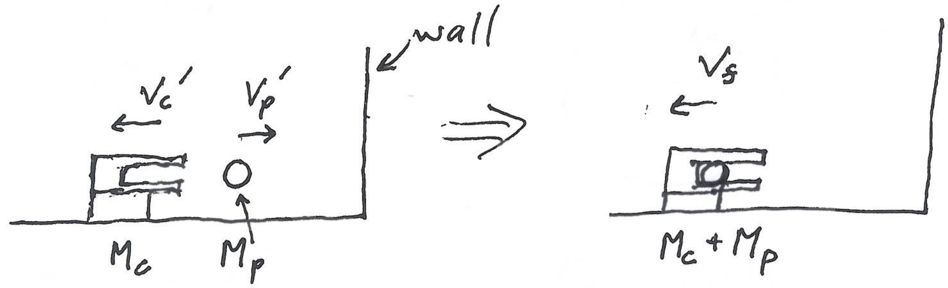
SMU Physics 1307 : Fall 2009

Exam 3

Problem 1 : The figure below shows a cannon of mass $M_c = 250\,\text{kg}$ which fires a projectile of mass $M_p = 30\,\text{kg}$ against an immovable (infinitely massive) wall at right. The explosion which propels the projectile adds $\Delta K = K' - K = 10^5$ J of kinetic energy to the system. The projectile then rebounds elastically and lodges back in the cannon. Assume that the surface on which the cannon rests is frictionless and neglect gravity entirely. Find the velocities v_c' and v_p' immediately after the cannon fires. Find the final velocity v_f of the combined system after the projectile lodges back in the cannon.



firing of Cannon: $K = 0 \quad K' = \frac{1}{2} M_c V_c'^2 + \frac{1}{2} M_p V_p'^2 = \Delta K$

$\Delta p = 0 \quad M_p V_p' + M_c V_c' = 0 \qquad \frac{1}{2} M_p V_p'^2 (1 + M_p/M_c) = \Delta K$

$$V_p' = \left(\frac{2\Delta K}{M_p}\right)^{1/2} \left(1 + M_p/M_c\right)^{-1/2} = \underline{77.2\ m/s}$$

$$V_c' = -M_p V_p'/M_c = \underline{-9.26\ m/s}$$

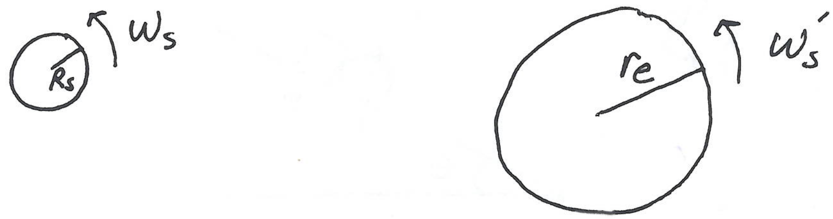
after rebound from wall: $V_p'' = -V_p'$

$\Delta p = 0 \qquad M_c V_c' + M_p V_p'' = (M_c + M_p) V_f$

$$V_f = \frac{M_c V_c' - M_p V_p'}{(M_c + M_p)} = \underline{-16.5\ m/s}$$

Problem 2 : The sun has a mass $M_s = 1.99 \times 10^{30}\,\mathrm{kg}$, and radius $R_s = 6.96 \times 10^{8}\,\mathrm{m}$. Assuming that it is a uniform sphere, its moment of inertia is $I_s = \frac{2}{5} M_s R_s^2$. The rotational period of the sun is about 25 days, for an angular velocity of $\omega_s = 2.9 \times 10^{-6}\,\mathrm{rad/s}$. Billions of years from now the sun will begin to increase in size as it burns the last of its hydrogen fuel and enters the last phases of its active life. Assume that no external torques act on the sun, and that it swells to a radius equal to that of the earth's orbit $r_e = 1.5 \times 10^{11}\,\mathrm{m}$ while remaining a uniform sphere. Find its new angular velocity ω_s', and find the change in kinetic energy $\Delta K = K' - K$ of the sun in this process.

sun now: sun later:



$$\Delta L = 0$$

$$I_s \omega_s = I_s' \omega_s' \qquad I_s = \tfrac{2}{5} M_s R_s^2$$

$$I_s' = \tfrac{2}{5} M_s r_e^2$$

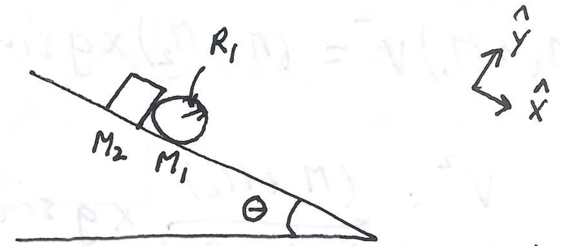
$$\omega_s' = \omega_s \frac{R_s^2}{r_e^2} = \underline{6.24 \times 10^{-11}\,\mathrm{rad/s}}$$

$$K = \tfrac{1}{2} I_s \omega_s^2 \qquad K' = \tfrac{1}{2} I_s' \omega_s'^2$$

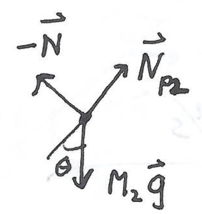
$$\Delta K = K' - K = \tfrac{1}{2} I_s \omega_s^2 \left(R_s^2/r_e^2 - 1 \right)$$

