/2} = 1017\ \mathrm{m/s}$$

Maximum radius : $K' = 0$

$$E = U' = -\frac{GM_eM_m}{r_{max}}$$

so,
$$-\frac{GM_eM_m}{2r_m} = -\frac{GM_eM_m}{r_{max}}$$

$$\underline{r_{max} = 2r_m = 7.7 \times 10^8\ \mathrm{m}}$$

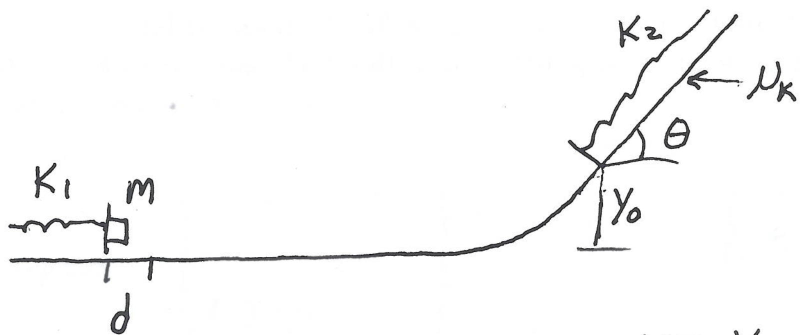
Hits earth :

$$E = \tfrac{1}{2}M_mV_c^2 - \frac{GM_eM_m}{R_e} = -\frac{GM_eM_m}{2r_m}$$

$$\tfrac{1}{2}V_c^2 = GM_e\left(\frac{1}{R_e} - \frac{1}{2r_m}\right)$$

$$\underline{V_c = 1.114 \times 10^4\ \mathrm{m/s}}$$

Problem 4 : The figure below shows a mass $m = 3\,\mathrm{kg}$ which initially rests against a spring with $k_1 = 200\,\mathrm{N/m}$ which is compressed by $d = 0.2\,\mathrm{m}$. The horizontal surface is entirely frictionless, but the hill at right (with angle $\theta = 30°$) has a frictional section with $\mu_k = 0.3$ that begins at a height $y_0 = 0.1\,\mathrm{m}$. The beginning of the frictional section coincides with the uncompressed position of a spring with $k_2 = 50\,\mathrm{N/m}$. Find the height y_m at which the mass comes to a stop. You will need to solve a quadratic equation.



$y = y_0 + x \sin\theta$

x: distance spring compressed

$K_0 = 0 \qquad E_0 = \frac{1}{2} K_1 d^2$

$K_1 = 0 \qquad E_1 = \frac{1}{2} K_2 x^2 + mgy$

$N = mg\cos\theta$

$E_1 - E_0 = -\mu_K N x = -\mu_K mg\cos\theta\, x$

so, $\frac{1}{2} K_2 x^2 + mg(y_0 + x\sin\theta) - \frac{1}{2} K_1 d^2 = -\mu_K mg\cos\theta\, x$

Quadratic: $A = \frac{1}{2} K_2 \qquad B = mg(\sin\theta + \mu_K\cos\theta) \qquad C = mgy_0 - \frac{1}{2}K_1 d^2$

positive root $\quad x = \left(-B + (B^2 - 4AC)^{1/2}\right)/2A$

$x = 0.045\,\mathrm{m}$

$y = y_0 + x\sin\theta = \underline{\underline{0.123\,\mathrm{m}}}$