$$= \underline{-1.62 \times 10^{36}\,\mathrm{J}}$$

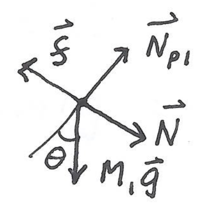
Problem 3 : The figure below shows a wheel of mass $M_1 = 1\,\mathrm{kg}$ and radius $R_1 = 0.02\,\mathrm{m}$ with $I_1 = M_1 R_1^2$ rolling down an inclined plane of angle $\theta = 30°$. Behind it is a block of mass $M_2 = 3\,\mathrm{kg}$ which has no frictional coupling with either the plane or the wheel, but exerts a normal force against the wheel. Find the acceleration $a = a_1 = a_2$ of the system, with positive a taken to be down the plane, as well as the angular acceleration α_1 of the wheel. Also find the normal force N exerted by the block on the wheel, as well as the force of friction f which the plane exerts on the wheel. Finding f does not required knowledge of the normal force from the plane but follows from the translational (down the plane) and rotational (with torques taken about the center of mass of the wheel) forms of Newton's second law. Also, find the velocity v_2 of the block when the system has moved a distance $x = 10\,\mathrm{m}$ down the plane.



block :



wheel :
(forces)



$\hat{x}$: $-N + M_2 g \sin\theta = M_2 a$

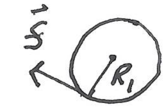
(ignore $\hat{y}$)

$\hat{x}$: $N - f + M_1 g \sin\theta = M_1 a$

(ignore $\hat{y}$)

combine: $(M_1 + M_2) g \sin\theta - f = (M_1 + M_2) a$

only $\vec{f}$ exerts torque about C.O.M.



$-f R_1 = I_1 \alpha_1 = M_1 R_1^2 \alpha_1$ $\quad a_1 = a = -R_1 \alpha$

$\underline{f = M_1 a}$ $\quad (M_1 + M_2) g \sin\theta = (2M_1 + M_2) a$

$a = \dfrac{g \sin\theta (M_1 + M_2)}{(2M_1 + M_2)} = \underline{3.92\ \mathrm{m/s^2}}$ $\quad f = M_1 a = \underline{3.92\,N}$

$N = M_2 g \sin\theta - M_2 a = \underline{2.94\,N}$

over for V_2 :

$$K + U = K' + U'$$

$$K' = U - U' = -(M_1 + M_2)\Delta y g$$

$$\Delta y = -x \sin\theta$$

$K = 0$ $\quad$ $\underline{V_1 = V_2 = V}$

$$K' = \tfrac{1}{2} M_2 V^2 + \tfrac{1}{2} M_1 V^2 + \tfrac{1}{2} I_1 \omega_1^2$$

$$V_1 = -R_1 \omega_1 = V$$

$$\tfrac{1}{2} I_1 \omega_1^2 = \tfrac{1}{2} M_1 R_1^2 \omega_1^2 = \tfrac{1}{2} M_1 V^2$$

$$K' = (\tfrac{1}{2} M_2 + M_1) V^2$$

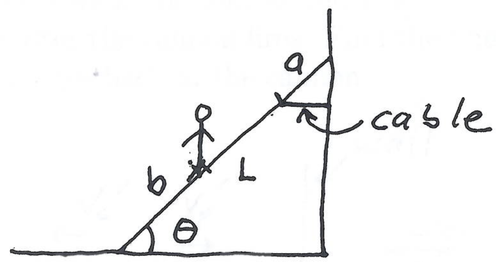
$$(\tfrac{1}{2} M_2 + M_1) V^2 = (M_1 + M_2) x g \sin\theta$$

$$V^2 = \frac{(M_1 + M_2)}{(\tfrac{1}{2} M_2 + M_1)} x g \sin\theta = 78.4\ m^2/s^2$$

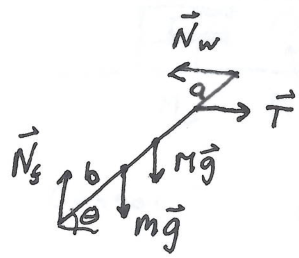
$$\underline{V = 8.85\ m/s}$$

Problem 4 :

The figure below shows a uniform beam of length $L = 5\,\text{m}$ and mass $M = 90\,\text{kg}$ which leans against a vertical wall and makes an angle of $\theta = 60°$ with the floor. Both the wall and the floor are frictionless, and thus exert only normal forces on the beam. The beam is attached to the wall by a horizontal cable at a point $a = 1\,\text{m}$ from the top of the beam. A person of mass $m = 70\,\text{kg}$ stands on the beam at point $b = 2\,\text{m}$ from the bottom of the beam. Find the tension T in the cable, and the normal forces N_w and N_f from the wall and floor, respectively.



L : entire length of beam
M : mass of beam
m : mass of person



$\hat{x}: \quad T - N_w = 0$

$\hat{y}: \quad N_f - (M+m)g = 0$

$\vec{\tau}$ about bottom:

$$-mg\,b\cos\theta - Mg\,\tfrac{L}{2}\cos\theta - T(L-a)\sin\theta + N_w L\sin\theta = 0$$

use $N_w = T$:

$$T a \sin\theta = \left(mgb + Mg\tfrac{L}{2}\right)\cos\theta$$

$$T = \frac{\left(mgb + Mg\tfrac{L}{2}\right)}{a\tan\theta} = \underline{2065\,N}$$

$$N_w = T = \underline{2065\,N}$$

$$N_f = (M+m)g = \underline{1568\,N}$$